SCHEDULING MULTIRATE CDMA USERS BASED ON AVERAGE POWER CONSUMPTION

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ABSTRACT OF THE THESIS

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Recent research efforts have attempted to modify standard multirate CDMA systems to better system performance, mostly in metrics of system capacity. With the availablility of wider bandwidths, most of these modifications attempt to employ the larger bandwidth to orthogonalize users via either frequency or time separation. This research focuses on just such a time-separation technique for a variable-processing-gain multirate CDMA system. The performance of this system is quantified in this thesis in terms of the total average power consumed by all system users. The notion of time frames and slots is adopted and scheduling techniques that assign users transmissions to slots are studied for potential system performance improvement. Three general classes of scheduling techniques is presented. The first assigns users to only one of the available orthogonal channels. The second allows for multiple channel access. And finally a third that utilizes a repeated transmission policy to minimize power consumption.

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Dedication

To my family, my friends who are family, and my adorable *chumuch*, Srishti.

Table of Contents

List of Figures

Chapter 1

Introduction

The marriage of multimedia traffic sources with wireless communication services has introduced new issues in the design of future systems. The integration of diverse data sources necessitates that wireless communication systems reliably transport multirate traffic. For systems that utilize Code Division Multiple Access (CDMA), past research efforts have provided various methods for fulfilling these multirate requirements; among them are systems based on variable processing gains, variable chip rates, and multi-codes [1]. More recent efforts have tried to modify the transmission and reception structure of such systems so as to improve overall performance, either in terms of system capacity or power consumption. More specifically, these pursuits employ either time-based or frequency-based separation of CDMA signals to better throughput and reliability. In this work, we examine exactly such a separation or "orthogonalizing" mechanism in the time-domain. The work here attempts to take a general approach to the problem of scheduling *multirate* CDMA signals into time slots for a system operating with variable processing gains. The frequency-dual to this problem will be the scheduling of multirate users to different bandwidths when operating with variable chip rates. We quantify system performance in terms of total average power consumed by all users in a single-cell system and thus wish to develop a scheduler that assigns multirate users to slots while minimizing this power consumption. Improvements in this metric in turn point to methods for more reliable reception as well as higher overall capacity.

We begin this chapter by first looking at the background issues in the study of a slotted CDMA system. In this regard, we focus in section 1.1 on previous research efforts that have attempted to modify standard CDMA systems so as to deal with multirate traffic. Then, in section 1.2, we introduce the system of concern in this research and summarize the scope of our examination of such a scheme. We conclude this chapter in section 1.3 by describing the structure of the thesis itself.

1.1 Previous Work

Recently, research efforts have been made that attempt to modify the basic transmission and reception mechanism of CDMA-based systems so as to enhance system operation. With goals to improve throughput and error rate performance, the modifications discussed in these specific works all attempt to separate CDMA signals in either the time or frequency domain. In [2], the authors attempt to use variable chip rates for frequency-based separation of signals in a dual-rate system. Variable chip rates operating on multirate requirements allow for different bandwidths among users. These different bandwidths are then in turn exploited for frequency-based rate separation. The results presented for such separation in a dual-rate system shows an enhancement in bit error rate performance. The crux of the work detailed in [3] and [4] describes a dual-rate system in which delay-tolerant CDMA signals are separated in time using a scheduling mechanism that administrates slotted CDMA transmissions. The subsequent results of applying such a scheduled CDMA system point to improvements in information throughput. The authors of [5] have proposed a wireless ATM system based on a transmission scheme they call S-CDMA (Scheduled CDMA). The authors suggest that in this ATM system, the CDMA signals transmitted by the supported mobile users must be time-slotted, though in an asynchronous fashion. Thus, with an aim not so different from [2], these slotted transmissions can be scheduled in such a manner so as to adhere to a user's Quality of Service (QoS) requirements in terms of bit error rates. Finally, in [6] yet another time-based separation of CDMA signals is addressed. In this instance, the focus is on employing CDMA signaling techniques that adhere to EC-MAC (Energy Conserving - MAC) protocol for user terminal power conservation. The fundamentals of this protocol also require an inherently time-based separation of uplink traffic.

1.2 Thesis Objectives

In this thesis, we focus our attention on a variable processing gain CDMA system which operates under the structure of time frames and slots. Users transmit their desired bits, i.e. packets, during assigned slots using CDMA signaling. A centralized scheduler collects minimum SIR (Signal-to-Interference Ratio) and rate requirements of each system user. Based on these values, the scheduler then computes the slot assignments and relays them to the individual mobiles. The main goal of this research is to determine the structure of this scheduler. Additionally, we would like to study the potential benefits of this time-based "orthogonalizing" of CDMA signals. This will be done by comparing when such a scheduled system outperforms the standard variable-processing-gain multirate CDMA system in terms of total average power consumption or equivalently, the number of users supported. The eventual goal of this work is to determine the conditions under which scheduling is desirable and to describe the mechanics of those schedulers that achieve power savings.

We have approached the construction of such a scheduler using two different formulations. In the first, we assume a distinct assignment in which each user transmits its information bits in one and only one of the slots in each frame. Though presented here under the premise of time division of CDMA signals, this approach directly parallels the assignment of CDMA users to distinct bands in systems with frequency bandwidth partitioning. In fact, as discussed here, the two cases are analytically equivalent. Furthermore, it is our belief that both these cases are simply just instances of the more general dynamic channel allocation problem in that the channels we are discussing are interference-based, orthogonal slots or bands. In the second formulation of the assignment of system users to slots, we relax the discrete assignment constraint by allowing users to transmit in multiple slots in a frame.

In studying these two approaches, we present heuristics that govern the operation of those schedulers that reduce power consumption compared to the standard variable processing gain CDMA system. The theoretical results of the one-slot-per-user assignment point to potential benefits in a scheduler that meets a load-balancing criterion for the discrete rate allocation scheme. The load-balancing constraint effectively equalizes the total power consumed in any one slot over all the slots. In addition to such a scheduler, we also study the implications of random assignments as well as reduced complexity assignment schemes that attempt to approximate load-balancing characteristics. The results of the relaxed assignment policy are, however, a bit more complicated. Due to the characteristics of this scheme, the relaxed assignment policy has potentially many user-to-slot allocations, especially when compared to the discrete scheme. Because of inherent complexities of the problem formulation, many of these assignments are local solutions to the minimum-power relaxed assignment problem. Thus we cannot rely on this formulation for an optimal solution but rather we may look upon it for means to study further average power reduction techniques in slotted systems.

In addition to this general slotted CDMA system, we also focused on a slightly more specific slotted CDMA scheme. Specifically, we studied an additional system in which each slotted transmission by a particular user constituted the same sequence of information bits. By requiring that each user transmit the same bits or packets in each assigned slot, we then were able to construct the optimal assignments in a manner very similar to that of allotting optimal user codes to standard (unslotted) CDMA users. In particular we were able to show that if users could scale their transmissions according to a particular sequence of values – defined by Welch Bound Equality (WBE) codes – a minimum sum of total average power slot assignment could be found for these users.

1.3 Thesis Outline

We begin our study into the possibilities of scheduling CDMA signals by first reviewing background information on the particularities of the system under analysis. This is done in Chapter 2 where we examine the basic mechanics of the variable-spreading-gain

CDMA system as it pertains to the slotted implementation. Along with this description, we also briefly investigate the feasibility and capacity issues of a slotted scheme. Chapters 3 and 4 focus on the discussion of the two separate scheduling scenarios of the slotted system. Both theoretical and experimental results are discussed here. The discrete one-slot-per-user allocation is presented with results in Chapter 3 where as Chapter 4 focuses on the relaxed version of this assignment problem. The repeated transmission system is described and studied with theoretical results in Chapter 5. Chapter 6 then concludes this presentation with a brief summary of our investigation and a discourse on possibilities of future analysis in this topic.

Chapter 2

System Description

This study of scheduling CDMA users begins with an understanding of the underlying communications channel. Multirate users can be supported on a CDMA-based system with modifications in either the spreading-gain, the chip-rate, or the code assignment to the diverse users [1]. Of the three, we focus here on a system that assigns different spreading gains to different rate users. This is achieved by keeping the system chip rate constant and giving users spreading gains based on their desired rate requirements. Practically speaking, a fixed basic chip rate is desirable so as to ensure that all users occupy the same bandwidth [7]. Such a multirate CDMA system is then modified to incorporate slotted transmissions. This involves superimposing a time-division structure consisting of a time frame and time slots during which users transmit their desired information bits or packets. Along with the details of the transmission scheme, one must also account for each user's rate and Quality of Service (QoS) requirements during transmission. We thus assume that system and user requirements met in conjunction allow for reliable end-to-end communications.

Once having formalized both the user and system requirements for reliable communications, we examine the exact nature of the signals transmitted during each slot by concentrating on the derivation of the Signal to Interference Ratio (SIR) for arbitrary user during any arbitrary slot. Aside from giving one an insight into the variable-processing-gain transmission in each slot, the SIR also provides a measurement of the particular user's achieved QoS during that slot. Each user desires a minimum SIR in each slot; this subsequently maps to a minimum QoS of that user via the bit error-rate (BER). With the SIR formulation and the minimum SIR requirement at hand, the minimum power levels for each user in each slot can be computed. After deriving these principles, we can then move towards devising a scheduling pattern for transmitting each user's desired packets in the slotted CDMA environment.

We begin this chapter by constructing the details of this slotted, variable-spreading-gain CDMA scheme. In section 2.1 we lay out the system requirements in terms of frames and slots and we characterize the variable-processing-gain CDMA signal. Section 2.2 analyzes the SIR of a user during a particular slot. The next section focuses on formulating the minimum power level solution for each user in each slot. We conclude this section by restating the system and user requirements in terms of these calculations. The last sections of this chapter address peripheral issues concerning the ramifications of slotted transmissions. In section 2.4, we examine similarities and differences between CDMA schemes that use time separation, i.e slotting, and those that utilize frequency separation of users. Finally, observe that traditional notions of CDMA signals are that system users have complete access to system bandwidth at all time. Slotting CDMA users therefore seems inconsistent with the usage of CDMA signaling. The final section of this chapter, section 2.5 attempts to briefly justify our time-orthogonalization of multirate users.

2.1 System Description

Consider a single-cell wireless system with system bandwidth W in which there are N mobiles wishing to transmit information bits, i.e. packets, to the local base station. The slotted, multirate CDMA system studied here refers to the uplink transmission mechanism that allows these N users to communicate with the local serving station. First, we introduce the notion of a time frame of duration T_f which consists of M slots each of duration T_s. Multiple users can transmit during each slot at different transmission rates. Such a system then requires that 1) each user is assigned to one or more slots per frame and 2) that it transmits only during its assigned slot(s).

Each user i has a minimum rate requirement so that it must send at least a certain number of bits per frame or equivalently r_i bits per second in a frame. Additionally, each user has a Quality of Service (QoS) requirement that it requires in its transmission stream. In the context of the link layer, this can be associated with a the bit error-rate (BER). As shown in Haykin [8], the BER is a particular function of the measured SIR of the transmitted signal. In fact we know that BER is a decreasing function in the measured SIR. Thus we see that the minimum QoS service requirement of the transmission stream of user i can then be rewritten as the minimum threshold SIR that user i must accomplish for reliable reception of its transmitted bits. In our analysis we further assume that the QoS constraints for each user, i.e. the desired BER, does not change over slots. In other words, regardless of the rate achieved and the power expended in each slot, the demodulation and detection of each bit at the receiver must meet that user's specified BER. Incorporating the system and user requirements, we can then visualize the slotted and unslotted scheme as in Figure 2.1.

In each slot only a given subset of the N users transmit their bit streams. Furthermore, each user sends its stream at different transmission rates. Since we are assuming a variable processing gain system with total available bandwidth W, we know that the chip interval is fixed for all users in all slots as $T_c = 1/W$. To accommodate the diverse rates, we let each user have different bit intervals, i.e. T_{ij} - the bit interval of user i in slot j. Thus the spreading gain of user i in slot j is: $G_{ij} = T_{ij}W$.

For such a system, we begin by assuming that the frame and slot lengths are such that *at the very least* the user with the maximum rate can be accommodated in a single slot. Equivalently, we assume the minimum processing gain in a slot is $W/Mr_{\max}$, where $r_{\max} = \max_i r_i$. This in turn implies that $W \geq Mr_{\max}$. Under this assumption, we wish to examine the performance of this slotted system over a frame and compare it with that of the multirate system described previously. More specifically we wish to determine if and when scheduling transmissions of multirate users into slots surpasses the efficiency of the standard system in terms of average power consumption.

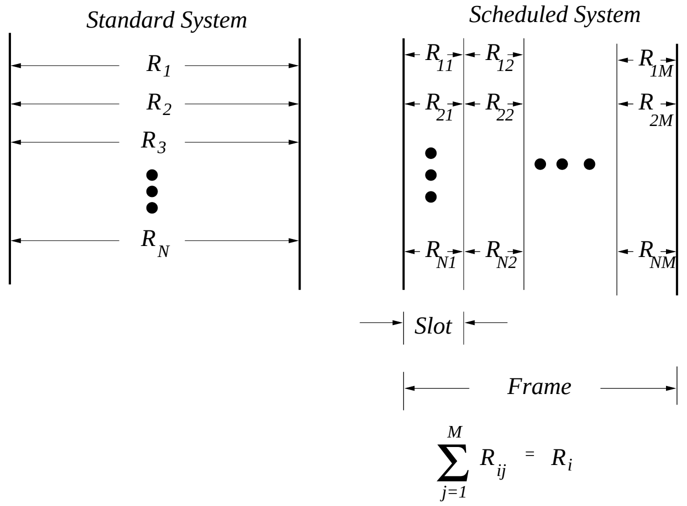

Figure 2.1: Comparison of unslotted and slotted multirate systems.

