# BeSpoken Protocol for Data Dissemination in Wireless Sensor Networks

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***Abstract*— For wireless sensor networks with many location-unaware nodes, we investigate a protocol, dubbed BeSpoken, that steers data transmissions along a straight path called a spoke. The protocol directs data transmissions by randomly selecting relays to retransmit data packets from crescent-shaped areas along the spoke axis. The resulting random walk of the spoke hop sequence may be modeled as a two dimensional Markov process. We propose design rules for protocol parameters that minimize energy consumption while ensuring that spokes propagate far enough and have a limited wobble with respect to the spoke axis.**

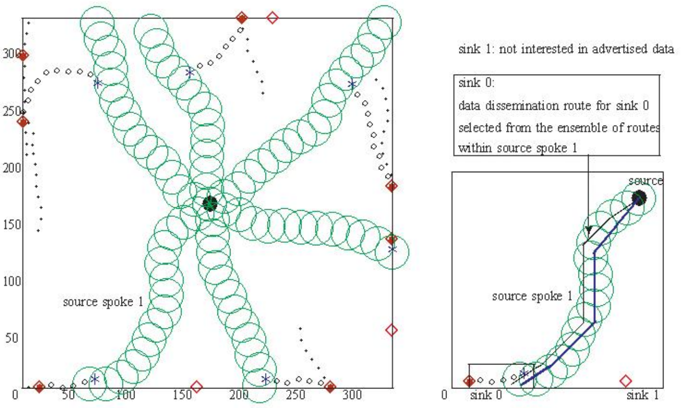


Fig. 1. The meaning of the name BeSpoken is twofold: the radial lines extending from the source form a pattern that resembles spokes of a wheel and, furthermore, spoke relays bespeak the source message. In this simulation snapshot, source spokes are shown as a sequence of relay transmission *ranges*, to illustrate the fact that each spoke is an ensemble of possible data routes (see the isolated spoke). The sequence of wireless transmission *relays* forming a productive sink spoke is denoted by tiny circles(see the boxed spoke). Unproductive sink spokes are represented by dots. The search success is marked by a $*$.

## I. Introduction

In wireless sensor networks, nodes that make observations, known as data sources, are frequently unaware of which data sinks have interest in their observations. In random sensor networks, composed of position unaware nodes, a data source is unable to send complete information to the sinks, in terms of both the nature of an event and its location. This problem is typical of sensor networks, due to their assumed simplicity and scarce energy resources. In particular, GPS, the most widely used positioning system, is a significant power consumer, and infeasible in environments with dense foliage or other clear-sky impediments. A wireless dissemination protocol uses a sequence of wireless transmissions that propagate the information from the source to the sinks. Motivated by the radial symmetry of isotropic wireless transmission, we propose a dissemination protocol, dubbed BeSpoken, that conveys information about an event to the sinks in an energy efficient manner, and whose spatial properties can simplify the source-location estimation. This protocol generates relatively straight-line trajectories called spokes, without requiring the nodes to have navigational information such as a GPS. We assume that the physical layer modulation and coding are designed to compensate for short-scale fading effects and, thus, our transmit power requirements depend only on distance-dependent propagation path loss. Even though in a sensor network environment data rates are low relative to the available bandwidth and interference is not a primary issue, still, our protocol mitigates the interference as it always selects only one node to retransmit. We envision simple sources while sinks are likely to be more capable in terms of *direction-of-arrival (DoA)* estimation, and in being location-aware and application-cognizant. As illustrated in Figure 1, a source disseminates data advertisements along the source spokes, and a sink sends a query along its spokes that may intersect the source spokes. Each intersection represents a successful search. The first sink spoke to reach one of the source spokes is called *productive.* Successful search is to be followed by the reinforcement of a route along the intersecting spokes and subsequent data dissemination. A GPS and DoA-enabled sink can determine the positions of the nodes along the productive sink spoke, and let them know of their positions. The other part of the enforced route can be learned based on the known position of the intersection nodes, provided that the direction of the source spoke is known. This can be achieved in a variety of ways, such as joint DoA estimation performed by the nodes surrounding the intersection, or by a mobile DoA-enabled sink which can move to the intersection and determine source spoke direction by polling the spoke nodes.

We consider BeSpoken as both a dissemination protocol, and a tool to build an infrastructure of relatively straight paths (spokes) whose direction and length can be learned with moderate effort. Both source and sink spokes, utilized to convey the information of an event from the source to the sink, may remain active for a relatively long time period. The spokes tessellate the sensor network space, providing a way to map subsequent events to areas between the known paths, and to aid efficient navigation toward the associated sources. It is not hard to envision how this framework may bootstrap a number of mechanisms for energy-efficient dissemination, load balancing and rapid data propagation in desired directions. Using the analogy of a network of roads and intersections that allows for navigation to randomly distributed places of interest, we observe that, as the number of events and interested sinks grows, the number of intersecting roads increases, providing alternative ways to direct the traffic and to use the resources in a more balanced way. Of course, protocols (outside the scope of this paper) are needed to manage the roads and redirect the traffic. This paper focuses on the mathematical model of the BeSpoken and its analysis for the purpose of spoke design.

