Engineering Notes

Imperfect Sampling Moments and Average SINR

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1 Introduction and System Model

In this short study we present an analysis of the system performance limitations due to imperfect sampling moments of the received signal. The performance will be described using average signal to interference and noise ratio (SINR), and we present the upper bound of the average SINR due to imperfect sampling moments. In the following we introduce a system model. In Section 2 we describe the assumed receiver, while in Section 3 we expand the results to a CDMA system (with HSDPA parameters). We analyze conventional single-input/single-output systems and briefly address receivers with multiple antennas.

We introduce a simple communication system where the transmitter sends a sequence of data symbols $d(k), (k = -\infty, \cdots, \infty)$, with the symbol period T_{sym}. Without loss of generality, we assume that $\mathrm{E}\left[|d(k)|^2\right] = 1$, and transmitted symbols are uncorrelate.

The channel consists of transmit and receive spectrum shaping filters with the additive white Gaussian noise (AWGN). Assuming that the transmit and receiver spectrum shaping filters have the frequency response $X_{tx}(jw)$ and $X_{rx}(jw)$, respectively, we denote the total frequency response of the channel as $H(jw) = X_{tx}(jw)\, X_{rx}(jw)$, and its impulse response as $h(t)$. As a common practical solution, the total response $H(jw)$ is selected to satisfy the Nyquist criterion [1]

$$\sum_{m=-\infty}^{\infty} H\left(j\left(w+m\frac{2\pi}{T_{sym}}\right)\right)=T_{sym}, \tag{1}$$

or in other words

$$h(mT_{sym})=\begin{cases}1 & m=0\\ 0 & m\neq 0\end{cases} \tag{2}$$

where m is an integer.

Further, the received signal is

$$r(t)=\sum_{k=-\infty}^{\infty} d(k)h(t-kT_{sym})+n(t), \tag{3}$$

where $n(t)$ corresponds to AWGN. The received signal $r(t)$ is sampled with the sampling rate

$$f_{smp}=\frac{1}{T_{smp}}=\frac{M}{T_{sym}}, \tag{4}$$

where the integer M is the oversampling factor. Consequently the i-th sample of the received signal is

$$r(iT_{smp}+\Delta T)=\sum_{k=-\infty}^{\infty} d(k)h(iT_{smp}+\Delta T-kT_{sym})+n(iT_{smp}+\Delta T), \tag{5}$$

where ΔT is the ambiguity of the sampling moment and it is modeled as an uniformly distributed random variable $\Delta T\in[-T_{smp}/2,T_{smp}/2]$.

2 Receiver Assumptions and Average SINR

We assume that the receiver has a knowledge of the impulse response $h(lT_{smp}+\Delta T)$ for $l=-K,\ldots,K$ (where K is sufficiently large to capture most of the energy of the impulse response $h(t)$). For example the estimate of $h(lT_{smp}+\Delta T)$ may be obtained from a pilot assisted estimation.

Up to this point the assumptions were quit general and did not restrict the performance of the system. Let us now introduce certain assumptions that lead to performance limitations for the given system model in (5).

Per each symbol interval, the receiver has M available samples. Receiver has to select one of those M samples, and based on that single sample to perform the detection (i.e., decide upon the transmitted data). Based on the estimate of $h(lT_{smp} + \Delta T)$ the receiver determines which sample has the largest magnitude,

$$L = \arg\max_{l} |h(lT_{smp} + \Delta T)|^2, \quad l = -K, \ldots, K, \tag{6}$$

and without loss of generality we assume that $L = 0$. Consequently, decision statistics for j-th transmitted symbol $d(j)$ is selected as

$$\hat{d}(j) = r(jT_{sym} + \Delta T) = \sum_{k=-\infty}^{\infty} d(k)h(\Delta T - (k-j)T_{sym}) + n(jT_{sym} + \Delta T), \tag{7}$$

Due to imperfect sampling moments (i.e., $\Delta T \neq 0$) inter symbol interference (ISI) is induced. We rewrite the above equation denoting specific terms as ISI

$$\hat{d}(j) = d(j)h(\Delta T) + \underbrace{\sum_{k=-\infty, k\neq j}^{\infty} d(k)h(\Delta T - (k-j)T_{sym})}_{\text{ISI}} + \underbrace{n(jT_{sym} + \Delta T)}_{\text{AWGN}}, \tag{8}$$

Note that in the idealized case of $\Delta T = 0$, $\hat{d}(j) = d(j) + n(jT_{sym})$, i.e., $h(\Delta T) = 1$ and $\text{ISI} = 0$. But in fact, due to the non ideal sampling moments (i.e., $\Delta T \neq 0$) $h(\Delta T) < 1$ and $\text{ISI} \neq 0$.