2.2 SIR Calculation

For user i in slot j, let R_{ij} represent the transmission rate and let P_{ij} be the received power. Denote the signature waveform of the ith user in slot j as $s_{ij}(t)$. Consider the received signal at the basestation during slot j when attempting to detect a bit transmitted by user i. Let K_j be the subset of users $\{1, 2, \ldots N\}$ who transmit (all or part of) their bits in slot j. Assuming synchronous transmissions by these users, the received signal $r(t)$ for detecting one of user i's bits in slot j is then

$$r(t) = \sqrt{P_{ij}} s_{ij}(t) b_i + \sum_{k \neq i, k \in K_j} \sum_{l=1}^{C} \sqrt{P_{kj}} s_{kj}(t - (l-1) \cdot T_{kj}) b_{k_l} + n(t) \tag{2.1}$$

Here b_i is the transmitted bit of user i that the base station wishes to detect. For users with $R_{kj} > R_{ij}$, b_{k_l} represents the lth transmitted bit of user k during the bit interval of user i in slot j, i.e. during T_{ij}. When $R_{kj} \leq R_{ij}$, $b_{k_l} = 0$ for $l \geq 2$. Thus we can say that $C = \lceil \frac{\max_k R_{kj}}{R_{ij}} \rceil$. The term $n(t)$ represents additive white Gaussian noise. Now let us assume that the signature waveforms for all the users are random binary sequences; in particular that they are pulses of height ± 1 with equal probability. Using a conventional (matched filter) receiver to detect the bit b_i of user i, we get the following statistic

$$\begin{aligned}
r_{ij} &= \int_0^{T_{ij}} s_{ij}(t) r(t)\, dt && (2.2)\\
&= \sqrt{P_{ij}} b_i \int_0^{T_{ij}} s_{ij}^2(t)\, dt && (2.3)\\
&\quad + \sum_{k \neq i, k \in K_j} \sum_{l=1}^{C} \sqrt{P_{kj}} b_{k_l} \int_0^{T_{ij}} s_{ij}(t) s_{kj}(t-(l-1)\cdot T_{kj})\, dt && (2.4)\\
&\quad + \int_0^{T_{ij}} n(t) s_{ij}(t)\, dt && (2.5)
\end{aligned}$$

The first term in (2.3) is the desired signal component of the received signal, the second term (2.4) represents the interference from all other transmitters, and finally the last term (2.5) is the interference experienced at the receiver due to the presence of background additive white Gaussian noise. Note that all b_{k_l} are independent random variables that take on values ± 1 with equal probability. The energy of the signal part is then ξ_S

$$\xi_S = \mathrm{E}\left[\left(\sqrt{P_{ij}} b_i \int_o^{T_{ij}} s_{ij}^2(t)\, dt\right)^2\right] = P_{ij} T_{ij}^2 = \frac{P_{ij}}{R_{ij}^2} \tag{2.6}$$

Note that we have not made an unit-energy signal assumption here though the results maybe easily extended as such. The energy of the interference contribution, ξ_I is

$$\begin{aligned}
\xi_I &= \mathrm{E}\left[\left(\sum_{k \neq i, k \in K_j} \sum_{l=1}^{C} \sqrt{P_{kj}} b_{k_l} \int_0^{T_j} s_{ij}(t) s_{kj}(t-(l-1)\cdot T_{kj})\, dt\right)^2\right] && (2.7)\\
&= \sum_{k \neq i, k \in K_j} \frac{P_{kj} G_{ij}}{W^2} && (2.8)\\
&= \sum_{k \neq i, k \in K_j} \frac{P_{kj}}{W R_{ij}} && (2.9)
\end{aligned}$$

Finally, the energy of the noise component of r_{ij}, ξ_N is

$$\begin{aligned}
\xi_N &= \mathrm{E}\left[\left(\int_0^{T_j} n(t) s_{ij}(t)\, dt\right)^2\right] && (2.10)\\
&= \eta_o T_{ij} && (2.11)\\
&= \frac{\eta_o}{R_{ij}} && (2.12)
\end{aligned}$$

where η_o is the power spectral density of the noise. The measured signal-to-interference ratio of user i in slot j is then, SIR_{ij}

$$SIR_{ij} = \frac{\frac{P_{ij}}{R_{ij}^2}}{\sum_{k\neq i, k\in K_j} \frac{P_{kj}}{WR_{ij}} + \frac{\eta_o}{R_{ij}}} \tag{2.13}$$

$$= \frac{W}{R_{ij}} \frac{P_{ij}}{\sum_{k\neq i, k\in K_j} P_{kj} + \eta_o W} \tag{2.14}$$

The result we obtained here for the SIR of user i in slot j matches that for the SIR of the standard unslotted multirate CDMA system in [9]. Please note that if user i was the only transmitter during slot j, then we can reduce (2.14) to:

$$SIR_{ij} = \frac{P_{ij}}{R_{ij}\eta_o} \tag{2.15}$$

2.3 Minimum Power Levels

Observe that if user i has a minimum QoS, or equivalently minimum SIR requirement, Γ_i, then one can determine the minimum power of this user such that $SIR_{ij} \geq \Gamma_i$. The procedure for calculating this minimum power vector for each user in a particular slot is very similar to that of calculating the minimum power vector of a variable-spreading-gain system without slotting and is outlined in [10]. We briefly extend these previous results to the slotted scheme in this section.

Denote each user's minimum rate requirement over a frame with the following vector: $\mathbf{r} = [r_1 \;\; r_2 \ldots r_N]^T$. In our discussion in this thesis we assume that each user has identical minimum SIR requirements. Denote this minimum SIR as Γ. Note that if a user desires $\Gamma' > \Gamma$, then we can effectively model this higher SIR requirement as a higher rate requirement due the measured SIR relation in (2.14). Thus if a user desires α times this identical SIR requirement, we can equivalently assign it the system SIR Γ while simultaneously increasing its rate requirement by a factor of α.

As discussed previously, the chip rate of each user is constant so that all users occupy the total system bandwidth, W. Let the vectors $\mathbf{P_j} = [P_{1j} \;\; P_{2j} \ldots P_{Nj}]^T$ and $\mathbf{R_j} = [R_{1j} \;\; R_{2j} \ldots R_{Nj}]^T$ represent the actual transmit powers and rates of each of the users during slot j, where $j = 1, 2, \ldots M$. The minimum SIR requirement can be stated as

$$\frac{W}{R_{ij}} \frac{P_{ij}}{\sum_{k \neq i, k \in K_j} P_{kj} + \eta_o W} \geq \Gamma \qquad \forall i = 1, 2, \ldots N \qquad \forall j = 1, 2, \ldots M \tag{2.16}$$

If we minimize the sum $\sum_{i=1}^{N} P_{ij}$ of the powers, of all the users in slot j subject to the constraint in (2.16), we get

$$P_{ij} = \frac{\eta_o W \frac{\gamma_{ij}}{\gamma_{ij}+W}}{1 - \sum_{k \in K_j} \frac{\gamma_{kj}}{\gamma_{kj}+W}} \tag{2.17}$$

where $\gamma_{ij} = \Gamma R_{ij}$. Note that it can be shown that this power assignment minimizes both the sum of total power for all users as well as the maximum power of any one of the N users in slot j, over all feasible power assignments [9]. Furthermore, note that if a user n is not assigned to slot j then in that slot its rate will be $R_{nj} = 0$ and its power will be $P_{nj} = 0$. We can thus write (2.17) in the following equivalent form

$$P_{ij} = \frac{\eta_o W \frac{\gamma_{ij}}{\gamma_{ij}+W}}{1 - \sum_{k=1}^{N} \frac{\gamma_{kj}}{\gamma_{kj}+W}} \qquad \forall i, j \tag{2.18}$$

Observe that these power levels are minimum only for those users sending in the *same* slot. The N users in this system can however "pick and choose" one or more of the M slots of the frame to transmit their bits as long as $\sum_{j=1}^{M} \gamma_{ij} = r_i \Gamma$. In words, users can transmit in any one of the M slots as long as the minimum rate and SIR requirements ($\mathbf{r}$ and Γ) are met. In fact, one can see that the standard, unslotted CDMA is just the slotted scheme with the assumption that $\gamma_{ij} = \frac{\Gamma r_i}{M}$, $\forall j$. Also note that over the entire frame a user i expends average power $\frac{1}{M} \sum_{j=1}^{M} P_{ij}$. The focus of this thesis is then to determine the overall minimum $\frac{1}{M} \sum_i \sum_j P_{ij}$ such that the rate and SIR requirements are met for all users over the M slots.

2.4 Bandwidth Partitioning

In this section we briefly examine the similarities between the slotted CDMA system and those proposed to implement a frequency-based separation of multirate CDMA signals. Such bandwidth partitioned systems have been proposed recently in [11] and [12] as a potential way to improve system performance. The problem that we have posed here with the slotted system is similar to bandwidth-partitioning in the following manner. Suppose that the total bandwidth for this frequency-based separation scheme is also W and that this W is partitioned into M non-overlapping bands, $B_1 = B_2 = \ldots = B_M = \frac{W}{M}$. Let each transmitting user in a particular band j utilize standard (unslotted) variable-spreading gain CDMA as a means to communicate to the basestation. Thus the average power consumed by a user i in band j such that $i \in K_j$ (where K_j is now the set of all users transmitting bits in band j) is

$$P_{ij} = \frac{\eta_o B_j \frac{\gamma_{ij}}{\gamma_{ij}+B_j}}{1 - \sum_{k \in K_j} \frac{\gamma_{kj}}{\gamma_{kj}+B_j}} \tag{2.19}$$

$$= \frac{\frac{\eta_o W}{M} \frac{\gamma_{ij}}{\gamma_{ij}+\frac{W}{M}}}{1 - \sum_{k \in K_j} \frac{\gamma_{kj}}{\gamma_{kj}+\frac{W}{M}}} \tag{2.20}$$

$$= \frac{\frac{\eta_o W}{M} \frac{M\gamma_{ij}}{M\gamma_{ij}+W}}{1 - \sum_{k=1}^{N} \frac{M\gamma_{kj}}{M\gamma_{kj}+W}} \tag{2.21}$$

The average power consumed by a user in T_f seconds can then computed by realizing that since the users in band j are operating without slots, their rates are R_{ij}/M, where R_{ij} is the rate that user i achieves in band j during an interval of time $[t, t+T_f]$. The average power consumed by a user i in band j during an interval of length T_f is then just

$$P_{ij} = \frac{\frac{\eta_o W}{M} \frac{\gamma_{ij}}{\gamma_{ij}+W}}{1 - \sum_{k=1}^{N} \frac{\gamma_{kj}}{\gamma_{kj}+W}} \tag{2.22}$$

This minimum power is exactly that of (2.18). The slotted scheme of M slots and band-partitioned scheme of M bands operating under the same bandwidth W consume the same average power over a given time interval for the same user rate assignments in each slot/band. In other words, let the set of users assigned to a particular slot j be the same as those assigned to a band b. If each user transmits at the same information rate in each of the two schemes and the two schemes have the same system bandwidth, then the average power measured over the interval $[t, t+T_f]$ is the same in both schemes.

The presentation of the bandwidth partitioned system has focused on allocation schemes that tend to assign a user to only one of the M bands. The parallel to that in the slotted scheme would be the assignment of each of the N users to one and only one of the M slots. This type of allocation is the focus of the next chapter.

2.5 Why Separate CDMA signals?

Before we begin to look at the different issues behind slotted CDMA, we must first briefly describe why one might wish to introduce this seemingly paradoxical notion of slotted CDMA. The idea of separating CDMA signals, be it in time or frequency, is in direct contradiction with the basic premise of CDMA: the simultaneous access of the entire system bandwidth by all system users. Though there are other justifications given in the previously referenced works [2], [3] and [4], we derive our reasons from the limitations of conventional power control schemes for such variable processing gain CDMA systems. As noted in [10], the power level assignment in standard unslotted systems are limited by the user with the lowest transmit power constraint, the greatest distance to the serving basestation, or the highest rate requirement. Our original impetus was to utilize slotting in large bandwidth systems to separate such "demanding" users in time slots and thereby improve performance for the other - if not all - system users. In other words, we start with a system with substantial system bandwidth so that even after an M slot division of the time frame, $\gamma_{\max} = \max_i \Gamma r_i < \frac{W}{M}$ (Γ and r_i are the required SIR and rate respectively of user i). For this system, we believe slotting those users who strain system performance because of their power or rate constraints will improve either the rate or power performance of the remaining users.

The presentation in this thesis, however, does not deal with the transmit power limits or associated link gains of the system users. On the other hand, we do uphold each users' minimum rate and SIR requirements. So, in our context, we note that conventional multirate power assignments are limited by the maximum rate requirement among all the supported users and our goal here is to show that system performance may be improved via slotting CDMA transmissions. We clarify this intuition with a simple example for the moment. (Note that the following chapters will provide more rigorous support for such a claim.) From [10], we are given that N multirate users can be supported by a system, i.e. their minimum SIR requirements can be met, if and only if

$$\sum_{i=1}^{N} \frac{\gamma_i}{\gamma_i + W} < 1 \tag{2.23}$$

where once again, $\gamma_i = \Gamma r_i$. This is obtained by realizing that if the interference in the system surpasses a threshold then the minimum SIR levels for the system users can no longer be met. Now, assume that there are only two slots in this system and that W is more than large enough to support the maximum γ_i in a slot by itself. Also assume that there is one user k who comprises most of the $\sum_{i=1}^{N} \frac{\gamma_i}{\gamma_i + W}$. If we utilize the second slot and separate this particular user from the others (i.e., let it transmit by itself in any one of the two slots), then the remaining users must adhere to the following in their own slot

$$\sum_{i=1, i \neq k} \frac{2\gamma_i}{2\gamma_i + W} < 1 \tag{2.24}$$

We can thus immediately see that depending on the exact values of γ_i:

1. Each user i in this second slot could possibly achieve higher γ_i and still meet the requirement in (2.24).

2. The second slot can potentially support additional users with $\gamma < \gamma_k$.

3. The second slot can maintain the same number of users with the identical rate requirements as above, but reduce the average power consumption.

What we have listed above is merely the intuition upon which we examined the slotted scheme. The subsequent chapters provide more justifications for these statements.

Chapter 3

Discrete Assignment Policy

Within the discipline of slotted CDMA transmissions, there exists a myriad of possible assignments of users to time slots. Within such a multitude of assignment schemes are those that optimize some metric of system performance. In the work presented here, the focus lies on establishing certain scheduling methodologies that attempt to minimize the overall total average power consumed by all users in the time frame T_f. The first type of scheduling mechanism studied relies on the assumption of a discrete slot assignment. By this we refer to the scenario in which each of N system users are assigned to one and only one of the M slots in each frame. This chapter concentrates on the principles of such an assignment policy. The alternate scheduling technique, the crux of chapter 4, allows users to be assigned to multiple slots in a frame. Since we have described our system thus far with a general number of users and slots, we begin this chapter by first addressing in section 3.1 how N and M affect these scheduling techniques. From here we move onto the central focus of the chapter, which is to describe the functions of the discrete assignment scheduler. This description is presented in sections 3.2, 3.3, and 3.4. The first of these sections deals with a pair of simple examples that provide a rudimentary understanding of the problem at hand. This is done by studying a system of two slots and three users. Section 3.3, on the other hand, deals with the more general problem of M slots and N users. In it, we propose a heuristic that reduces the total average power consumed for any other discrete assignment that does not balance or equalize the load carried in any two pairs of slots over the M slots. Load-balancing over two slots is achieved when the total load carried in the two slots is as equal or balanced as possible. The result then is that the power levels in each of the two slots are also equalized. This heuristic simultaneously imposes a pairwise load-balancing criterion on the assignments and also reduces power consumption. With simple arguments presented in this same section, one can then show that this load-balancing assignment of the N system users to the M time slots is non-polynomial (NP) complete. For our problem, we note that the load-balancing scheduler is simply too complex for implementation. To circumvent the complexity of this scheduler, we introduce in section 3.4 two cases of simpler sub-optimal assignments. One set of assignments places users into slots in a random manner. The other set attempts to approximate the results of the load-balancing scheduler by two different assignment schemes. In one we place users to slots in order of decreasing rate and in the second we assign each user to the least loaded slot. In section 3.5 we analyze the performance of such discrete schedulers versus that of standard multirate CDMA. For small-order problems, we compare all discussed schemes. For larger problems (with many slots and significantly many diverse users), one is forced to eliminate the load-balancing scheduler from the comparison due to its complexity. In these examples, we thus deal with only the reduced-complexity, sub-optimal performance, including comparisons to standard multirate CDMA.

3.1 Scheduling and the Relationship between N and M

A basic method of scheduling users to slots in a frame is a one-slot-per-user assignment. Here each user is assigned to only one of the M slots. Before deriving the conditions for such a scheduler, let us first make some observations concerning the relationship between M and N. Due to the constraints of this problem formulation, if $M \geq N$ (more slots than users), then each user will be scheduled to transmit in its own slot. Such an assignment results from the principles of orthogonality [13]. In fact, for $M = N$, this assignment would be the absolute minimum even if one were to remove the discrete assignment requirement. Once again this is a consequence of orthogonal signaling.