## II. Related Research

Flooding is the simplest dissemination strategy, nevertheless it typically yields excessive communication [1] and can lead to a "broadcast storm" of redundant transmissions [8], unless a mitigating technique is employed [13]. The idea of data source localization enabled by the dissemination protocol properties is absent from flooding-based dissemination techniques. Several papers consider spatial properties of the dissemination route. Different forms of spatially constrained random walk are discussed in [2], [4], [11], while the idea of a trajectory-based dissemination is presented in [6], [9]. None of these dissemination approaches enable unknown source localization.

Finally, an important body of work, including [3], [5], [7], [12], studies forward-progress routing, however, only for location-aware wireless networks.

## III. System Model and Problem Statement

We consider a dense wireless network with a uniform spatial distribution of nodes. The *BeSpoken* protocol organizes a sequence of fixed-power *relay* transmissions that propagate the source message hop-by-hop, without positional or directional information. The hop relays form a *spoke* which may deviate from the radial *spoke axis*. Each spoke hop is organized using a sequence of two control message transmissions followed by the hop data transmission. We define the transmission range as the maximum distance from the source at which nodes can reliably receive a packet. Assuming radially symmetric attenuation (isotropic propagation), the area in which the transmitted packet is reliably received is a disk of a given radius. We use the same transmission power for both data and control packets, but different coding rate and/or modulation format, so that the communication rate for control messages is lower and translates to a longer range.

In this work, we examine networks in which the sinks are distributed uniformly along the perimeter of the network area, forming the *sink population*. We define the *event popularity* as the fraction of the sink population interested in the event relative to the total number of nodes in the network. Figure 1 illustrates a network where the event popularity is close to $10^{-4}$. Since the exact positions of interested sinks are not known, we conjecture that the likelihood of successful data search would increase if both the source and the sinks spawn several equally spaced radial spokes. We suggest optimizing the number of radial spokes depending on the popularity of the event, so that the energy consumed in the search is minimized. This optimization and the energy accounting are outside the scope of this document. Note that different sinks form their spokes at different times, so that the intersection with a previously created productive sink spoke represents a successful search (see two leftmost sinks on Figure 1). By reinforcing the intersecting spokes for data transmission, sink gets access to a collection of nodes that learn of the advertised event by virtue of being within the transmission range of a spoke relay, as illustrated in Figure 1. A sink can form a data dissemination path by selecting a sequence of nodes belonging to this collection, while respecting an inherent order defined by the spoke. Hence, every spoke is an ensemble of routes that are all available to the sink for optimizing network utilization and meeting application-driven requirements.

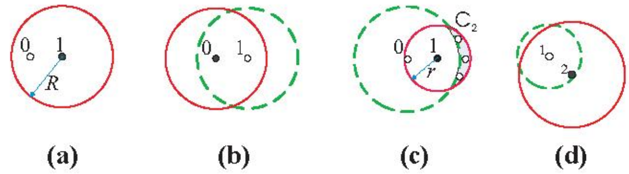


Fig. 2. BeSpoken Protocol: At each protocol stage, the current transmission range is denoted with the full circle while the previous range is denoted with a dashed circle.

### A. BeSpoken Protocol

The BeSpoken protocol implements a recursive process illustrated in Figure 2 in the following way:

- **(a)** The leading relay (node 1) sends an RTS (request to send) control packet with range $R = rq$ where $q = 2 - \epsilon$, for small $\epsilon$.
- **(b)** The pivot (node 0) sends a BTS (block to send) control packet with range $R$.
- **(c)** The leading relay transmits the data packet with range $r$ and becomes the new pivot. The region in which nodes receive this data packet but do not receive the preceding BTS packet forms the *1-st hop crescent* $C_2$.
- **(d)** A random node from the crescent $C_2$ becomes the new leading relay by transmitting a new RTS. The process returns to (a) with node 1 as the pivot and node 2 as the leading relay.

This recursive process is initialized by assigning the role of the pivot to the source node which transmits the data packet with a range $r$. The first node which receives the data packet and gets access to the medium becomes the first leading relay. The underlying ALOHA-type Carrier Sense Multiple Access MAC protocol would resolve any collisions, which would, with some delay incurred, eventually result in one random node transmitting the RTS packet. The vector from node 0 to node 1 defines the spoke axis. The crescent subtending angle determines how much the spoke may deviate from the spoke axis direction. The parameter $q = R/r$ determines the maximum crescent subtending angle. We would like to choose parameters $r$ and $q$ so that the spoke extends to distance $d$ with probability $p$, and that the expected number of hops before the spoke deviates too much from the spoke axis direction is sufficient to reach the distance $d$. Note that the energy per hop grows as $r^{\alpha}$, where $\alpha \geq 2$ is the propagation loss coefficient, so that the total energy per spoke grows as $dr^{\alpha-1}$. Thus, minimizing the transmission range $r$ corresponds to a minimum energy objective.