Based on the above, the average SINR is

$$\text{SINR} = \frac{\text{E}_{\Delta T}[|h(\Delta T)|^2]}{\sum_{k=-\infty, k\neq j}^{\infty} \text{E}_{\Delta T}[|h(\Delta T - (k-j)T_{sym})|^2] + N_o} \tag{9}$$

where $N_o = \text{E}[|n|^2]$. The upper bound of the average SINR is obtained for $N_o = 0$.

Let us now present numerical results for a system whose impulse response corresponds to the raised cosine function (Figure 1, for the roll-off factor 0.22 which is typical for UMTS [2]).

In Figure 2 we present the average SINR upper bound as a function of oversampling, for different roll-off factors. We note that the bound is larger for higher oversampling and larger role-off factors. In Figure 3 we present the average SINR as a function of the signal to background noise (SNR), ISI excluded, for the four-time oversampling and the role-off factor 0.22. For high SNRs, the ceiling effect due to the imperfect sampling is obvious. Linear portion of the characteristic is approximately up to SNR = 13dB, where SINR drops 1dB with respect to SNR (SINR=12dB). In other words, up to that point background noise is dominant source of noise in the system. For higher SNRs than 13dB eventual effort to mitigate the effects of imperfect sampling moments may start to pay off.

Let us now address a receiver with multiple receive antennas. As a common practical solution, the sampling in each receiver branch is driven by the same clock source, and the samples are

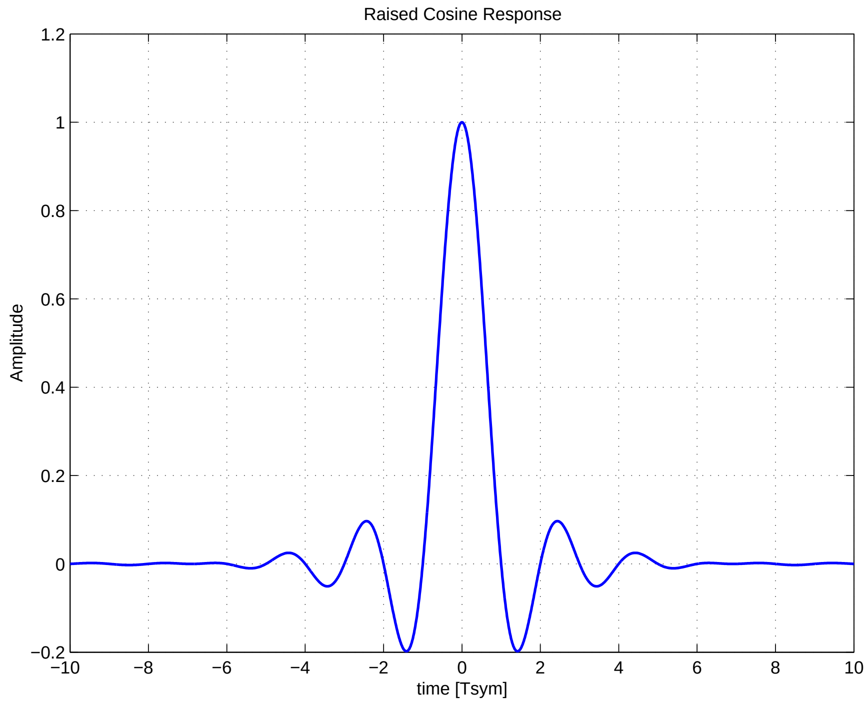

Figure 1: Raised cosine impulse response, the roll-off factor is 0.22.