For the case $M > N$, it can be shown that the total average power consumed by the N users in such a discrete assignment is the same as that for an assignment that allows a user to transmit over multiple slots. To see this, let us begin a fixed N and M such that $M > N$. Call the set of all users in the system U. Scheduler 1 assigns users to slots discretely, i.e. one-slot-per-user. Scheduler 2, on the other hand, can divide users' transmissions over multiple slots. Once again relying on previous results from orthogonal signaling, we realize that both the schedulers will minimize power by separating users into different slots. Scheduler 1 merely places the N users in N different slots, leaving the remaining $M-N$ empty (resource inefficient but nonetheless minimal in the power metric). Scheduler 2 meanwhile takes a subset of the N users and assigns them a singular slot. The data streams of the remaining users, members of the subset denoted by K ($K \subset U$), are then divided over the remaining $M-N$ slots so that each of these users' transmitted bits are the only bits transmitted during the respective slot. In other words, scheduler 2 may assign a user i, where $i \in K$, l slots ($l \leq M-N$) over which to transmit its bits. Specify these l slots of user i by the set L. Then to minimize the total average power, no other user is assigned to these same l slots. Furthermore, the average power that user i exhausts in each of these l slots is

$$P_{ij} = \Gamma_{ij} R_{ij} \eta_o \qquad j \in L \tag{3.1}$$

Note that the minimum rate requirement demands that $\sum_j R_{ij} = r_i$. Thus over the entire frame, user i expends average power

$$\begin{aligned} \bar{P}_i &= \frac{1}{M}\sum_{j=1}^{M} P_{ij} && (3.2)\\ &= \frac{1}{M}\sum_{j\in L} \Gamma_{ij} R_{ij} \eta_o && (3.3)\\ &= \frac{1}{M} r_i \Gamma_i \eta_o && (3.4) \end{aligned}$$

Therefore, we note that the average power expended by each user $i \in K$ during the entire frame is described by (3.4). Now, for scheduler 1 in this $M > N$ scenario, each user $i \in U$ will be assigned its own slot, say slot s. Observe that the power in each slot $j \neq s$ will be 0. Thus the average power consumed by the user i in this case will be

$$\bar{P}_i = \frac{1}{M}\sum_{j=1}^{M} P_{ij} = \frac{1}{M} r_i \Gamma_i \eta_o \tag{3.5}$$

Comparing the results of (3.4) and (3.5), we see that any user i in this system with $M > N$ consumes the same average power if it is assigned its own slot or set of slots.

Since previous principles all dictate that orthogonal signaling minimizes power consumption, we deduce that for cases of $M \geq N$, the scheduling results for either policy (discrete or non-discrete) result in the same power consumption regardless of the slot assignment. These results are in fact fairly non-extraordinary. For the remaining discussion in this thesis we examine the more interesting case of $M < N$. We begin here in this chapter by looking at the discrete assignment policy for $M < N$.

3.2 Two Slots and Three Users

Before examining the general scenario of N users and M slots, let us take a detour into a simpler arrangement. In this section we describe two examples of a system comprised of three users and two slots, $N = 3$ and $M = 2$. In the first example we assume that two of these three users are identical in their requirements. The second example deals with the optimal assignment of users to slots when each of the three users have different rate requirements.

Example 1: 2 Identical Users

Consider three users in a system, each with the same desired required SIR level, Γ. In this first example, we look at the case when two of the users (user 1 and 2) have an identical minimum rate requirement *over a slot*, say r, and the third user (user 3) has a requirement βr *over a slot.* Under these assumptions we analyze the power consumption of three schemes described in Figure 3.1. The first two schemes comprise the only two possible slots assignments for the slotted system limited to one slot per user. The third scheme is the standard multirate system of [10]. Note that the desired rates in the third scheme are half that of the slotted scheme.

Let us begin by comparing schemes 1 and 2. In scheme 1, the two equal rate users transmit simultaneously during one slot and the third user transmits by itself in the

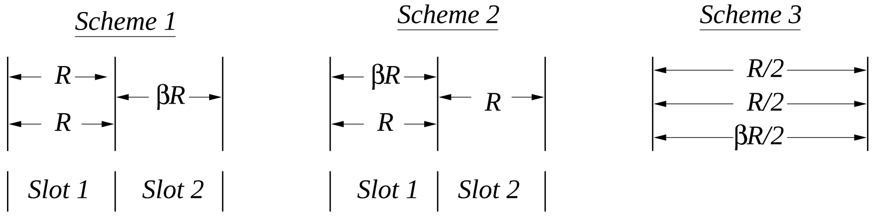

Figure 3.1: The three schemes of Example 1

second slot. Using the minimum power allocations per slot given in (2.17), we see that

$$P_{11}^* + P_{21}^* = \frac{2\gamma\eta_o W}{W-\gamma} \tag{3.6}$$

$$P_{31}^* = 0 \tag{3.7}$$

$$P_{12}^* = P_{22}^* = 0 \tag{3.8}$$

$$P_{32}^* = \beta\gamma\eta_o \tag{3.9}$$

where $\gamma = r\Gamma$ and P_{ij}^* is the minimum power for user i in slot j. In the second scheme, the minimum power levels during the respective slots are

$$P_{11}^* = \gamma\eta_o \tag{3.10}$$

$$P_{21}^* = P_{31}^* = 0 \tag{3.11}$$

$$P_{12}^* = 0 \tag{3.12}$$

$$P_{22}^* + P_{32}^* = \frac{\gamma\eta_o W^2 + \gamma\beta\eta_o W^2 + 2\gamma^2\beta\eta_o W}{W^2 - \gamma^2\beta} \tag{3.13}$$

The average power consumption of each of the user i, $\bar{P}_i^*$ over the frame in the either schemes is then computed as

$$\bar{P}_i^* = \frac{1}{2}\sum_{j=1}^{2} P_{ij}^* \qquad\qquad i = 1, 2, 3 \tag{3.14}$$

The average power consumed by all users in scheme 1, $\bar{P}_{s_1}$, and scheme 2, $\bar{P}_{s_2}$ are then just the summation of the $\bar{P}_i^*$ over i

$$\bar{P}_{s_1} = \frac{\beta\gamma\eta_o}{2} + \frac{\gamma\eta_o W}{W-\gamma} \tag{3.15}$$

$$\bar{P}_{s_2} = \frac{\gamma\eta_o}{2} + \frac{\gamma\eta_o W^2 + \gamma\beta\eta_o W^2 + 2\gamma^2\beta\eta_o W}{2(W^2 - \gamma^2\beta)} \tag{3.16}$$

Comparing $\bar{P}_{s_1}$ and $\bar{P}_{s_2}$, it can be shown that scheme 1 consumes less average power than scheme 2 if and only if $\beta > 1$. Conversely, for $0 < \beta < 1$ scheme 2 is more efficient.

Now let us assume that $\beta > 1$ and compare the minimum slotted assignment, i.e. scheme 1, with the unslotted transmissions of scheme 3. Using the results of (2.17), we see that

$$\bar{P}_{s_3} = \frac{\eta_o W \left(\left(\frac{\gamma/2}{\gamma/2+W} \right) + \left(\frac{\gamma/2}{\gamma/2+W} \right) + \left(\frac{\beta\gamma/2}{\beta\gamma/2+W} \right) \right)}{1 - \frac{\gamma/2}{\gamma/2+W} - \frac{\gamma/2}{\gamma/2+W} - \frac{\beta\gamma/2}{\beta\gamma/2+W}} \tag{3.17}$$

This comparison attempts to quantify when $\bar{P}_{s_1} > \bar{P}_{s_3}$. After some algebraic manipulation we can show that for $\bar{P}_{s_1} > \bar{P}_{s_3}$, we must have

$$\frac{2\gamma W + \beta\gamma(W-\gamma)}{2(W-\gamma)} > \frac{W(2\gamma(\beta\gamma + 2W) + \beta\gamma(\gamma + 2W))}{4W^2 - 2\beta\gamma^2 - 2\gamma W} \tag{3.18}$$

This can be further simplified to

$$\beta^2(2\gamma^2 - 2\gamma W) + \beta(4\gamma W - 8W^2) + 4W^2 > 0 \tag{3.19}$$

Let $g(\beta) = \beta^2(2\gamma^2 - 2\gamma W) + \beta(4\gamma W - 8W^2) + 4W^2$. The roots of $g(\beta)$ are

$$\beta_{1,2} = \frac{4W^2 - 2\gamma W \pm 2W\sqrt{4W^2 - 2\gamma W - \gamma^2}}{2\gamma(\gamma - W)} \tag{3.20}$$

Of the two roots indicated above, it can be shown that only one is positive. More specifically,

$$\beta_1 = \frac{4W^2 - 2\gamma W + 2W\sqrt{4W^2 - 2\gamma W - \gamma^2}}{2\gamma(\gamma - W)} > 0 \tag{3.21}$$

Now we observe that $g(\beta)$ has one negative and one positive root and $g(0) = 4W^2 > 0$. Thus, $g(\beta) > 0$ for positive β, implies that $\bar{P}_{s_1} > \bar{P}_{s_3}$ if and only if $0 < \beta \leq \beta_1$. Now we briefly examine β_1; in particular, we determine if and when β_1 takes on values greater than 1. Solving thru $\beta_1 > 1$ results in the following inequality:

$$2W^3 - 3W^2\gamma + \gamma^3 < 0 \tag{3.22}$$

Setting $h(\gamma) = 2W^3 - 3W^2\gamma + \gamma^3$, we observe that $h(\gamma)$ has a root at $\gamma = -2W$ and a double root at $\gamma = W$. Since, $h(0) = 2W^3 > 0$, we see that $h(\gamma) < 0$ for $\gamma < -2W$. Since $\gamma < -2W$ is impossible, we see that β_1 is not greater than 1. We were originally comparing these two schemes assuming $\beta > 1$. Consequently, we see that scheme 1 is minimum to scheme 3 regardless of the value of β.

Please note, however, that the feasibility of scheme 3 requires that the values of β that we can compare must be such that

$$\beta < \frac{W(2W-\gamma)}{\gamma^2} = \beta_{m3} \tag{3.23}$$

This is obtained by requiring that denominator of $\bar{P}_{s_3}$ is positive. In other words only these values (3.23) of β will ensure a positive power solution for scheme 3. The limitations for scheme 1 in terms of values of β lie in the minimum processing gain as discussed before, i.e. $\beta < \frac{W}{2\gamma} = \beta_{m1}$. Quickly, comparing β_{m1} and β_{m3}, we see that $\beta_{m3} > \beta_{m1}$. So the result of comparing schemes 1 and 3 is that scheme 1 minimizes power consumption for $1 \leq \beta < \beta_{m1}$ and scheme 3 is "optimal" for $\beta_{m1} \leq \beta < \beta_{m3}$ by default due to the infeasibility of scheme 1.

Finally consider the comparison between scheme 2 and scheme 3 with the assumption that $\beta < 1$. Once again we begin by calculating the conditions when $\bar{P}_{s_2} > \bar{P}_{s_3}$. After some simplification, $\bar{P}_{s_2} > \bar{P}_{s_3}$ when

$$\frac{\gamma(2W^2 - \gamma^2\beta + \beta W^2 + 2W\beta\gamma)}{2(W^2 - \gamma^2\beta)} > \frac{\gamma W(3\beta\gamma + 4W + 2W\gamma)}{2(2W^2 - \gamma W - \gamma^2\beta)} \tag{3.24}$$

Reducing this further we get

$$W\beta^2\gamma^4 + W^2\beta^2\gamma^3 + W\beta\gamma^4 + \beta^2\gamma^5 - 2W^3\gamma^2 - 2W^2\gamma^3\beta > 0 \tag{3.25}$$

Let $g(\beta)$ equal the expression on the left-hand side. The roots of $g(\beta)$ are

$$\beta_{1,2} = \frac{2\gamma W - \gamma^2 \pm W\sqrt{\gamma^4 + 4\gamma^3 W + 12\gamma^2 W^2 + 8\gamma W^3}}{2(\gamma^3 + \gamma^2 w + \gamma w^2)} \tag{3.26}$$

Analysis of $\beta_{1,2}$ for varying values of γ and W indicates only one positive root, namely

$$\beta_1 = \frac{2\gamma W - \gamma^2 + W\sqrt{\gamma^4 + 4\gamma^3 W + 12\gamma^2 W^2 + 8\gamma W^3}}{2(\gamma^3 + \gamma^2 w + \gamma w^2)} > 0 \tag{3.27}$$

Observe that for $\gamma = W$, this root is $\beta_1 = 1$. This means that for $\gamma = W$, $\bar{P}_{s_2} > \bar{P}_{s_3}$ if and only $\beta \geq 1$. Our initial assumption, however, was that $\beta < 1$. In the case of $\gamma = W$, we can state that the feasible, scheduled transmission of scheme 2 consumes less power than the unslotted scenario of scheme 3. Now for $\gamma = 0$, we see that the root above, β_1, approaches infinity. In fact, for a fixed W, the root β_1 is given by the

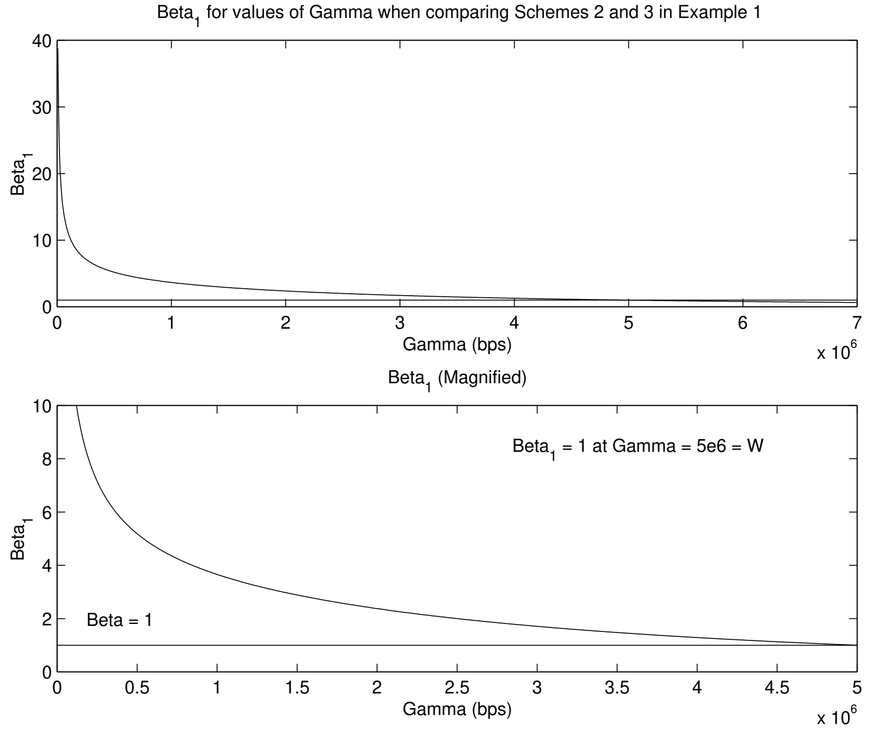

Figure 3.2: Top: β_1 vs. γ when comparing schemes 2 and 3 from Example 1. Bottom: Same curve as above but for smaller range of β_1 and γ.

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curve in Figure 3.2. From this curve, we observe that the values of β_1 for $\gamma < W$ are all greater than 1. Thus, for $\gamma < W$, $\bar{P}_{s_2} > \bar{P}_{s_3}$ if and only if $\beta > \beta_1$ where $\beta_1 > 1$. We can therefore conclude that a feasible implementation of scheme 2 consumes less power than scheme 3.

Now we determine the values of β for which scheme 2 is in fact feasible. To obtain position power allocations for the users in scheme to, we require that denominator in 3.16 be positive. Thus we want $\beta < W^2/\gamma^2$. Let $\beta_{m2} = W^2/\gamma^2$. The feasibility of scheme 2 was stated earlier as $\beta < \beta_{m1}$. A quick comparison shows that $\beta_{m2} < \beta_{m3}$ and $\beta_{m1} < \beta_{m2}$.