### B. Markov Process Model and Problem Formulation

To describe the effects of the data and control ranges $r$ and $R$, we evaluate the spoke behavior with respect to the constraints:

- **Outage:** the probability that a spoke dies before reaching a distance $d$ is small,

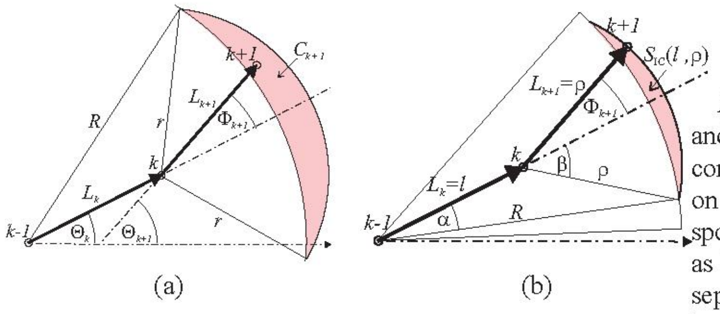


Fig. 3. **(a)** At hop $k+1$, node $k+1$ is distance $L_{k+1}$ from node $k$ and the current spoke direction is $\Theta_{k+1} = \Theta_k + \Phi_{k+1}$. **(b)** Given $L_k = l$ and $L_{k+1} = \rho$, the angular hop displacement $\Phi_{k+1}$ is constrained to the interval $-\beta \leq \Phi_{k+1} \leq \beta$ where the maximum angular displacement at hop $k+1$ is $\beta = \beta(l, \rho)$. The shaded area denotes the interior crescent of area $S_{\mathrm{IC}}(l, \rho)$.

- **Wobbliness:** the deviation of the instantaneous spoke direction with respect to the spoke axis is within defined limits.

Figure 3(a) depicts hops $k$ and $k+1$. At the completion of hop $k$, the length $L_k$ denotes the *current hop length* and the angle $\Theta_k$ denotes the *current spoke direction*. At hop $k+1$, the *angular hop displacement* $\Phi_{k+1}$ changes the current spoke direction in that

$$\Theta_{k+1} = \Theta_k + \Phi_{k+1} = \sum_{i=1}^{k+1} \Phi_i \tag{1}$$

The spoke direction process $\{\Theta_k\}$ depends on the hop length process $L_k$, which we characterize next.

From Figure 3(b) we observe that given $L_k = l$ and $L_{k+1} = \rho$ the control circle of radius $R$ centered at node $k-1$ and the circle of radius $\rho$ centered at node $k$ specify a radius $\rho$ arc for the possible positions of node $k+1$. The endpoints of this radius $\rho$ arc constrain the angular hop displacement $\Phi_{k+1}$ to the interval $-\beta \leq \Phi_{k+1} \leq \beta$ where the maximum angular displacement is $\beta = \beta(l, \rho)$. Applying the law of cosines to the complementary angle $\pi - \beta(l, \rho)$ yields

$$\cos\beta(l, \rho) = \frac{R^2 - \rho^2 - l^2}{2l\rho}. \tag{2}$$

We also observe that the region between the radius $R$ control circle and the radius $\rho$ arc defines an *interior crescent*, shown as the shaded area in Figure 3(b). From geometric arguments, it can be verified that the area of this interior crescent is

$$S_{\mathrm{IC}}(l, \rho) = 2\rho^2\beta(l, \rho) - 2R^2\alpha(l, \rho) + Rl\sin\alpha(l, \rho) \tag{3}$$

where $\alpha(l, \rho)$ is found from the law of cosines to satisfy $\cos\alpha(l, \rho) = (R^2 - \rho^2 + l^2)/(2lR)$.

$L_{k+1}$ can vary from a minimum value of $R - L_k$ to a maximum value of $r$. The induced interior crescent $C_{k+1}$ in Figure 3(a) has an area $S_c(L_k) = S_{\mathrm{IC}}(L_k, r)$. We note that $C_{k+1}$, termed the *current crescent*, is the set of all possible positions of the node $k+1$.

For design purposes we assume that the spatial distribution of network nodes is a planar Poisson point process of intensity $\lambda = 1$. Thus, a current crescent yields a candidate set for node $k+1$ with cardinality $Z_k$ that is, conditionally, a Poisson random variable with conditional expected value

$$E[Z_k | L_k = l_k] = S_c(l_k). \tag{4}$$

A spoke stops at stage $k$ when the crescent $C_k$ is empty and thus spoke generation is a transient process. The outage constraint depends only on the crescent sizes $S_c(L_k)$ but not on the hop direction process $\Theta_k$. On the other hand, the spoke wobbliness depends on the $\Theta_k$ but is meaningful only as long as each current crescent $C_k$ is non-empty. Thus, we separate the analysis of the outage and wobbliness constraints by formally defining $\{L_k\}$ as a fictitious process that never encounters an empty crescent.