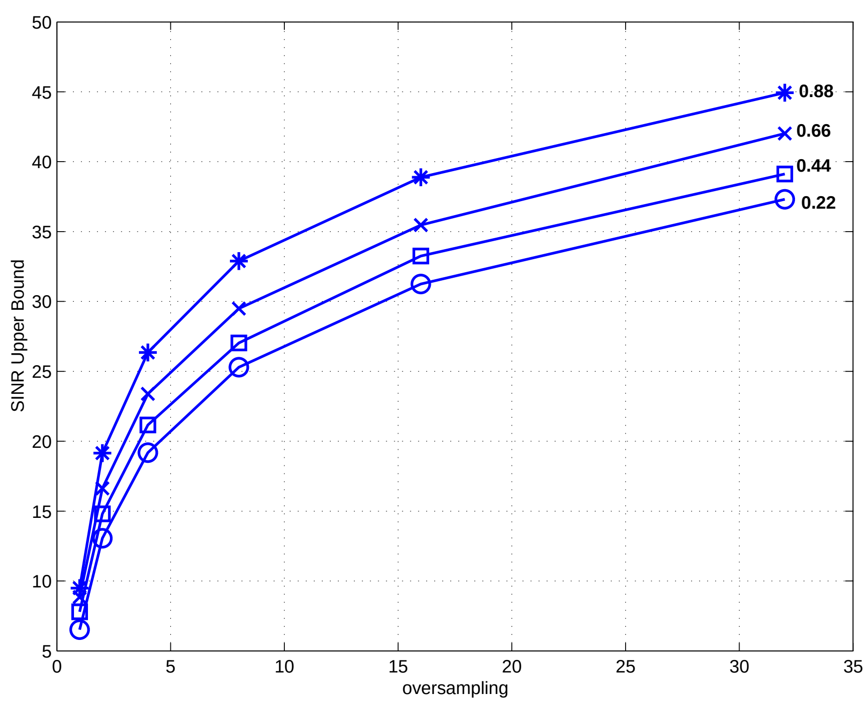

Figure 2: The average SINR upper bound vs. oversampling, equation (9) for $N_o = 0$, for different roll-off factors 0.22, 0.44, 0.66 and 0.88.

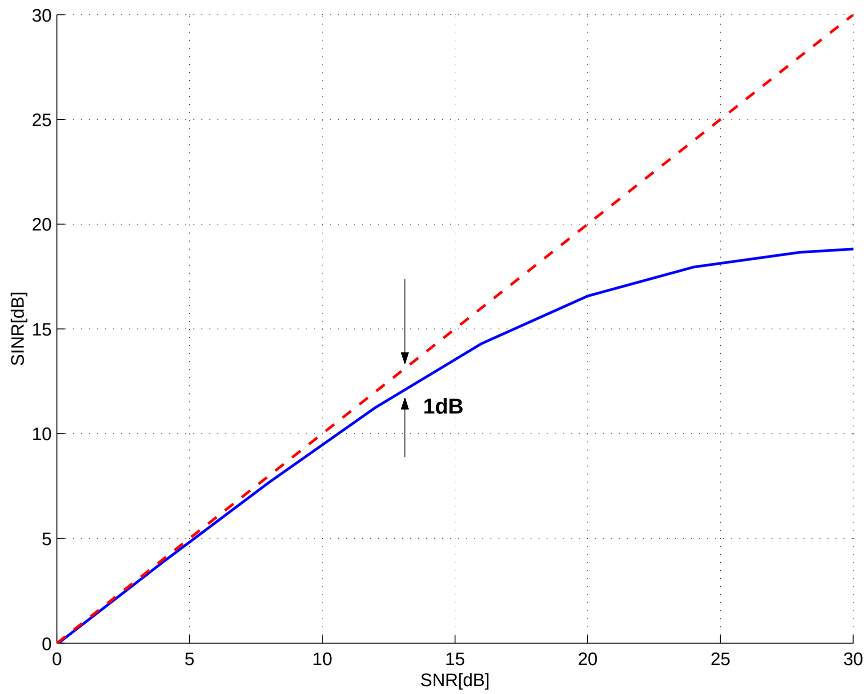

Figure 3: The average SINR vs. signal to background noise (ISI excluded), four-time oversampling, roll-off factor 0.22, solid line corresponds to imperfect sampling, dashed line corresponds to ideal sampling moments $\Delta T = 0$.

taken at the same moment, i.e, resulting in the same ΔT across the receiver antennas. Based on this assumption, it can be shown that multiple receivers do not help in lowering the ISI, i.e., the receiver diversity can not improve the average SINR upper bound.

3 CDMA Systems

Regarding CDMA systems, we assume the spreading factor G, and N as a number of ortogonal codes in the system. In addition, we assume that each code has the same power assigned to it. It can be shown that SINR in (9) becomes

$$\mathrm{SINR} = \frac{G\,\mathrm{E}_{\Delta T}[|h(\Delta T)|^2]}{N\,\sum_{k=-\infty,k\neq j}^{\infty}\mathrm{E}_{\Delta T}[|h(\Delta T-(k-j)T_{sym})|^2]+N_o} \tag{10}$$

The above SINR corresponds to SINR after the despreading, i.e., per one CDMA code. For HSDPA system we assume , $G = 16$ and $N = 12$ (10 data, 1 pilot and 1 control channel codes). The role-off factor is 0.22. In Figure 4 and 5 we present the average SINR upper bound and SINR vs. SNR, respectively. Compared to the results in Figure 2 and 3, we notice a gain in the average SINR upper bound. The gain corresponds to the spreading factor $G = 16$ devided by the number of codes $N = 12$.