Let us approach the system feasibility from a different point of view. We begin with the assumption that the system bandwidth is large enough to support the maximum rate user under the slotted scheme. In other words, β and γ are such that the slotted scheme can easily accommodate these users in a slot. The feasibility of scheme 2 is

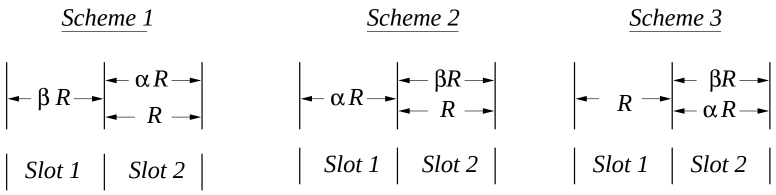

Figure 3.3: Three possible ways that three users may transmit with two available slots

simply that $\gamma < \frac{W}{\sqrt{\beta}} = \gamma_2$. Scheme 3 on the other hand is feasible if and only if

$$\gamma < \frac{W}{2\beta}(\sqrt{1+8\beta} - 1) = \gamma_3 \tag{3.28}$$

Comparing γ_2 and γ_3, we see that $\gamma_2 > \gamma_3$ for $\beta < 1$. Similarly scheme 1 is feasible for $\gamma < W = \gamma_1$. Comparing γ_1 and γ_3, we get $\gamma_1 > \gamma_3$ if $\beta > 1$. These two comparisons state that if the users are slotted in an intelligent manner (i.e., with average power reduction), then the range of γ's over which the slotted scheme remains feasible is larger than that for the standard, unslotted multirate system. This supports the assertion made at the end of Chapter 2.

Example 2: 3 Different Users

This example surveys the case in which the rate requirements of each of the three users is different for $M = 2$, $N = 3$. Again the assumption is that the SIR requirements are the same for each user. In this example we only focus on the minimum assignment for the slotted system. Figure 3.3 depicts the three possible slot assignments.

Using analysis as in Example 1, we see that scheme 1 requires less average power then scheme 2 if

$$\frac{\beta\gamma\eta_o}{2} + \frac{2\gamma^2\alpha\eta_o W + \gamma\eta_o W^2 + \alpha\gamma\eta_o W^2}{2(W^2 - \alpha\gamma^2)} < \frac{\alpha\gamma\eta_o}{2} + \frac{2\gamma^2\beta\eta_o W + \gamma\eta_o W^2 + \beta\gamma\eta_o W^2}{2(W^2 - \beta\gamma^2)} \tag{3.29}$$

It can be shown via straight forward algebra that (3.29) is true if $\alpha < \beta$. Next, we compare assignments 1 and 3. Once again scheme 1 consumes less average power over the frame than scheme 3 if

$$\frac{\beta\gamma\eta_o}{2}+\frac{2\gamma^2\alpha\eta_o W+\gamma\eta_o W^2+\alpha\gamma\eta_o W^2}{2(W^2-\alpha\gamma^2)}<\frac{\gamma\eta_o}{2}+\frac{2\gamma^2\beta\alpha\eta_o W+\alpha\gamma\eta_o W^2+\beta\gamma\eta_o W^2}{2(W^2-\alpha\beta\gamma^2)} \tag{3.30}$$

Again it can be shown that the statement in (3.30) is true if $\beta > 1$. Thus combining this result with that for the scheme 1 and scheme 2 comparison, we see that scheme 1 minimizes the sum of average powers if and only if $\beta > 1$ and $\alpha < \beta$.

The result of these two examples indicates that to minimize the total average power consumption over a frame, the slot assignments should be such that the highest rate user transmits by itself in a slot and the other two users share the remaining slot. Simply put, we should isolate the high rate user into its own slot. Also note that if $\alpha = \beta = 1$, the three schemes would be identical and the average power consumed in each of the cases would be the same.

3.3 General Problem

Let us now turn to solving the discrete assignment problem for a general $M < N$. Due to the one-slot-per-user limitation, we see that P_{ij} is non-zero for only one j for each user i. This requirement can be stated as

$$P_{ij}^* = \frac{\frac{\delta_{ij}\gamma_i}{\delta_{ij}\gamma_i+W}}{1-\sum_{k=1}^{N}\frac{\delta_{kj}\gamma_k}{\delta_{kj}\gamma_k+W}} \tag{3.31}$$

where δ_{ij} is an indicator function which equals 1 if user i is assigned to transmit in slot j and 0 otherwise. Note that $\gamma_i = \Gamma r_i$ where r_i is the rate requirement *over a slot* (and in this case over a frame as well) and Γ is the minimum SIR requirement for that user's data stream. Observe that in the standard unslotted scheme, user i's rate requirement would be r_i/M in each slot. The minimum sum of average powers for the entire frame can then be found as the solution to the following

$$\min_{\delta}\frac{1}{M}\sum_{j=1}^{M}\frac{\eta_o W\sum_{i=1}^{N}\frac{\delta_{ij}\gamma_i}{\delta_{ij}\gamma_i+W}}{1-\sum_{k=1}^{N}\frac{\delta_{kj}\gamma_k}{\delta_{kj}\gamma_k+W}} \tag{3.32}$$

$$\text{s.t.} \qquad \sum_{j=1}^{M}\delta_{ij}=1 \qquad \forall i=1,2,\ldots N \tag{3.33}$$

In fact we may further simplify the notations in this problem by realizing that if $\delta_{ij} = 1$ for a particular slot, that user in that slot contributes $\frac{\gamma_i}{\gamma_i+W}$ in that slot and zero otherwise. Thus we can write the problem statement above as

$$\min_{\delta} \frac{1}{M} \sum_{j=1}^{M} \sum_{i=1}^{N} \frac{\eta_o W \frac{\delta_{ij}\gamma_i}{\gamma_i+W}}{1 - \sum_{k=1}^{N} \frac{\delta_{kj}\gamma_k}{\gamma_k+W}} \tag{3.34}$$

$$\text{s.t.} \quad \sum_{j=1}^{M} \delta_{ij} = 1 \qquad \forall i = 1, 2, \ldots N \tag{3.35}$$

3.3.1 Single Rate System

In a single rate system, each user has the same desired rate requirement, γ. We can therefore reduce the problem statement above to

$$\min_{\delta} \frac{1}{M} \sum_{j=1}^{M} \frac{\frac{\eta_o W\gamma}{\gamma+W} \sum_{i=1}^{N} \delta_{ij}}{1 - \frac{\gamma}{\gamma+W} \sum_{i=1}^{N} \delta_{ij}} = \min_{\delta} \frac{1}{M} \sum_{j=1}^{M} \frac{\eta_o W\gamma \sum_{i=1}^{N} \delta_{ij}}{\gamma + W - \gamma \sum_{i=1}^{N} \delta_{ij}} \tag{3.36}$$

$$\text{s.t.} \quad \sum_{j=1}^{M} \delta_{ij} = 1 \tag{3.37}$$

We see that $\sum_{i=1}^{N} \delta_{ij}$ now represents the number of single rate users assigned to slot j. Let $N_j = \sum_{i=1}^{N} \delta_{ij}$, then constraint 3.37 can be rewritten as $\sum_{i=1}^{M} N_j = N$, giving us the new single-rate system problem statement

$$\min_{mbN} f(\mathbf{N}) = \frac{\eta_o}{M} \sum_{j=1}^{M} \frac{N_j}{1 - N_j} \tag{3.38}$$

$$\text{s.t.} \quad g(\mathbf{N}) = \sum_{j=1}^{M} N_j = N \tag{3.39}$$

This too is a discrete optimization problem, where each N_j must take on an integral value. To understand the mechanics of this problem, we begin by assuming that N is divisible by M. Under this assumption, we now try to the solve the relaxed version of this problem, i.e. with the integer solution assumption relaxed. In other words, we assume that these N_j's can take on real values. This second assumption now allows us to use techniques from constraint minimization of continuous functions to derive an optimal solution. Observe that the discrete solution to the problem above is a subset of the solutions described by this continuous case. Using the Lagrange multiplier, λ, for the linear constraint in (3.39), we get

$$\frac{1}{(1-N_j)^2} = \lambda \qquad \forall j = 1, 2, \dots M \tag{3.40}$$

Equation (3.40) says that all N_j's are equal. This taken in conjunction with the linear constraint above implies that $N_j = N/M$, $\forall j$. Since we started out with the assumption that N and M are divisible, we can easily deduce that this would be the solution for the discrete optimization problem as well, since the N_j's are integral in this solution.

In the case that N and M are not divisible, we make the argument that $N\lfloor \frac{N}{M} \rfloor$ of these identical users should be assigned to each slot. The remaining users can each then be placed arbitrarily in any of the M slots so that there are $N - NM\lfloor \frac{N}{M} \rfloor$ slots with $N\lfloor \frac{N}{M} \rfloor + 1$ simultaneous transmitters and the remaining with $N\lfloor \frac{N}{M} \rfloor$ simultaneous users. To show validation for this argument, we first observe that if users are not assigned to slots as in the manner prescribed above, there will be at least one pair of slots, say slots j_1 and j_2 such that $N_{j_1} - N_{j_2} > 1$. In that case, the average power consumed by these two slots will be

$$P_{\text{old}} = \frac{W\eta_o}{2}\left(\frac{N_{j_1}}{1+\frac{W}{\gamma}-N_{j_1}} + \frac{N_{j_2}}{+\frac{W}{\gamma}-N_{j_2}}\right) \tag{3.41}$$

Let $S = N_{j_1} + N_{j_2}$. Now fix S and redistribute the number of users in the two slots so that there are now N'_{j_1} and N'_{j_2} users in slots j_1 and j_2 respectively. Once again, since S is fixed, $N'_{j_1} + N'_{j_2} = S$. The power consumed with this distribution of users is then

$$P_{\text{new}} = \frac{W\eta_o}{2}\left(\frac{N'_{j_1}}{1+\frac{W}{\gamma}-N'_{j_1}} + \frac{N'_{j_2}}{1+\frac{W}{\gamma}-N'_{j_2}}\right) \tag{3.42}$$

We can now determine when the change in power, $\Delta = P_{\text{old}} - P_{\text{new}}$ is greater than zero. With some algebra, we observe that $\Delta > 0$ if and only if $N'_{j_1}N'_{j_2} > N_{j_1}N_{j_2}$. We can maximize the product $N'_{j_1}N'_{j_2}$ under the constraint $N'_{j_1} + N'_{j_2} = S$. The result for S even is $N'_{j_1} = N'_{j_2} = S/2$ and for S odd is $|N'_{j_1} - N'_{j_2}| = 1$. Thus for the two slots, power consumption is minimized if the number of users in each slots is as balanced or equal as possible. Comparing the number of users in each of the M slots two slots at a time, we can then extend this result over all M slots. In other words, the number of users in each of the M slots should be as equal or balanced as possible. The result then is that for N not divisible by M, the difference in the number of users in each slot is either zero or one.

3.3.2 Multirate Problem

In trying to solve the discrete assignment problem, we employ a process that effectively attempts to minimize each of the following terms in the cost function above

$$\frac{\eta_o W \sum_{i=1}^{N} \frac{\delta_{ij}\gamma_i}{\gamma_i+W}}{1-\sum_{k=1}^{N} \frac{\delta_{kj}\gamma_k}{\gamma_k+W}} \tag{3.43}$$

The discussion here takes an arbitrary user-to-slot assignment and improves it power performance by a process of pairwise-load-balancing. Though we are not able to prove the global optimality of this process, we are able to show that due to its inherent properties, it achieves pairwise load-balancing among the M slots while reducing total power. In other words, the resulting assignment scheme equalizes the total sum of rates carried in pairs of slots for all pair. For simplification, let $a_j = \sum_{i=1}^{N} \frac{\delta_{ij}\gamma_i}{\gamma_i+W}$. The cost function in terms of these a's, $f(\mathbf{a})$, is then

$$f(\mathbf{a}) = \frac{\eta_o W}{M} \sum_{j=1}^{M} \frac{a_j}{1-a_j} \tag{3.44}$$

Let us now specifically state what we mean by pairwise load-balancing. If we take a pair of slots, say slots j_1 and j_2 and, with the given set of users in these two slots, interchange the user-to-slot assignments (i.e., swap users between the two slots) so that $|a_{j_1} - a_{j_2}|$ is minimum, then we have pairwise load-balanced the two given slots.

In proving the load-balancing power reduction, we employ a result similar to the one shown for the single-rate user scenario when N was not divisible by M. Though the following theorem and proof are presented as a procedural mechanism that reduces power consumption via pairwise-load-balancing, the actual process of balancing loads between two slots over all pairs of slots turns out to be complex. The discussion of this complexity is left for later.

Theorem 1

For an arbitrary slot assignment $\{a_1, a_2, \ldots a_M\}$ redistributing users to slots so as to achieve pairwise load-balancing reduces the cost function $f(\mathbf{a})$.

Proof:

We begin this proof by extending the discussion in the previous section concerning load balancing between two slots with identical users to this multirate case. We are given an arbitrary assignment of users to slot so that the total load in slot 1 is a_1, slot 2 is a_2, and so on. Let us examine any two slots in the frame, say slot j_1 and j_2. We are therefore given the subset of the N system users assigned to these two slots. In these two slots, the given arbitrary assignment implies the following average power over these slots is

$$P_{\text{old}} = \frac{\eta_o W}{2} \left(\frac{a_{j_1}}{1 - a_{j_1}} + \frac{a_{j_2}}{1 - a_{j_2}} \right) \tag{3.45}$$

Now let us assume that the given subset of users in slots j_1 and j_2 are swapped between the two slots so that the load distribution is now a'_{j_1} and a'_{j_2}. The average power over the two slots under this new distribution is

$$P_{\text{new}} = \frac{\eta_o W}{2} \left(\frac{a'_{j1}}{1 - a^1_{j_1}} + \frac{a'_{j_2}}{1 - a'_{j_2}} \right) \tag{3.46}$$

Let $s = a_{j_1} + a_{j_2} = a'_{j_1} + a'_{j_2}$, i.e. the total load in the two slots is fixed as s. For this constraint, we once again note that $\Delta = P_{\text{old}} - P_{\text{new}} > 0$ if and only if $a'_{j_1} a'_{j_2} > a_{j_1} a_{j_2}$. For $a'_{j_2} = s - a'_{j_1}$, we wish to determine when $a'_{j_1}(s - a'_{j_1})$ is maximum. The plot of this product is shown in Figure 3.4. Observe that the maximum occurs at $a'_{j_1} = s/2$. Also note that all the discrete assignments of the given subset of users in the two slots are merely a collection of finite points along this curve. We thus see that the optimal distribution of users between the two slots is one in which the loads in the two slots are as equal (close to $s/2$) as possible. Another way to state this for the discrete assignments here, is that the optimal distribution of the given set of users over the two slots is one in which $|a'_{j_1} - a'_{j_2}|$ is minimum. There is therefore a power reduction in this pairwise load-balancing. We now note that performing this pairwise load-balancing over all pairs

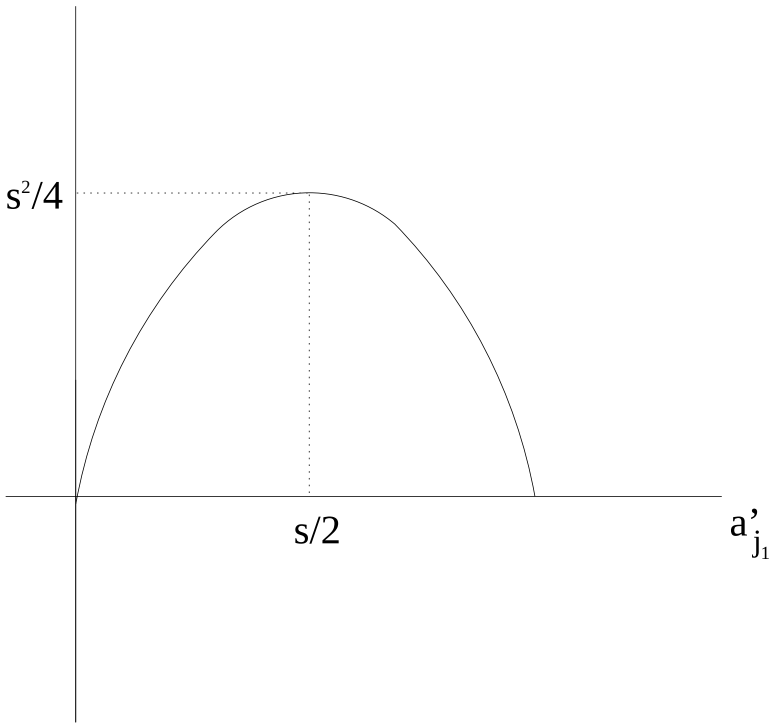

Figure 3.4: Plot of $a'_{j_1}(s - a'_{j_1})$.

of slots in the M slots then reduces total power, i.e. reduces the overall cost function $f(\mathbf{a})$.