Under the fictitious process model, the position of node $k+1$ will be uniformly distributed over the crescent $C_{k+1}$. From Figure 3 we see that, given the current hop length $L_k = l_k$, the arc of radius $\rho$ has length $2\rho\beta(l_k, \rho)$. The conditional probability that we find node $k+1$ in the annular segment of width $d\rho$ along the arc of radius $\rho$ is $2\rho\beta(l_k, \rho)d\rho/S_c(l_k)$. It follows that the conditional pdf of the next hop length $L_{k+1}$ given $L_k = l_k$ is

$$f_{L_{k+1}|L_k}(\rho | l_k) = \frac{2\rho\beta(l_k, \rho)}{S_c(l_k)} \qquad R - l_k \leq \rho \leq r, \tag{5}$$

and zero otherwise. We note that (5) provides a complete characterization of the fictitious process $\{L_k\}$.

We observe that all positions along the radius $\rho$ arc in Figure 3 are equiprobable locations for node $k = 1$. Thus, given the sequence $\{L_k\}$, the angular hop displacements $\{\Phi_k\}$ form a sequence of conditionally independent uniform random variables with the conditional pdf

$$f_{\Phi_{k+1}|L_k, L_{k+1}}(\phi | l_k, l_{k+1}) = \frac{1}{2\beta(l_k, l_{k+1})}, \tag{6}$$

for $|\phi| \leq \beta(l_k, l_{k+1})$, and zero otherwise. We note that equations (1), (5), and (6) describe a Markov process model for the evolution of a spoke. In particular, the current angle sequence $\{\Theta_k\}$ is a random walk process modulated by the Markov process $\{L_k\}$. To describe wobbliness, we define

$$T_{\varphi_o} = \min\{n : |\Theta_n| \geq \varphi_o\}. \tag{7}$$

to be the first time that the spoke goes off-course.

With respect to outage, a spoke stops at hop $k$ when the crescent $C_k$ is empty, i.e., $Z_k = 0$. Since the nodes obey a planar Poisson process, it follows from (4) that the conditional probability the crescent $C_k$ is empty is

$$\Pr\{Z_k = 0 | L_k = l_k\} = e^{-S_c(l_k)}. \tag{8}$$

We define

$$D = \min\{n : Z_n = 0\} \tag{9}$$

as the first time the process encounters an empty crescent.

An ideally straight spoke would require $\eta = \lceil d/r \rceil$ hops to reach the distance $d$. For analytical tractability, instead of requiring the spoke to travel distance $d$ with high probability, we require it to travel $\eta$ hops with high probability. The design goal is simply to minimize $r$ subject to the constraints

$$\Pr\{D \leq \eta\} \leq p, \tag{10}$$

$$E[T_{\varphi_o}] \geq \eta. \tag{11}$$

The outage constraint (10) dictates that the crescent area $S_c(L_k)$ be sufficiently large while the wobbliness constraint (11) requires that the subtending angles $\beta(L_k, r)$ of each crescent be sufficently small. These requirements can be met by a careful choice of the data range $r$, and the ratio $q = R/r$.

However, the analysis of (10) is challenging due to the complex way in which the hop length process $\{L_k\}$ evolves with time. In particular, a small $L_k$ will create a small crescent; this induces a support set $[R - L_k, r]$ for $L_{k+1}$ that excludes small hop lengths in the interval $[R - r, R - L_k)$. As illustrated in Figure 4, an imaginary coil is attached between a fixed pivot and a moving leading relay: when contracted, it pulls the leading relay's data circle inside the blocking control circle, exposing only a tiny area with possible relays. Note that the next hop length has to be long (close to $r$), if the relay is found in this tiny area. At the other extreme, when the coil is completely relaxed to length $r$, it exposes the largest possible area. This reduces the likelihood of an empty crescent yet it increases the likelihood of the next hop length being small. This oscillatory effect illustrates the importance of the Markov property for the hop length evolution model (5).

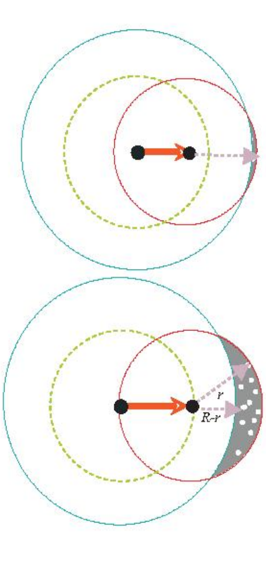

Fig. 4. Spring-coil analogy

## IV. Protocol Parameter Design

This section develops closed-form expressions for the protocol parameters that satisfy the outage and wobbliness constraints (10) and (11), based on a Markov Chain model that approximates the Markov process described above.