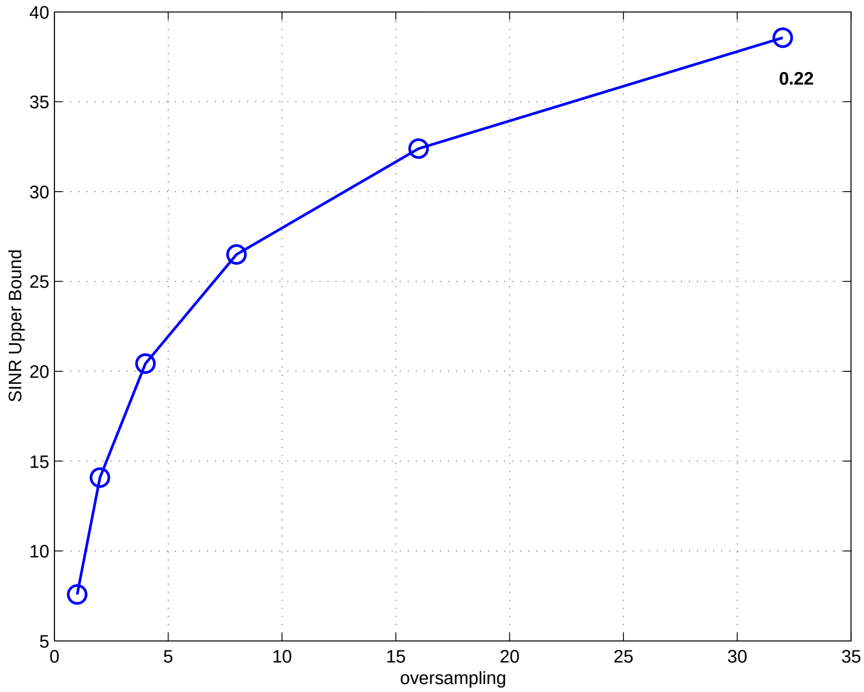

Figure 4: The average SINR upper bound vs. oversampling, equation (10) for $N_o = 0$, HSDPA system, role-off factor is 0.22, $G = 16, N = 12$.

4 Conclusion

In this short study we presented results corresponding to the average SINR in the case of the particular receiver assumptions and imperfect sampling moments. We analyzed the average SINR upper bound and we show that is getting larger when oversampling and/or roll-off factor is increased. We also showed that for lower SNRs, the effect of the imperfect sampling is less important and the background noise dominates. For higher SNRs, the imperfect sampling and induced ISI is more dominant and should be mitigate using improved receiver schemes (e.g., adaptive equalization). The equivalent result we presented for a CDMA system using the HSDPA parameters. We argued that the receiver antenna diversity can not improve the average SINR upper bound.

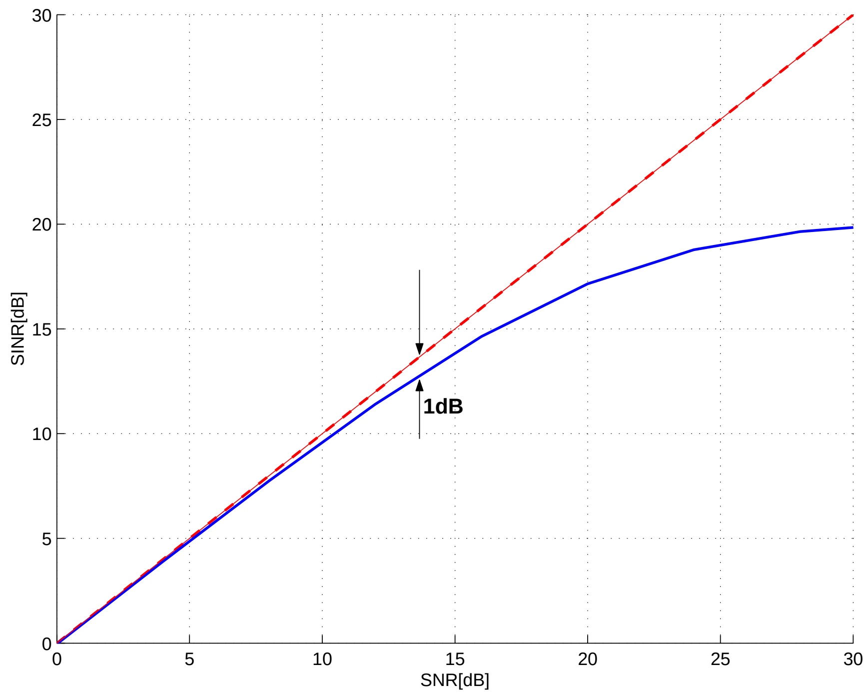

Figure 5: The average SINR vs. signal to background noise (ISI excluded), four-time oversampling, HSDPA system, role-off factor is 0.22, $G = 16$, $N = 12$, solid line corresponds to imperfect sampling, dashed line corresponds to ideal sampling moments $\Delta T = 0$.

References

[1] J. G. Proakis, *Digital Communications.* New York: McGraw-Hill, 3rd ed., 1995.

[2] H. Holma and A. Toskala, eds., *WCDMA for UMTS.* Wiley, first ed., 2000.