Pairwise load-balancing over all slots can be repeatedly performed until we cannot pairwise improve $|a'_{j_1} - a'_{j_2}|$ for any two slots j_1 and j_2. This repeated implementation eventually converges to a fixed point. The resulting power distribution is a *local* not global minimum to the discrete assignment under study here. To show the local optimality of the pairwise load-balancing scheme we employ a specific example. Suppose that for a three slot system with six users, the load requirements for each user are: 0.70, 0.41, 0.38, 0.33, 0.30, 0.299. Note that the load requirement is the ratio: $\gamma/(\gamma + W)$. Let us implement pairwise load-balancing on these users so that the fixed point results as in the left part of Figure 3.5. Observe that we cannot swap any set of users between any two slots and equalize the total loads in the slots further. For this pairwise load-balanced solution, we see that $a'_1 = 0.999$ and $a'_2 = a'_3 = 0.71$. For this three slot system with the given rate requirements, we also propose a second distribution of users to slots as indicated in the right half of Figure 3.5. In this case, $a^*_1 = 0.70$,

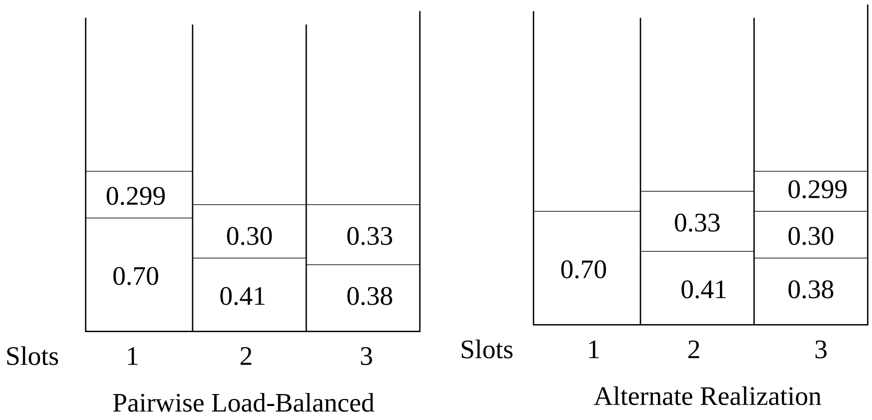

Figure 3.5: User-to-Slot assignments for given example.

$a_2^* = 0.74$, and $a_3^* = 0.979$. We now evaluate $f(\mathbf{a}')$ and $f(\mathbf{a}^*)$. The result is, $f(\mathbf{a}') = 334.6322$ $(\eta_o W)$ and $f(\mathbf{a}^*) = 17.2662$ $(\eta_o W)$. The pairwise load-balancing convergence point is not the global minimum in this case. We can therefore conclude that the fixed point solution of the pairiwse load-balancing procedure leads to a local minimum for the discrete assignment case.

3.3.3 Complexity of Pairwise Load-Balancing Assignment

Let us briefly discuss the complexity of the pairwise load-balancing algorithm. If there are K users between a pair of slots, the complexity of determining the load balanced solution is of order $O(K^2)$ [14], [15]. Note that in a worst-case scenario, the intial distribution of users maybe that all N system users are assigned between a pair of slots. Depending on the given arbitrary user distribution, it may however require only one pairwise load-balancing to achieve an over all pairwise load-balanced assignment. Regardless, to determine this we would need to go over all pairs of slots at least once. Since there are M choose 2 pairs of slots, we would need at the minimum M choose 2 times the N^2 complexity of a singular pair load-balancing mechanism. Furthermore, we may have to repeatedly cycle through all the pairs of slots a multiple number of times. To set an upperbound on the number of times we may have to cycle through all the pairs of slots, we define first an ordered set $d = \{d_{11}, d_{12}, d_{13}..., d_{1M}, d_{23}, d_{24}..., d_{(M-1)M}\}$. The elements of this define a pair of slots over which one may attempt load-balancing.

There are, as noted above,, M choose 2 members in this set. Let D be the number of permutations of the set d. It is, therefore, possible that we may have to cycle through the pairs of users D times. The result therefore is an alogrithm that is implementationally complex realistic systems in which the number of users can be of order 10^2 with 5 or more slots. For this reason, we look towards suboptimal schemes as a means to approximate the pairwise load-balancing heuristic.

3.4 Possible Sub-Optimal Schemes

Due to the complexity of the pairwise load-balancing scheduler, we can devise alternate methods for assigning each user to one and only one slot. First we look at random schedulers. The first one places each user into one of the M slots with equal probability. For such a scheduler we can compute the average power spent over a frame as, $\bar{P}_{rand}$

$$\bar{P}_{rand} = \frac{(M-1)^{N-1}}{M^N}\left(\sum_{i=1}^{N}\frac{\beta_i}{1-\beta_i}\right) + \frac{(M-1)^{N-2}}{M^N}\left(\sum_{i=1}^{N}\sum_{k\neq i}\frac{\beta_i+\beta_k}{1-\beta_i-\beta_k}\right) \quad (3.47)$$

$$+\frac{(M-1)^{N-3}}{M^N}\left(\sum_{i=1}^{N}\sum_{k\neq i}\sum_{n\neq k}\frac{\beta_i+\beta_k+\beta_n}{1-\beta_i-\beta_k-\beta_n}\right) + \ldots \quad (3.48)$$

$$\frac{1}{M^N}\left(\frac{\sum_{i=1}^{N}\beta_i}{1-\sum_{i=1}^{N}}\right) \quad (3.49)$$

where $\beta_i = \frac{\gamma_i}{\gamma_i+W}$. Note, however, that this random allocation of users to slots (which we will refer to as random *slot* assignment) can lead to allocations which potentially leave empty slots. This is both bandwidth and (in the case of $M < N$) power inefficient. Thus as a second random scheme, we look a scheduler that effectively employs random "water-filling" in the number of interferers per slot. This scheduler places $\lfloor \frac{N}{M} \rfloor$ users in each slot. These users are selected randomly from the set of N users. The remaining users are then assigned to slots in a random manner. For this scheme (referred to as the random *user* assignment), we have produced only experimental results presented a bit later.

The other set of suboptimal schemes are those that attempt to equalize the load in each slot (thereby approximating the load-balancing algorithm). One such scheme is to assign each system user one at a time to the least lightly loaded slot. Another version of this is to first order the system users in terms of their desired rates and then assign users to the slot with the minimum load. Yet another is described by the following "zig-zag" strategy:

1. For the N system users with rate requirements γ_i, sort these rates in increasing order.

2. Label the slots 1 through M. Set counter $c = 0$.

3. Place the M highest users starting from slot 1 to slot M. Increment counter,c.

4. If $c = \lfloor \frac{N}{M} \rfloor$, set $d = 1$ and then skip to step 6. Otherwise, place the next M sorted users in slots starting from slot M to slot 1. Increment counter c.

5. If $c = \lfloor \frac{N}{M} \rfloor$, set $d = 0$ and skip to step 6. Otherwise, place the next M sorted users starting from slot 1 through M. Increment counter c. Goto step 4.

6. If $d = 1$, then place the last remaining, sorted users in slots starting from slot M down. If $d = 0$, place the remaining users in slots starting from slot 1 up.

The effect of this "zig-zag" placement is shown in Figure 3.6. Also note that the three slot example studied in the previous section also proves that the zig-zag scheme does not converge to the global minimum. The pairwise load-balanced fixed point distribution shown in that example is also the zig-zag distribution for the given users. The pairwise load-balanced distribution given in the example is also the sorted minimum slot assignment for these users as well.

Before we begin to look at the system performance of the optimal and these suboptimal schemes, we observe that for identical rate users, i.e. the single-rate system, the pairwise load-balanced, the random user assignment, the "zig-zag", the sorted and on-line minimum slot assignment schemes all produce identical total-average power consumption over the frame.

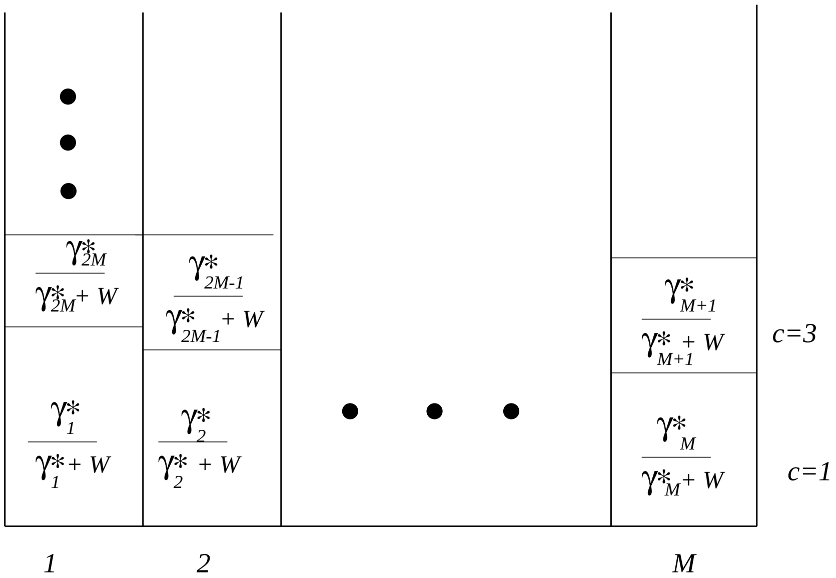

Figure 3.6: Depiction of the Zig-Zag Scheme. γ^* = Sorted γ requirement.

3.5 Performance Analysis

In this chapter we posed a few scheduling techniques that adhere to the discrete assignment policy: M-fold minimum bin-packing technique, the random slot assignment, the random user assignment, the "zig-zag" approach, and finally the sorted and on-line versions of the minimum slot assignment scheme. In this section, we now apply these techniques to an experimental system and determine their merits.

3.5.1 Smaller Systems

First, we begin with smaller systems because of our limitation in computing the pairwise load-balanced assignment scheme for complex scenarios. In these systems, we begin by comparing all proposed schemes to the standard CDMA assignment. The system bandwidth in this case is 10 MHz and user rates are chosen at random from the following rate vector: $\mathbf{R}$ = [9.6 14.4 28.8 36.6 54 128 144 256 360] Kbps and each user's desired SIR level is 6db. The time frame is divided into 3 slots and we analyze this small system for N=4 to N=10. The top graph in Figure 3.7 depicts the ratio of the average power consumed by the applied scheme to the average power consumed under standard, unslotted CDMA. The exact rate requirement for this particular experiment was: [28.8 144 360 9.6 256 28.8 360 14.4 14.4 9.6] Kbps. We immediately see that the random slot assignment case really suffers in terms of any potential power benefits. This, as we indicated earlier, has to do with inefficient assignments that leave slots empty or heavily slot one slot while the remaining slots carry very few low rate users. Due to its poor performance in a such a basic, small system, we now preclude the random slot assignment from further analysis. In Figure 3.7, we look at the ratios of the three other assignment schemes when compared to standard CDMA. The power levels for the random user assignment were accrued by averaging fifty slot assignments over the given set of rate requirements. We see that for this system, even the performance of the random user to slot assignment betters the power consumption of standard CDMA. The zig-zag scheme, which effectively assigns the same number of user per slot as the random scheme, improves power consumption when compared to the random scheme because it incorporates some information about user rate requirements into the slot assignment. The best power consumption over these schemes is observed under pairwise load-balanced assignment. This is essentially due to the load-balancing process described earlier in the chapter. One could treat any of these other assignment schemes as arbitrary given slot assignments. Their power performance can then be improved by performing the swaps indicated in the proof of Theorem 1.

Note that though the systems in Figure 3.7 indicate power savings compared to standard CDMA, the actual portion of savings is really not all that substantial. This savings increases if the rate requirements and number of slots are held the same and the total available bandwidth reduced. The results for this are in Figure 3.8. The top figure is for a system with $W = 5$MHz while the bottom is for a system with $W = 2$MHz. Note that we hold the user rate requirements over a slot fixed in each of these systems.

3.5.2 Larger Systems

Now we move onto larger systems, i.e. systems with significantly more users. Here we analyze two different types of user requirements: identical and multirate. As we

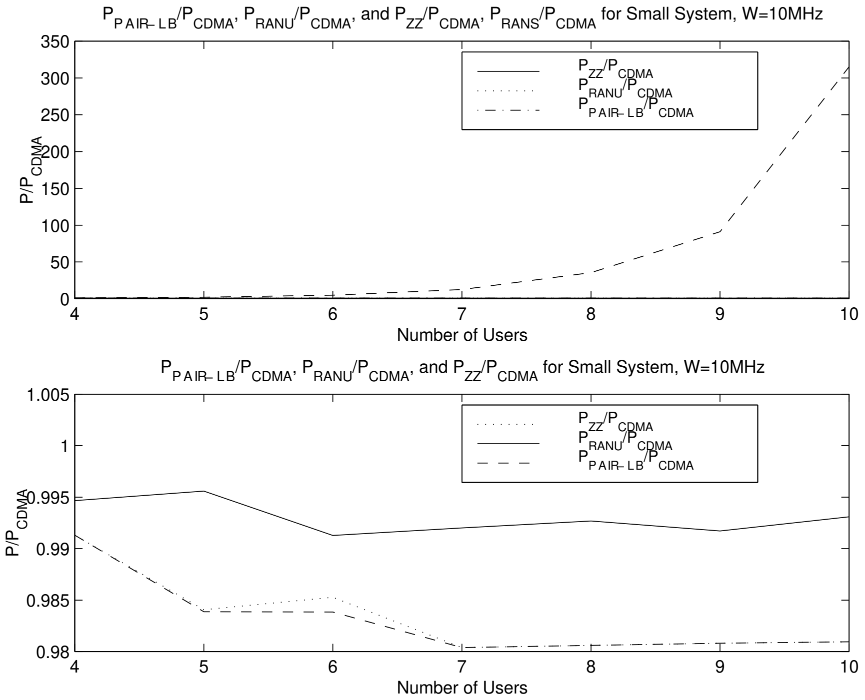

Figure 3.7: Top: Relative average power performance of the four schemes studied for a small system with W=10Mhz. Note that due to the high power levels of the random slot assignment case, the curves of the other systems are "hidden" as they produce ratios less than 1. Bottom: relative average power performance of the Pairwise Load-Balanced, Zig-Zag, and Random User Schemes for a small system with W=10Mhz.