### A. Finite State Ergodic Markov Chain Model

We start by quantizing the $L_k$ process, yielding the $m$-state Markov chain $\hat{L}_k$. We first select a chain state set that quantizes the process state space $[R - r, r]$, then describe a mapping from the process state space to the chain state set and, last, describe the resulting chain probability transition matrix. We define $\{h_1, \ldots, h_m\} \subseteq [R - r, r]$ to be the chain state set. Without loss of generality, we assume that $h_0 = R - r < h_1 < h_2 < \ldots < h_m = r$. As illustrated in Figure 5, whenever the $k$th hop Markov chain state is $\hat{L}_k = h_i$, the corresponding next process hop length is $L_{k+1} \in \mathcal{I}_i = [R - h_i, r]$, where $\mathcal{I}_i$ is the *next hop span* and its length $|\mathcal{I}_i|$ is also the width of the corresponding quantized crescent $\hat{C}_k$ of area $c_i = S_c(h_i)$. $L_{k+1}$ is quantized to state $h_j$ whenever $\hat{L}_{k+1} \in \mathcal{I}_{ij}$ where

$$\mathcal{I}_{ij} = \mathcal{I}_i \cap (h_{j-1}, h_j]. \tag{12}$$

Note that the set $\{\mathcal{I}_{ij} : j = 1, \ldots, m\}$ partitions $I_i$ and serves as a set of quantization intervals for $L_{k+1}$ when $\hat{L}_k = h_i$. This quantization mapping is illustrated in Figure 5 where $L_{k+1} \in \mathcal{I}_{42}$ is extended to reach the quantized node position marked with a grey circle at $\hat{L}_{k+1} = h_2$. The chain proceeds by declaring a fictitious node at the quantized position as the new leading relay. As depicted in Figure 5, a quantization interval $\mathcal{I}_{ij}$ corresponds to the strip of area

$$d_{ij} = \begin{cases} \int_{R-h_i}^{h_j} 2\rho\beta(h_i, \rho)\, d\rho, & j = j^*(i), \\ \int_{h_j-\Delta}^{h_j} 2\rho\beta(h_i, \rho)\, d\rho, & j > j^*(i), \end{cases} \tag{13}$$

(and zero otherwise), and of width $|\mathcal{I}_{ij}|$ within the crescent $\hat{C}_k$ of area $c_i = \sum_j d_{ij}$. Here $j^*(i) = \min\{j : h_j > R - h_i\}$ is the index of the leftmost non-empty quantization interval within $I_i$.

As shown in Figure 5, $c_{ij} = S_{\mathrm{IC}}(h_i, h_j)$ is the *quantized interior crescent area* formed by the control circle (of radius $R$) centered at the $k$th hop relay and a circle of radius $h_j$ centered at node $k + 1$ at distance $\hat{L}_k = h_i$. Note that $c_{ij} < c_{i(j+1)} \cdots < c_{im}$, where $c_{ij} = 0$ for $j < j^*(i)$, $c_{im} = c_i$, and $d_{ij} = c_{ij} - c_{i(j-1)}$. The hop-length transition probabilities $P_{ij} = \Pr\left\{\hat{L}_{k+1} = h_j | \hat{L}_k = h_i\right\} =$

$$\Pr\{L_{k+1} \in \mathcal{I}_{ij} | L_k = h_i\} = \frac{d_{ij}}{c_i}. \tag{14}$$

follow from the uniformity of Poisson spatial distribution of nodes and since the fictitious process assumes that the crescent $\hat{C}_k$ is not empty. Intuitively, when $m$ is sufficiently large, the ergodic Markov chain will approximate well the ergodic Markov process. We consider Markov chain models with both uniform and non-uniform quantization of $[R - r, r]$. With only $m = 2$ levels, the uniform quantization lacks accuracy. However, a carefully chosen two-state chain provides a useful non-uniform quantization model. The transition matrix for both two-state systems is

$$\mathbf{P} = \begin{bmatrix} 0 & 1 \\ c_{21}/c_2 & (c_2 - c_{21})/c_2 \end{bmatrix}, \tag{15}$$

since $c_{12} = c_1$ and $c_{22} = c_2$.

*1) Uniform Quantization Model:* In this model, the hop-length states $\{h_i\}$ uniformly quantize the process state space $[R - r, r]$ so that $h_i = R - r + i\Delta$, where $\Delta = (2r - R)/m$ is the quantization interval. Furthermore, $j^*(i) = m - i$ so that the next-hop quantization intervals $\mathcal{I}_{ij}$ satisfy $\mathcal{I}_{ij} = (h_j - \Delta, h_j]$ for $j > m - i$ and are empty for $j \leq m - i$. The transition probabilities are now

$$P_{ij} = \frac{c_{ij} - c_{i(j-1)}}{c_i}, \quad i + j > m, \tag{16}$$

and $P_{ij} = 0$ whenever $i + j \leq m$ follows since, in that case, $(h_{j-1}, h_j]$ and $\mathcal{I}_i = [R - h_i, r]$ intersect in at most one point.

For example, the uniformly quantized $m = 2$ Markov chain has $\Delta = r - R/2$, $h_1 = R - r + \Delta = R/2$ and $h_2 = r$, and, accordingly, $c_1 = S_c(R/2)$ and $c_2 = S_c(r)$.

*2) Non-Uniform Quantization Model for the Outage Constraint:* The proposed non-uniform quantization, two-state Markov chain model has a simpler definition with $c_1 = 1$, and $c_2 = S_c(r)$. The corresponding set of hop length states

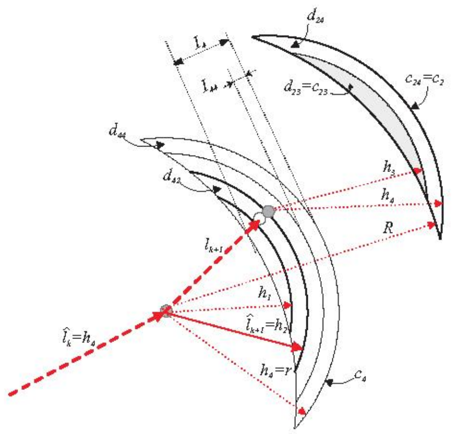

Fig. 5. Ergodic Finite State Markov Chain: quantization example for a four-state chain ($m = 4$): $\hat{L}_k = h_4 = r$ results in the first crescent $\hat{C}_k$ of area $c_4$ partitioned into four strips of total area $c_4 = d_{41} + d_{42} + d_{43} + d_{44}$; $L_{k+1} \in \mathcal{I}_{42}$, quantized to $\hat{L}_{k+1} = h_2$, is followed by a crescent $\hat{C}_{k+1}$ of area $c_2$ and a hop span $I_2 = [R - h_2, r]$ which is (uniformly) quantized into a crescent of area $d_{23} = c_{23}$ (shaded region) and a crescent strip $d_{24} = c_2 - c_{23}$ (the unshaded area).

includes $h_2 = r$ and $h_1$, which is a solution of $c_1 = 1 = S_c(h_1)$. Hence, the next hop partition mapping satisfies $d_{21} = c_{21}$, and $d_{22} = c_2 - c_{21}$. Let $R/2 > h_1 = S_c^{-1}(1) > R - r$, and, in this case, we have that $j^*(1) = 2$, $d_{11} = 0$, and $d_{12} = c_1 = 1$. The non-uniform partitioning differs from the uniform in that $c_2 \gg c_1$ and $c_{22} \gg c_{21}$ for large enough $r$. The rationale behind such a design follows in the next subsection.

### B. Model for the Outage Constraint

Here we evaluate the outage probability for given $q = R/r$, in order to evaluate the associated outage constraint (10). Recall that (10) is decoupled from the wobbliness constraint (11), that provides the value of $q$, a measure of the maximum crescent subtending angle. For the $m$-state Markov chain, let us denote the event that the first $\eta$ crescents $\hat{C}_k$, $k = 1, \cdots, \eta$, are not empty as $A_\eta = \{\min_{k \leq \eta} Z_k > 0\}$. The probability that the crescents $\hat{C}_1, \ldots, \hat{C}_\eta$ are not empty, and that the system is in state $j$ at time $\eta$ is denoted $\kappa_j^{(\eta)} = \Pr\left\{\hat{L}_\eta = h_j, A_\eta\right\}$. Using Markovity of $\hat{L}_k$ and conditional independence of $Z_k$ given $\hat{L}_k$, it is straightforward to show that

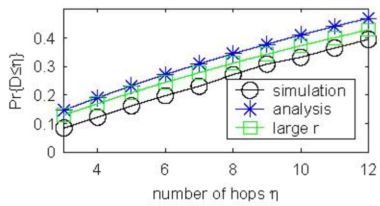


Fig. 6. Outage probability curves

$$\kappa_j^{(\eta)} = \sum_{i=1}^{m} e_j P_{ij} \kappa_i^{(\eta-1)} \tag{17}$$

where $e_j = 1 - \exp(-\lambda c_j)$ is the probability of a non-empty crescent while in state $j$. Let us define the $m \times m$ matrix $\breve{\mathbf{P}}$ where $\breve{P}_{ij} = P_{ij} e_j$ is the conditional probability to transition from state $i$ to state $j$, and that the resulting crescent of area $c_j$ is not empty. Note that (15) implies $P_{11} = \breve{P}_{11} = 0$. In addition, by defining the vector $\kappa^{(\eta)} = [\kappa_1^{(\eta)}, \cdots, \kappa_m^{(\eta)}]$, (17) becomes $\kappa^{(\eta)} = \kappa^{(\eta-1)} \breve{\mathbf{P}}$. Recursively, we obtain $\kappa^{(\eta)} = \kappa^{(1)} \breve{\mathbf{P}}^{\eta-1}$. Given the initial state $m$, we see that $\kappa_i^{(1)} = 0$ for $i < m$ and $\kappa_m^{(1)} = e_m$. Thus, $\kappa^{(\eta)} = [0 \ \cdots \ e_m] \breve{\mathbf{P}}^{\eta-1}$, As $\Pr\{A_\eta\} = \sum_{i=1,\cdots,m} \kappa_i^{(\eta)} = \kappa^{(\eta)} [1 \ \cdots \ 1]^T$, the probability that the spoke will stop at or before hop $\eta$ (assuming that the chain always starts in state $h_m$) becomes $\Pr\{D \leq \eta\} =$

$$1 - \Pr\{A_\eta\} = 1 - [0 \cdots e_m] \breve{\mathbf{P}}^{(\eta-1)} [1 \cdots 1]^T . \tag{18}$$