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indicated earlier, when the system users are all identical (single-rate requirement), the five schemes (pairwise load-balanced, random users, zig-zag, minimum slot, and on-line minimum slot assignments) all collapse into a singular assignment scheme in terms of power consumption. Under such conditions, we perform average power calculations for two types of schemes. In both these scenarios we assume a system bandwidth of 10MHz, user rates of 64Kbps, and required SIR of 6 db. The difference lies in the number of slots M. In one we assume $M = 3$ and in the second $M = 5$. The result of these two experiments are indicated in the graphs of Figure 3.9. In both the cases we have run the experiment up till the point the system "collapses", i.e. the interference becomes significant enough so that target SIR's are no longer attainable by the users. First we see that the slotted systems are both able to support more users than the standard systems before "collapsing." Specifically, the $M = 3$ system collapses under standard CDMA when $N = 115$ while the slotted system is able to support two additional users before

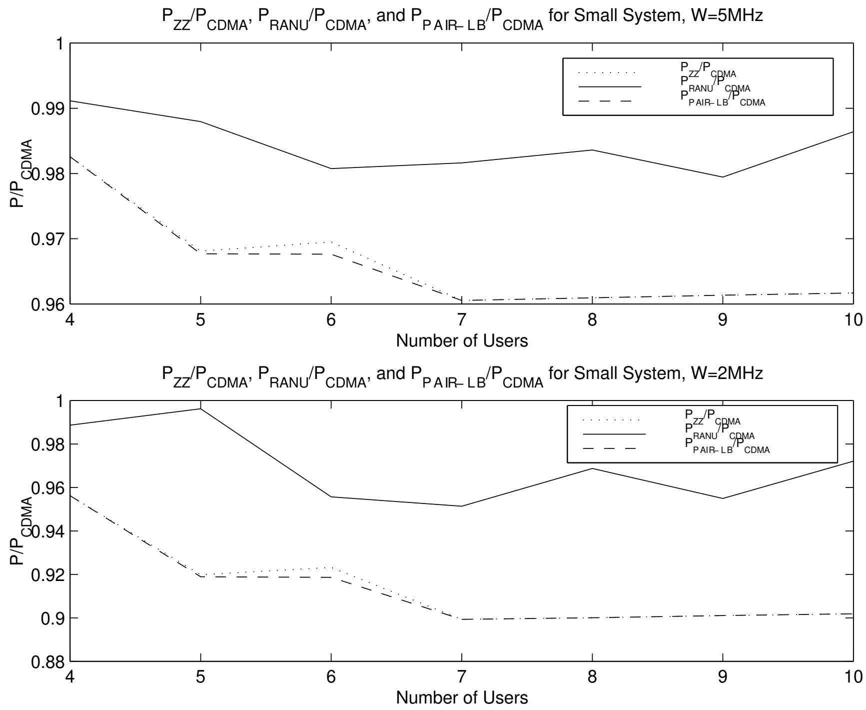

Figure 3.8: Top: Relative average power performance of the Pairwise Load-Balanced, Zig-Zag, and Random User Schemes for a small system with W=5Mhz. Bottom: Relative average power performance of the same schemes in a system with W=2MHz.

becoming infeasible at $N = 117$. For $M = 5$, the slotted system can support 196 users while the standard CDMA system holds $N = 192$. This is just a direct consequence of the capacity of a single-rate CDMA system as studied in [16]. In particular, number of users supported in a single rate CDMA system is

$$N = 1 + \frac{W}{r\Gamma} \tag{3.50}$$

where r is the desired rate of each user and Γ the minimum SIR. Since slotting is equivalent to bandwidth partitioning, we can effectively compare the number of users supported by an unslotted, single-rate system (N_s) to that in an M-part system (N_M) as

$$\begin{aligned} N_S &= 1 + \frac{MW}{r\Gamma} && (3.51) \\ N_M &= M\left(1 + \frac{W}{r\Gamma}\right) && (3.52) \end{aligned}$$

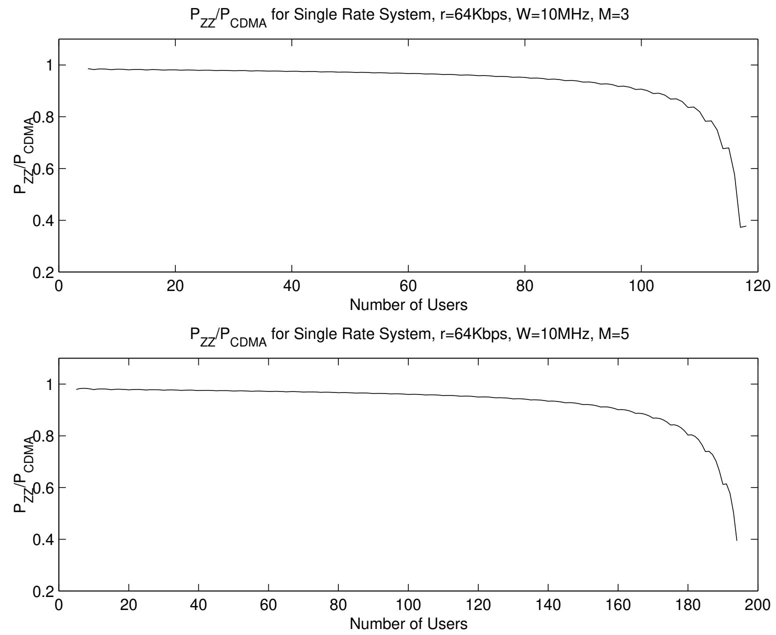

Figure 3.9: Relative Power Performance of Slotted Scheme (Zig-Zag) compared to standard CDMA for large system with identical users demanding rate 64Kbps. The top figure is for a system with $M = 3$ while the bottom is for a system with $M = 5$ slots.

Comparing N_S and N_M, we see that for $N_S - N_M = M - 1$. Note that equations (3.51) and (3.52) ignore background noise, as do the results indicated in Figure 3.9. Now observe that these differences are, practically speaking, trivial. Carrying $M - 1$ extra users does not amount to a significant gain in capacity. Thus both these schemes are comparable in performance at lower loads.

In the top graph in Figure 3.9, we see the ratio between the average power consumed by the slotted system with $M = 3$ versus that of the standard CDMA system. The bottom graph is for the same system but with $M = 5$. The ratios in both cases clearly indicate that for all values of N, the slotted system reduces total average power. The significant power savings, however, are seen at heavy system load. The measurable difference between the two systems in Figure 3.9 lies in the number of users supported. The capacity for both the CDMA and the slotted scheme increased. This, however, resulted from an extension of the time frame. In other words, the difference in the capacity for $M = 3$ and $M = 5$ is due to our definition of the rate requirement. In the two slotted systems, we have maintained that each user will transmit 64Kbps *in a slot.*

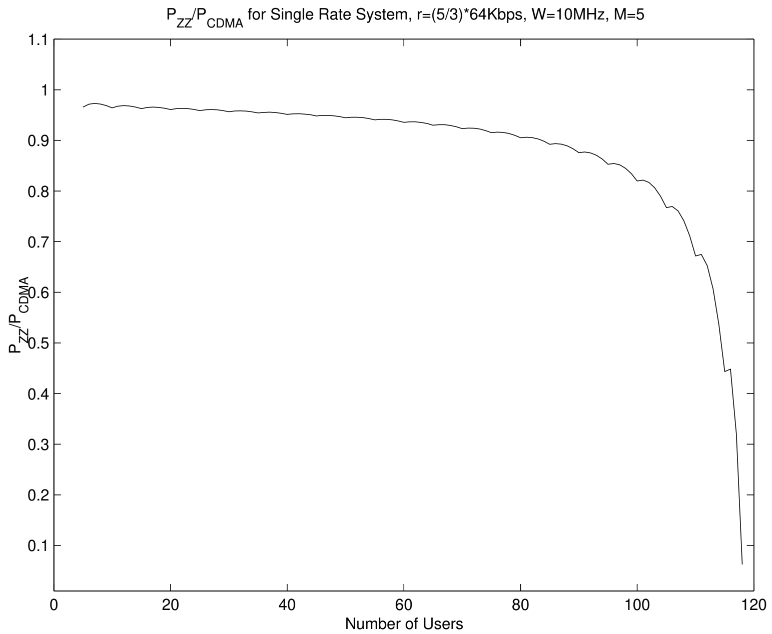

Figure 3.10: Relative power performance of slotted scheme (Zig-Zag) compared to standard CDMA for large system with identical users demanding rate (5/3)64Kbps. This is the result when one fixes the information rate over a fixed frame length.

If, on the other hand, the user rate requirements are kept fixed over a fixed frame length, then the power performance behaves as the curve in Figure 3.10. Here the users rate requirements are increased for the shorter slot lengths, i.e. the rate requirements are scaled up by a factor of 5/3. The result is that the CDMA and slotted scheme capacity for this particular system with $M = 5$ remains as it was for $M = 3$. The difference in the system performance lies in the average power consumption which decreases in going from $M = 3$ to $M = 5$ due to additional orthogonal dimensions.

Another large-scale system analyzed here dealt with multirate users. These users were once again assigned random rate requirements from the vector $\mathbf{R}$ described earlier. Due to the complexity of the pairwise load-balanced scheme, we are unable to study its system performance in this scale. Furthermore, as stated previously, the random slot assignment has extremely poor performance in small systems and we cannot anticipate any improvements as there are more and more users. The complexity of accruing reliable statistics for the random user assignment also makes the analysis of it power consumption difficult in large size systems. For this reason, we have provided only the comparison between the zig-zag scheme and the standard, unslotted CDMA system. Figure 3.11 indicates the ratio (P_{ZZ}/P_{CDMA}) between the average power consumed in the zig-zag scheme and the unslotted multirate CDMA system over a frame. The average power points were calculated by averaging the performance over a hundred random rate assignments. In fact, for this scheme we have also accumulated the number of times that the either the CDMA scheme or the zig-zag scheme was feasible for each of the hundred different rate assignments over the different numbers of system users. The result is provided as the bottom plot in Figure 3.11. The standard CDMA system has N users transmitting in each slot with rate γ_i/M. For each N, we can then treat the total load in each slot as $\sum_{i=1}^{N} \frac{\gamma_i/M}{\gamma_i/M+W}$. Now each $\frac{\gamma_i/M}{\gamma_i/M+W}$ is a random variable since each user's rate requirement is chosen randomly from the rate vector $\mathbf{R}$. Thus the total load in a slot for a CDMA system is a sum of random variables which we can then approximate as a Gaussian random variable with mean and variance as defined

$$\mu = \mathrm{E}\left[\sum_{i=1}^{N} \frac{\gamma_i/M}{\gamma_i/M+W}\right] = N\mathrm{E}\left[\frac{\gamma_i/M}{\gamma_i/M+W}\right] = \frac{N}{L}\sum_{j=1}^{L} \frac{R_j\Gamma}{R_j\Gamma+MW} \tag{3.53}$$

$$\sigma^2 = \frac{N}{L}\sum_{j=1}^{L} \left(\frac{R_j\Gamma}{R_j\Gamma+MW}\right)^2 - \mu^2 \tag{3.54}$$

where Γ is the target SIR, which we assume in this experiment to be 6db, and L is the number of possible rate requirements, which in this case was 9. The probability that any (and thus all) slot is feasible for the standard CDMA system is then

$$p = \mathrm{P}\left[\sum_{i=1}^{N} \frac{\gamma_i}{\gamma_i+MW} < 1\right] = \Phi\left(\frac{1-\mu}{\sigma}\right) \tag{3.55}$$

where Φ is the CDF of the normal distribution. We can then finally calculate the expected number of feasible allocations as Np. The result of this Gaussian approximation for the expected number of feasible assignments is also indicated in the feasiblity plot in Figure 3.11.

In examining the results in Figure 3.11 vis-a-vis those for the identical rate users in Figure 3.9, we see that slotting users in multirate systems has significantly more advantages than slotting users in single rate systems. We noted in the single-rate system that though there were power savings for slotting users, substantial benefits were seen only

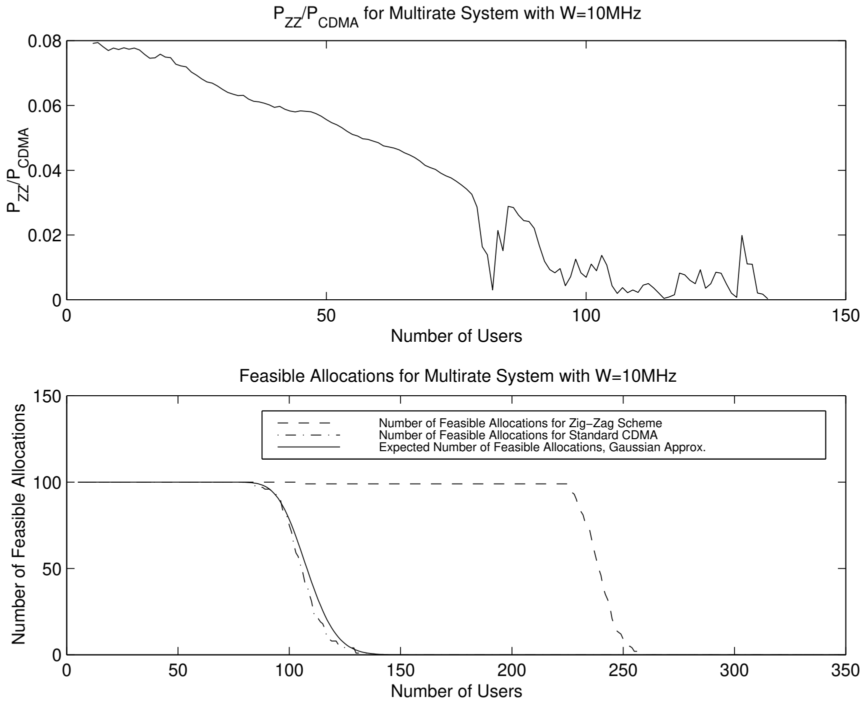

Figure 3.11: Top: Relative power performance of slotted scheme (Zig-Zag) compared to standard CDMA for large system with randomly assigned user rates. Bottom: The number of feasible allocations obtained for the slotted and unslotted schemes over a 100 different random rate requirements.

at unrealistically heavy loads. On the other hand, for multirate systems average power consumption easily reduced by 90scenarios when moving from the standard unslotted system to the slotted one. Another measure of this savings was then an increase in the number of users the system could support.

Chapter 4

Relaxed Assignment Policy

An alternate structure for scheduling multirate CDMA users to time slots is based on a relaxation of the discrete assignment studied in the previous chapter. In other words, this alternate scheduler does not have to adhere to the one-slot-per-user condition and thus may opt to assign a user to multiple slots in a frame. This multi-slot allocation for a particular user is achieved by distributing the transmission of its data stream to potentially many slots in the frame. Due to the difficulty of CDMA acquisition [17], [18], it maybe beneficial for the system to assign such users contiguous slots in a frame. The discussion here, however, is not bound by such a restriction. In analyzing the relaxed assignment problem formulation in this chapter, we first note that there are inherent complications in pin-pointing an optimal minimum power assignment over the frame for this system model. In fact, a close study of the governing equations shows that available optimization techniques produce multiple solutions to the scheduling problem, among them the standard, unslotted CDMA transmission scheme. The aim in this chapter therefore is not to develop an algorithm which produces an optimal user-rate-to-slot assignment but rather to examine possible power reduction techniques in a slotted CDMA system under the guise of a relaxed assignment scheduler.

This effort begins in section 4.1 with the statement of the relaxed assignment problem in terms of the minimum total average power cost function and its accompanying system constraints. Added to the discussion in this section are observations that clearly state the mathematical hurdles in obtaining an optimal scheduling heuristic for this policy. In section 4.2, we describe some interesting analytical (not necessarily optimal) results for this scheduler. Then in section 4.3, we address a relaxed allocation scheme for single rate users that can further improve the average power performance when compared to both the standard, unslotted system and the discrete allocation scheme from the previous chapter. Finally in section 4.4 we present performance results that pertain to the discussion in this chapter.

4.1 Relaxed Problem Statement

Recall from Chapter 3, that the discrete problem statement for a minimum average power scheduling assignment was

$$\min_{\delta} \frac{1}{M} \sum_{j=1}^{M} \sum_{i=1}^{N} \frac{\eta_o W \frac{\delta_{ij}\gamma_i}{\delta_{ij}\gamma_i + W}}{1 - \sum_{k=1}^{N} \frac{\delta_{kj}\gamma_k}{\delta_{kj}\gamma_k + W}} \tag{4.1}$$

$$\text{s.t.} \quad \sum_{j=1}^{M} \delta_{ij} = 1 \qquad \forall i = 1, 2, \ldots N \tag{4.2}$$

In this problem statement we had assumed that the δ_{ij}'s were indicator functions that were set to 1 if the particular user i was assigned to slot j and 0 otherwise. The relaxed assignment problem effectively "loosens" the constraint on these δ_{ij}'s. In other words, users can now transmit fractions of their required rates in multiple slots, as long as the total transmitted rate meets the minimum requirement, i.e. the linear constraint above is upheld. To simplify notation, we now set $\beta_{ij} = \frac{\delta_{ij}\gamma_i}{\delta_{ij}\gamma_i + W}$. Thus we write the equivalent problem statement as

$$\min_{\beta} f(\beta) = \frac{1}{M} \sum_{j=1}^{M} \frac{\sum_{i=1}^{N} \beta_{ij}}{1 - \sum_{k=1}^{N} \beta_{kj}} \tag{4.3}$$

$$\text{s.t.} \quad g_i(\beta) = \sum_{j=1}^{M} \frac{\beta_{ij}}{1 - \beta_{ij}} = \frac{\gamma_i}{W} \qquad \forall i = 1, 2, \ldots N \tag{4.4}$$

where each β_{ij} can take on values in the interval $[0, \frac{\gamma_i}{\gamma_i + W}]$. For our discussion we assume that $\beta = [\beta_{11}, \ , \ \beta_{12} \ldots \beta_{NM}]$ is a point in the NM-dimensional hypercube whose boundaries are defined by the interval above. The cost function is then $f(\beta) : \Re^{NM} \to \Re^1$. First we show that this cost function above is convex in β.

Theorem 2:

The cost function $f(\beta)$ of the relaxed assignment problem is convex in each β_{ij}.

Proof:

We prove the convexity of the cost function by way of analyzing two similar functions. First we show that a function $h(x) = \frac{x}{1-x}$ is convex for $x \in [0,1)$. The convexity of $h(x)$ is noted as follows. First observe that,

$$\begin{aligned}
\alpha \quad & h(x_1) + (1-\alpha)h(x_2) - h(\alpha x_1 + (1-\alpha)x_2) && (4.5)\\
&= \frac{\alpha x_1}{1-x_1} + \frac{(1-\alpha)x_2}{1-x_2} - \frac{\alpha x_1 + (1-\alpha)x_2}{1-\alpha x_1 - (1-\alpha)x_2} && (4.6)\\
&= \alpha x_1 \left(\frac{1}{1-x_1} - \frac{1}{1-\alpha x_1 - (1-\alpha)x_2}\right) && (4.7)\\
&\quad +(1-\alpha)x_2 \left(\frac{1}{1-x_2} - \frac{1}{1-\alpha x_1 - (1-\alpha)x_2}\right) && (4.8)\\
&= \alpha x_1 \left(\frac{(1-\alpha)(x_1-x_2)}{(1-x_1)(1-\alpha x_1 - (1-\alpha)x_2)}\right) && (4.9)\\
&\quad +(1-\alpha)x_2 \left(\frac{\alpha(x_2-x_1)}{(1-x_2)(1-\alpha x_1 - (1-\alpha)x_2)}\right) && (4.10)
\end{aligned}$$

Without a loss of generality, assume $x_2 > x_1$. Therefore,

$$\begin{aligned}
\alpha h(x_1) \quad + \quad & (1-\alpha)h(x_2) - h(\alpha x_1 + (1-\alpha)x_2) && (4.11)\\
= \quad & \left(\frac{\alpha(1-\alpha)(x_2-x_1)}{1-\alpha x_1 - (1-\alpha)x_2}\right)\left(\frac{x_2}{1-x_2} - \frac{x_1}{1-x_1}\right) && (4.12)\\
= \quad & \frac{\alpha(1-\alpha)(x_2-x_1)^2}{(1-\alpha x_1 - (1-\alpha)x_2)(1-x_2)(1-x_1)} && (4.13)\\
\geq \quad & 0 && (4.14)
\end{aligned}$$

Thus, $h(x)$ as described above is convex in $x \in [0,1)$. Now let us define a new function $z(\mathbf{x}) = h(w(\mathbf{x}))$, where $w(\mathbf{x})$ is a linear sum: $\sum_{i=1}^{N} x_i$ and $h(w) = \frac{w}{1-w}$. The second partial of z with respect to x_i and x_j is identical for all i and equal to

$$\begin{aligned}
\frac{\partial^2 z}{\partial x_i \partial x_j} &= \frac{\partial}{\partial x_i}\left(\frac{dh}{dw} \cdot \frac{\partial w}{\partial x_j}\right) && (4.15)\\
&= \frac{2}{(1-\sum_{i=1}^{N} x_i)^3} && (4.16)
\end{aligned}$$

Observe that value of (4.16) is positive for all $x \in [0, 1)$. We then see that the Hessian of z is composed of positive, identical entries, implying that it is positive definite. This then implies that z is convex in each x_i.

Our original cost function $f(\beta)$ in (4.3) can be written as

$$f(\beta) = \frac{\eta_o W}{M} \sum_{j=1}^{M} z_j(\beta_{\mathbf{j}}) \tag{4.17}$$

where $\beta_{\mathbf{j}} = [\beta_{1j}\ \beta_{2j}\ \ldots\ \beta_{Nj}]^T$. From the argument above, we know that each z_j is convex in β_{ij}. Now we simply have to show that the sum of convex functions z_j over j is convex. Let each $h_i(\mathbf{x})$ be any convex function in each component of $\mathbf{x}$. Also let $z(\mathbf{x}) = \sum_i h_i(\mathbf{x})$. Then:

$$\begin{aligned} z(\alpha\mathbf{x}_1 + (1-\alpha)\mathbf{x}_2) &= \sum_i h_i(\alpha\mathbf{x}_1 + (1-\alpha)\mathbf{x}_2) && (4.18) \\ &\leq \sum_i \alpha h_i(\mathbf{x}_1) + (1-\alpha)h_i(\mathbf{x}_2) && (4.19) \\ &= \alpha z(\mathbf{x}_1) + (1-\alpha)z(\mathbf{x}_2) && (4.20) \end{aligned}$$

Thus the sum of convex functions is convex. This then implies that the cost function of the relaxed assignment problem is in fact convex in β_{ij}.

Now we move on to the constraint set. Specifically, we would like to determine if the constraint set is convex in β or not. For clarity, we note that a set A is convex if for all $x_1 \in A$ and $x_2 \in A$, $\alpha x_1 + (1-\alpha)x_2$ ($\forall \alpha \in [0, 1]$) is also an element in A. If the constraint set in the relaxed assignment problem above meets this condition, then the relaxed problem statement of (4.3) is just the minimization of convex function over a convex set which implies the existence of a unique minimum solution [19]. In our case, however, it can be shown that the constraint set is not convex. In particular we do this by providing a counter-example. The details of this are provided in the following theorem and proof.

Theorem 3:

The constraint set $\{\beta | g_i(\beta) = \gamma_i/W\ \forall i = 1, 2, \ldots N\}$ is non-convex.

Proof:

Note that constraint set is comprised of N mutually exclusive sets each defined by $B_i = \{\mathbf{b} = [\beta_{i1}, \beta_{i2}, \ldots \beta_{iM}] | g_i(\mathbf{b}) = \gamma_i/W\}$. The overall constraint set is just $B = \bigcup_{i=1}^{N} B_i$. We will now prove the above theorem via the following counter-example:

Assume that we have a two slot system and that for any arbitrary user i, $\mathbf{b}_1 = [\beta_{i1} \quad \beta_{i2}]$ and $\mathbf{b}_2 = [\beta'_{i1} \quad \beta'_{i2}]$ are elements in the constraint set B_i. Thus $\mathbf{b}_1$ and $\mathbf{b}_2$ satisfy the constraints:

$$\frac{\beta_{i1}}{1-\beta_{i1}} + \frac{\beta_{i2}}{1-\beta_{i2}} = \frac{\gamma_i}{W} \tag{4.21}$$

$$\frac{\beta'_{i1}}{1-\beta'_{i1}} + \frac{\beta'_{i2}}{1-\beta'_{i2}} = \frac{\gamma_i}{W} \tag{4.22}$$

First solve thru for β_{i1} and β'_{i1}:

$$\beta_{i1} = \frac{\beta_{i2}W - \gamma_i + \gamma_i\beta_{i2}}{2\beta_{i2}W - \gamma_i - W + \gamma_i\beta_{i2}} \tag{4.23}$$

$$\beta'_{i1} = \frac{\beta'_{i2}W - \gamma_i + \gamma_i\beta'_{i2}}{2\beta'_{i2}W - \gamma_i - W + \gamma_i\beta'_{i2}} \tag{4.24}$$

Now if $\alpha\mathbf{b}_1 + (1-\alpha)\mathbf{b}_2$ is in B_i, then:

$$\alpha\beta_{i1} + (1-\alpha)\beta'_{i1} = \frac{(\alpha\beta_{i2} + (1-\alpha)\beta'_{i2})W - \gamma_i + \gamma_i(\alpha\beta_{i2} + (1-\alpha)\beta'_{i2})}{2(\alpha\beta_{i2} + (1-\alpha)\beta'_{i2})W - \gamma_i - W + \gamma_i(\alpha\beta_{i2} + (1-\alpha)\beta'_{i2})} \tag{4.25}$$

for all $\alpha \in [0,1]$. But we note that solving this equation for α gives us only two solutions: $\alpha = 0$ and $\alpha = 1$. Thus B_i has been shown to be non-convex for any arbitrary user i in the two slot case. This in turn implies that we can show that B_i is non-convex for all users. This subsequently brings us to the result that $B = \bigcup_{i=1}^{N} B_i$ is non-convex since each B_i is non-convex.

Due to the two theorems above, the resulting relaxed assignment problem has potentially many local extrema points [19]. This comes about because the convex cost function and the non-convex constraint set. A rigorous discussion of such a situation is provided in [20]. Note that though a global minimum is difficult to compute in such a case, there has been some literature available for global minimization under these conditions. These techniques aim at obtaining a rather rough estimate of the global minimum. Some preliminary discussion of this is provided in [21] with extensive algorithms provided in [22], [23], and [24]. Some of these algorithms employ a penalty functions, others a random search method. The author in [23] assumes the objective function and constraints to be Lipschizian. For solving auxiliary minimization problem involving functions in the constraints, a grid of feasible points is constructed. The objective function is then minimized over this grid. For our purposes, however, the implication of these convexity characteristics is simply that we cannot find an unique optimal point. We are therefore left only with possibility of studying such a formulation for deducing further power reduction techniques, not necessarily the optimal power reduction method.

4.2 Observations concerning the Relaxed Assignment Problem

Though we already know that there are clear obstacles towards finding an optimal solution to the problem statement in (4.3), let us nevertheless attempt to solve this minimization problem employing the techniques of constrained optimization. Let the vector $\lambda \in \Re^N$ be the Lagrange multipliers associated with the equality constraint $\mathbf{g}(\beta) = [\frac{\gamma_1}{\gamma_1+W} \ \ldots \frac{\gamma_N}{\gamma_N+W}]^T$. And let $\mu \in \Re^{NM}$ and $\nu \in \Re^{NM}$ be the Lagrange multipliers associated with the upper and lower bound constraints respectively. Thus, due to the Kuhn-Tucker conditions, we know that at the optimal β^*:

$$\mathbf{l}(\beta^*) = \nabla f(\beta) + \lambda^T \nabla \mathbf{g}(\beta) + \mu + \nu \quad = \quad \mathbf{0} \tag{4.26}$$

$$\mu^T(\beta - \mathbf{w}) \quad = \quad 0 \tag{4.27}$$

$$\nu^T \beta = 0 \tag{4.28}$$

$$\mu \geq \mathbf{0} \tag{4.29}$$

$$\nu \leq \mathbf{0} \tag{4.30}$$

where $\mathbf{l}(\beta^*)$ is the Lagrangian for the optimization problem under consideration and $w_{ij} = \frac{\Gamma r_i}{\Gamma r_i + W}$. These first-order necessary conditions for the relaxed assignment problem allow us to make the following observation:

Theorem 4

The standard, unslotted multirate CDMA allocation for the relaxed assignment problem satisfies the Kuhn-Tucker conditions stated in (4.26)-(4.30).

Proof

In the standard, unslotted multirate CDMA system, users transmit in each and every slot at a rate allocation: $\beta_{ij} = \frac{\frac{\gamma_i}{M}}{\frac{\gamma_i}{M}+W}$. We note that according to the Kuhn-Tucker conditions for the relaxed assignment problem, if each user transmits during each and every slot, then due to (4.27) and (4.28), $\mu = \mathbf{0}$ and $\nu = \mathbf{0}$. The resulting conditions imply:

$$\frac{1-\beta_{11}}{1-\sum_{k=1}^{N}\beta_{k1}} = \frac{1-\beta_{12}}{1-\sum_{k=1}^{N}\beta_{k2}} = \ldots = \frac{1-\beta_{1M}}{1-\sum_{k=1}^{N}\beta_{kM}} \tag{4.31}$$

$$\frac{1-\beta_{21}}{1-\sum_{k=1}^{N}\beta_{k1}} = \frac{1-\beta_{22}}{1-\sum_{k=1}^{N}\beta_{k2}} = \ldots = \frac{1-\beta_{2M}}{1-\sum_{k=1}^{N}\beta_{kM}} \tag{4.32}$$

$$\vdots \tag{4.33}$$

$$\frac{1-\beta_{N1}}{1-\sum_{k=1}^{N}\beta_{k1}} = \frac{1-\beta_{N2}}{1-\sum_{k=1}^{N}\beta_{k2}} = \ldots = \frac{1-\beta_{NM}}{1-\sum_{k=1}^{N}\beta_{kM}} \tag{4.34}$$

If we assign each $\beta_{ij} = \frac{\frac{\gamma_i}{M}}{\frac{\gamma_i}{M}+W}$, then each of the conditions (4.31) - (4.34) is satisfied as is the initial assumption that each user contributes a non-zero β_{ij} in each slot.

The second-order conditions for the standard, unslotted CDMA transmission can now be analyzed to determine if this particular extrema is a local maximum or minimum. This is done in Appendix 1 and results point out that this transmission scheme is a local minimum. However, we know from the analysis in the previous chapter that the standard CDMA point is not a global minimum for this problem since we were able to produce results minimum to this standard CDMA point using certain discrete assignments. Plugging in particular boundary conditions, i.e. a particular subset of all possible discrete assignments into the relaxed assignment problem will also satisfy the Kuhn-Tucker conditions above. However, we know from our analysis in the previous chapter that not all these assignments will yield a superior power performance when compared to the standard CDMA point. The random slot assignment scheme studied earlier is an example of this. At the same time, we also know that there a set of discrete assignments that are minimum to the CDMA operating point, such as the load-balancing, the zig-zag, etc.