The following asymptotic (large $r$) analysis of the outage probability (18) is based on the two state non-uniform quantization model (15). Let $\lambda_1 > \lambda_2$ be the two eigenvalues of $\breve{\mathbf{P}}$ in (18) based on (15). The eigenvalue $\lambda_1$ describes the rate at which the outage probability increases with the number of hops, while the negative eigenvalue $\lambda_2$ describes the oscillatory, self-recovery mechanism depicted in Figure 4. Let $r \gg 1$ and $q$ be close to two. Then $c_{22} = c_2 \gg c_1 = 1$ in (15). Now, $\lambda_1$ is close to one, while $\lambda_2$ one is close to zero. Furthermore, by combining (18) and (10), we show that, for a spoke to reach $\eta$ hops with probability $p$, given $q$, the range is required to be

$$r \geq 1 / \sqrt{\exp(1) \left(1 - (1-p)^{\frac{1}{\eta-1}}\right) f(q)}, \tag{19}$$

where $f(q) = S_c(r)/r^2$. Note that (3) implies that $f(q)$ is the crescent area for unit $r$, i.e. the ratio $S_c(r)/r^2$ does not depend on $r$. Figure 6 illustrates that (19) matches well the simulation results for large $r$.

### C. MMRW Model for Wobble Constraint

Here we evaluate the expected number of hops a spoke will make before it goes off course in order to evaluate the associated wobbliness constraint (11). The spoke goes off-course at hop $T_{\varphi_o}$ whenever the current angle $\Theta_k$ in (1) exceeds one of the following two thresholds $\phi_o$ and $-\phi_o$, as described by (7). Here we consider $\Theta_k$ as a random walk modulated by the ergodic Markov chain $\hat{L}_k$, and $T_{\varphi_o}$ is the corresponding stopping time. To evaluate (7), the transform domain analysis of Markov modulated random walks [10] dictates that we first define the conditional moment generating functions of the incremental angular displacement $\Phi_{k+1}$ from (1)

$$g_{ij}(\omega) = E\left[\exp\left(\Phi_{k+1}\omega\right) | \hat{L}_k = h_i, \hat{L}_{k+1} = h_j\right] \tag{20}$$

$$= \frac{1}{2\varphi_{ij}} \int_{-\varphi_{i,j}}^{\varphi_{i,j}} \exp(\phi\omega) \, d\phi = \hbar(\varphi_{ij}\omega), \tag{21}$$

for $\omega$ in a convergence region $(\omega_-, \omega_+)$, where $\hbar(x) = \frac{\sinh x}{x}$ and $\varphi_{ij} = \beta(h_i, h_j)$.

Let $\Gamma_{ij}(\omega) = P_{ij} g_{ij}(\omega)$ denote the elements of the matrix $\mathbf{\Gamma}(\omega)$. The Perron-Frobenius theorem (see e.g., [10]) dictates that its largest eigenvalue $\sigma(\omega)$ is real and positive. The elements of the corresponding right eigenvector $\nu(\omega) = [\nu_1(\omega) \cdots \nu_m(\omega)]^T$ are also real and positive.

Next, we define the product martingale [10]

$$M_k(\omega) = \frac{\exp(\omega \, \Theta_k) \, \nu_{i(k)}(\omega)}{\sigma^k(\omega) \, \nu_{i(0)}(\omega)} \tag{22}$$

where $i(k)$ is the random state index of the chain at time $k$, and the random variable $\nu_{i(k)}(\omega)$ is the $i(k)$-th element of the right eigenvector. Following [10, Chapter 7.7], $T_{\varphi_o}$ is also a stopping rule for the martingale $M_k(\omega)$ relative to the joint process $\{M_k(\omega), L_k;\}$. Hence, following [10, Lemma 6] and the *optional sampling theorem* [10, Theorem 6] we have

$$E\left[M_{T_{\varphi_o}}(\omega)\right] = E\left[\frac{\exp(\omega\Theta_{T_{\varphi_o}})\nu_{i(T_{\varphi_o})}(\omega)}{\sigma(\omega)^{T_{\varphi_o}}\nu_{i(0)}(\omega)}\right] = 1, \quad (23)$$

for $\omega \in (\omega_{-}, \omega_{+})$. The form (23) is an extension of the *Wald identity* to Markov modulated random walks.

The random variable $\Theta_{T_{\varphi_o}}$ is either $-\varphi_o$ or $\varphi_o$, assuming that there is no overshoot. We address the problem of overshoot later. By symmetry arguments, the first and second moments of $\Theta_{T_{\varphi_o}}$ are

$$E\left[\Theta_{T_{\varphi_o}}\right] = 0 \quad (24)$$

$$\mathrm{var}\left[\Theta_{T_{\varphi_o}}\right] = E\left[{\Theta_{T_{\varphi_o}}}^2\right] = \varphi_o^2. \quad (25)$$

We evaluate the second derivative of (23) with respect to $\omega$ at $\omega = 0$ to obtain the expected number of hops until the hop angle hits the threshold as

$$E\left[T_{\varphi_o}\right] = \frac{\mathrm{var}\left[\Theta_{T_{\varphi_o}}\right] + E\left[\mu_{i(T_{\varphi_o})}(\omega)\right]\Big|_{\omega=0} - \mu_{i(0)}(\omega)\Big|_{\omega=0}}{\frac{\sigma''(\omega)}{\sigma(\omega)}\Big|_{\omega=0}}, \quad (26)$$

where $\mu_i(\omega) = \nu_i''(\omega)/\nu_i(\omega)$. One can show that, for $m = 2$, the denominator of (26) is $\frac{\sigma''(\omega)}{\sigma(\omega)}\Big|_{\omega=0} = \frac{1}{3}\left(\pi_2 P_{22}\varphi_{22}^2 + \pi_1 P_{11}\varphi_{11}^2 + \pi_1 P_{12}\varphi_{12}^2 + \pi_2 P_{21}\varphi_{21}^2\right)$, where $\pi_i$, $i = 1,2$ are the elements of the row vector of stationary state probabilities $\overline{\pi} = [\pi_i]_{(1\times m)}$. Its direct generalization to an $m$ state model has a form $\mathrm{var}[\theta]_p = \overline{\pi}P^{(v)}u^T$, where, $P^{(v)} = \left[P_{ij}^{(v)}\right]_{(m\times m)}$ with elements $P_{ij}^{(v)} = \left(P_{ij}\varphi_{ij}^2\right)/3$, and a unit vector $u = [1...1]_{(1\times m)}$. It can be shown that for small crescent subtending angles relative to the threshold $\varphi_o$, we can ignore the terms $E\left[\mu_{i(T_{\varphi_o})}\right]$ and $\mu_{i(0)}$ in (26) resulting in:

$$E\left[T_{\varphi_o}\right] = \frac{\mathrm{var}\left[\Theta_{T_{\varphi_o}}\right]}{\mathrm{var}[\theta]_p}. \quad (27)$$

By assuming identical distributions of the random walk overshoot and undershoot we extend the numerator of (27) to include overshoot terms as

$$\mathrm{var}\left[\Theta_{T_{\varphi_o}}\right] = \varphi_o^2 + 2\varphi\left(\overline{\pi}P^{(a)}u^T\right) + 1/2\left(\overline{\pi}P^{(v)}u^T\right), \quad (28)$$

where $P^{(a)} = \left[P_{ij}^{(a)}\right]_{(m\times m)}$ and $P_{ij}^{(a)} = P_{ij}\varphi_{ij}/3$.

The overshoot-inclusive variant of (27) for a multi-state chain yields values that match the simulation results closely and consistently. The derivation details are omitted due to space limitation. Finally, Figure 7 displays the design algorithm for BeSpoken parameters. Note that (27) expresses the expected stopping time as a function of $q$ only.

## V. Results and Conclusion

We propose a protocol that generates spokes, relatively straight-line data dissemination trajectories, without requiring the nodes to have navigational information. Analysis of a Markov-modulated random walk model for the spoke results

Given the desired distance $d$, and the angle threshold $\varphi_o$
$n = 1, \ldots \infty$
(a) Calculate $q^*$ assuming $n = E\left[T_{\varphi_o}\right]$ from (27)
(b) Given $q = q^*$ from (a), calculate $r^*$ from (19)
(c) If $d/r^* < n$, goto (a) else BREAK.

Fig. 7. Iterative Design Algorithm for BeSpoken parameters

in a design for protocol parameters, necessary to produce sufficiently long and straight enough trajectories. We support our analysis with simulation results. We simulate a stationary network with nodes deployed over a square region with density $\lambda = 1$. The coordinates of the nodes are generated as iid uniform random variables. To obtain simulation statistics we extend a large number of spokes to follow the same direction (as in Figure 8), over several different network realizations. We collect the statistics for the number of spokes that ended upon running into an empty crescent, and those that were stopped for going off-course. The collected statistics confirm the validity of the design with respect to imposed constraints. In fact, Figure6 illustrates that, for large $r$, (19) matches better the simulation results than (18) with $m = 38$ quantization levels.

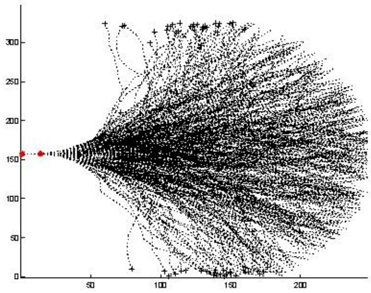

Fig. 8. A sample of 500 spokes directed eastward, designed to reach network perimeter.

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