Next, we probe a bit further into the problem of assigning users to slots based on the relaxed assignment policy. In this observation, we assume that all users have been appointed optimal β_{ij}'s over all the slots. Now an additional user enters the system. Without disrupting the transmission schedule of the currently transmitting users, how does the system incorporate the new user in an optimal fashion?

To solve this assignment problem, we begin with the assumption that the new user is assigned to transmit only over two of the M slots and determine when and what the optimal assignment would be. Next, we extend the available slots to 3 and recalculate the optimal assignment. Finally we deduce the optimal assignment if the user was allowed to transmit in any subset of the M slots.

β_{ij} assigned over two slots

Assume that in the optimal solution, user i is assigned non-zero values of β_{ij} in two of the M slots. Without a loss of generality, we call these two slots 1 and 2. We start by assuming that in slot 1 and 2, user i is assigned β_{i1} and β_{i2} respectively. The cost function ($f(\beta) = \frac{1}{M}\sum_{j=1}^{M} \frac{\eta_o W \sum_{l=1}^{N} \beta_{lj}}{1-\sum_{k=1}^{N} \beta_{kj}}$) evaluated at this $\beta = \beta^*$ assignment can then be written as:

$$f(\beta^*) = \frac{\eta_o W}{M}\left(\frac{\sum_{k\neq i}\beta_{k1} + \beta_{i1}}{1-\sum_{k\neq i}\beta_{k1} - \beta_{i1}} + \frac{\sum_{k\neq i}\beta_{k2} + \beta_{i2}}{1-\sum_{k\neq i}\beta_{k2} - \beta_{i2}} + \sum_{j=3}^{M}\frac{\sum_{k\neq i}\beta_{kj}}{1-\sum_{k\neq i}\beta_{kj}}\right) \quad (4.35)$$

Now assume that all other users k, i.e. $k \neq i$, have been optimally assigned to the M slots. Let $a_j = \sum_{k\neq i}\beta_{kj}$. Based on these preassigned, optimal a_j's ($j = 1, 2, \ldots M$), we would like to first determine the value of β_{i1} and β_{i2} that minimize the cost function. In doing this analysis, we assume the rate and SIR requirements of user i such that it can be accommodated in the most weighted slot, i.e. the slot with the largest sum a_j. In other words, let us assume $\max_j a_j + \frac{\gamma_i}{\gamma_i+W} < 1$ over $j = 1, 2, \ldots M$, which further implies:

$$\gamma_i < W\left(\frac{1}{\max_j a_j} - 1\right) \qquad \text{over } j = 1, 2, \ldots M \tag{4.36}$$

Note, however, that if $\max_j a_j \leq 0.5$, then this constraint says $\gamma_i \leq \alpha W$, where $\alpha \geq 1$. Since the value of γ_i is clearly limited by W, we can state the upper bound on a feasible γ_i as

$$\gamma_i \leq \min\left(W\left(\frac{1}{\max_j a_j} - 1\right),\ W\right) \tag{4.37}$$

Having established these constraints on the problem statement, we would now like to determine the solution to the following optimization:

$$\min_{\beta_{i1}\beta_{i2}} \frac{a_1 + \beta_{i1}}{1 - a_1 - \beta_{i1}} + \frac{a_2 + \beta_{i2}}{1 - a_2 - \beta_{i2}} + \sum_{j=3}^{M} \frac{a_j}{1 - a_j} \tag{4.38}$$

$$\text{s.t.} \qquad \frac{\beta_{i1}}{1 - \beta_{i1}} + \frac{\beta_{i2}}{1 - \beta_{i2}} = \frac{\gamma_i}{W} \tag{4.39}$$

Since we assume a_j for $j \geq 3$ have been optimally assigned, we can reduce the problem statement to:

$$\min_{\beta_{i1}\beta_{i2}} \frac{a_1 + \beta_{i1}}{1 - a_1 - \beta_{i1}} + \frac{a_2 + \beta_{i2}}{1 - a_2 - \beta_{i2}} \tag{4.40}$$

$$\text{s.t.} \qquad \frac{\beta_{i1}}{1 - \beta_{i1}} + \frac{\beta_{i2}}{1 - \beta_{i2}} = \frac{\gamma_i}{W} \tag{4.41}$$

After some straightforward calculation, we see that the optimal β_{i1} and β_{i2}, denoted as β_{i1}^* and β_{i2}^* are:

$$\beta_{i1}^* = \frac{a_2(\gamma_i + W) - a_1 W}{(2W + \gamma_i)a_2} \tag{4.42}$$

$$\beta_{i2}^* = \frac{a_1(\gamma_i + W) - a_2 W}{(2W + \gamma_i)a_1} \tag{4.43}$$

We see that $\beta_{i1}^* > 0$ for any $\gamma_i > W\left(\max\left(\frac{a_1}{a_2},\ \frac{a_2}{a_1}\right) - 1\right)$. By combining with the result in (4.37), the overall constraint on γ_i is

$$\left(\max\left(\frac{a_1}{a_2},\ \frac{a_2}{a_1}\right) - 1\right) < \frac{\gamma_i}{W} \leq \min\left(\left(\frac{1}{\max_j a_j} - 1\right),\ 1\right) \tag{4.44}$$

If γ_i satisfies this constraint, then the total sum of powers in slots 1 and 2 are P_1^* and P_2^*, respectively

$$P_1^* = \frac{a_1 + \beta_{i1}^*}{1 - a_1 - \beta_{i1}^*} = \frac{Wa_1 - a_2(W + \gamma_i) - a_1 a_2(2W + \gamma_i)}{a_1 a_2(2W + \gamma_i) - W(a_1 + a_2)} \tag{4.45}$$

$$P_2^* = \frac{a_2 + \beta_{i2}^*}{1 - a_2 - \beta_{i2}^*} = \frac{Wa_2 - a_1(W + \gamma_i) - a_1 a_2(2W + \gamma_i)}{a_1 a_2(2W + \gamma_i) - W(a_1 + a_2)} \tag{4.46}$$

Note the following observations about P_i^*, P_2^*, β_{i1}^*, and β_{i2}^*:

- For $a_1 \neq a_2$, $P_1^* \neq P_2^*$. Conversely for $a_1 = a_2$, $P_1^* = P_2^*$.
- For $a_1 > a_2$, we observe $P_1^* > P_2^*$ and $\beta_{i1}^* < \beta_{i2}^*$. Similarly, if $a_2 > a_1$, then $P_2^* > P_1^*$ and $\beta_{i2}^* < \beta_{i1}^*$.
- For $\max\left(\frac{a_1}{a_2}, \frac{a_2}{a_1}\right) \geq 2$, the lower bound in (4.44) is greater than W. Once again we note that $\gamma_i > W$ is infeasible.

Based on these observations and the constraint (4.44), we see that the assignment of β_{i1} and β_{i2} is not simply that listed in (4.42) and (4.43). In particular, we need to examine further for the cases when $\gamma_i < W\left(\max\left(\frac{a_1}{a_2}, \frac{a_2}{a_1}\right) - 1\right)$ and when $\max\left(\frac{a_1}{a_2}, \frac{a_2}{a_1}\right) \geq 2$.

If we impose the boundary conditions $\beta_{i1} \in \left[0, \frac{\gamma_i}{\gamma_i+W}\right]$ and $\beta_{i2} \in \left[0, \frac{\gamma_i}{\gamma_i+W}\right]$ into the optimization problem stated in (4.40) and (4.41), then we see that the solution for $\gamma_i < W\left(\max\left(\frac{a_1}{a_2}, \frac{a_2}{a_1}\right) - 1\right)$ is

$$\beta_{ij} = 0 \quad \text{for} \quad j = \arg\max_j (a_1, a_2) \tag{4.47}$$

$$\beta_{ik} = \frac{\gamma_i}{\gamma_i + W} \quad \text{for} \quad k \neq j \tag{4.48}$$

Clearly, if $\max\left(\frac{a_1}{a_2}, \frac{a_2}{a_1}\right) \geq 2$, then any feasible γ_i cannot satisfy the constraint will thus also lead to β_{ij} solutions as in (4.47) and (4.48).

Grouping these results, we see therefore for this two-slotted assignment of user i's transmission requirements, the optimal β_{ij} allocations are such that β_{ij} have discrete solutions. Explicitly

$$\beta_{i1} = 0, \beta_{i2} = \frac{\gamma_i}{\gamma_i + W} \quad \text{for} \quad \gamma_i < W\left(\frac{a_1}{a_2} - 1\right) \quad \forall \frac{a_1}{a_2} \geq 1 \tag{4.49}$$

$$\beta_{i1} = \frac{\gamma_i}{\gamma_i + W}, \beta_{i2} = 0 \quad \text{for} \quad \gamma_i < W\left(\frac{a_2}{a_1} - 1\right) \quad \forall \frac{a_2}{a_1} \geq 1 \tag{4.50}$$

$$\beta_{i1} = \beta_{i1}^*, \beta_{i2} = \beta_{i2}^* \quad \text{otherwise} \tag{4.51}$$

Therefore, we see that except for a range when $1 \leq \max\left(\frac{a_1}{a_2}, \frac{a_2}{a_1}\right) < 2$ and $\gamma_i > W\left(\max\left(\frac{a_1}{a_2}, \frac{a_2}{a_1}\right) - 1\right)$, the optimal β_{ij} is a discrete assignment.

Intuitively, this results says that if the difference between the optimal weights a_1 and a_2 is large enough compared to the γ_i desired by user i, then place this user in the slot with the smaller sum a_j. Otherwise, if the difference between a_1 and a_2 is small, then there is a large set of values of γ_i that satisfy the lower-bound defined earlier. If the desired γ_i is greater than this lower-bound, then split the desired rate of the user over the two slots as in (4.42) and (4.43). Note that these were optimal solutions assuming fixed a_j.

β_{ij} assigned over 3 slots

Here we extend the assignment of user i transmission request over 3 slots. In other words, we wish to find the optimal β_{ij} where $j = 1, 2, 3$ for the following problem

$$\min_{\beta_{i1}\beta_{i2}} \frac{a_1 + \beta_{i1}}{1 - a_1 - \beta_{i1}} + \frac{a_2 + \beta_{i2}}{1 - a_2 - \beta_{i2}} + \frac{a_3 + \beta_{i3}}{1 - a_3 - \beta_{i3}} \tag{4.52}$$

$$\text{s.t.} \quad \frac{\beta_{i1}}{1 - \beta_{i1}} + \frac{\beta_{i2}}{1 - \beta_{i2}} + \frac{\beta_{i3}}{1 - \beta_{i3}} = \frac{\gamma_i}{W} \tag{4.53}$$

Let us now detail the solution to this problem using the principles of multivariable minimization. To develop the answer (as done in the two slot case), we will employ a Lagrange multiplier λ for the equality constraint. Thus the optimal β_{ij}'s (once again denoted as: β_{i1}^*, β_{i2}^*, and β_{i3}^*) must solve the following set of equations

$$\frac{1}{(1 - a_1 - \beta_{i1}^*)^2} = \frac{\lambda}{(1 - \beta_{i1}^*)^2} \tag{4.54}$$

$$\frac{1}{(1 - a_2 - \beta_{i2}^*)^2} = \frac{\lambda}{(1 - \beta_{i2}^*)^2} \tag{4.55}$$

$$\frac{1}{(1 - a_3 - \beta_{i3}^*)^2} = \frac{\lambda}{(1 - \beta_{i3}^*)^2} \tag{4.56}$$

$$\frac{\beta_{i1}^*}{1 - \beta_{i1}^*} + \frac{\beta_{i2}^*}{1 - \beta_{i2}^*} + \frac{\beta_{i3}^*}{1 - \beta_{i3}^*} = \frac{\gamma_i}{W} \tag{4.57}$$

Solving (4.54), (4.55), and (4.56) for λ, gives the following

$$\beta_{i2}^* = 1 - \frac{a_2}{a_1} + \frac{a_2 \beta_{i1}^*}{a_1} \tag{4.58}$$

$$\beta_{i3}^* = 1 - \frac{a_3}{a_1} + \frac{a_3 \beta_{i1}^*}{a_1} \tag{4.59}$$

Substituting these results into (4.57) and then solving backwards for β_{i2}^* and β_{i3}^*

$$\beta_{i1}^* = \frac{a_2a_3(\gamma_i+2W) - W(a_1a_2+a_1a_3)}{a_2a_3(\gamma_i+2W)} \tag{4.60}$$

$$\beta_{i2}^* = \frac{a_1a_3(\gamma_i+2W) - W(a_1a_2+a_3a_2)}{a_1a_3(\gamma_i+2W)} \tag{4.61}$$

$$\beta_{i3}^* = \frac{a_1a_2(\gamma_i+2W) - W(a_1a_3+a_2a_3)}{a_1a_2(\gamma_i+2W)} \tag{4.62}$$

Here $\beta_{i1} > 0$ if and only if $\gamma_i > W\left(\max\left(\frac{a_1}{a_2}+\frac{a_1}{a_3},\ \frac{a_2}{a_1}+\frac{a_2}{a_3},\ \frac{a_3}{a_1}+\frac{a_3}{a_2}\right)-2\right)$. Assuming γ_i satisfies this, the minimum sum of powers in each slot are then

$$P_1^* = \frac{W(a_1a_3+a_1a_2) - (\gamma_i+2W)a_2a_3 - (\gamma_i+3W)a_1a_2a_3}{a_1a_2a_3(\gamma_i+3W) - W(a_1a_2+a_1a_3+a_2a_3)} \tag{4.63}$$

$$P_2^* = \frac{W(a_2a_1+a_2a_3) - (\gamma_i+2W)a_1a_3 - (\gamma_i+3W)a_1a_2a_3}{a_1a_2a_3(\gamma_i+3W) - W(a_1a_2+a_1a_3+a_2a_3)} \tag{4.64}$$

$$P_3^* = \frac{W(a_3a_1+a_3a_2) - (\gamma_i+2W)a_1a_2 - (\gamma_i+3W)a_1a_2a_3}{a_1a_2a_3(\gamma_i+3W) - W(a_1a_2+a_1a_3+a_2a_3)} \tag{4.65}$$

Without loss of generality, let us assume that slot 3 is most weighted, i.e. a_3 is largest. Then it is required that $\gamma_i > W\left(\frac{a_3}{a_1}+\frac{a_3}{a_2}\right)$. Since γ_i is upper-bounded by W, we require: $2 \le \frac{a_3}{a_1}+\frac{a_3}{a_2} < 3$. Since each a_3/a_1 and a_3/a_2 is greater than 1, we can thus deduce β_{ij} is non-zero over three slots if $1 \le \frac{a_3}{a_1} < 2$ *and* $1 \le \frac{a_3}{a_2} < 2$. If, on the other hand, $\frac{a_3}{a_1} \ge 2$, $\frac{a_3}{a_2} \ge 2$ and $\max\left(\frac{a_2}{a_1},\ \frac{a_1}{a_2}\right) < 2$ and $\gamma_i > W\left(\max\left(\frac{a_2}{a_1},\ \frac{a_1}{a_2}\right)-1\right)$, then by the arguments used in the previous section, user i should transmit in slots 1 and 2 only according to (4.42) and (4.43). Under any other feasible condition, we should assign β_{ij} such that it is non-zero (and equal to $\frac{\gamma_i}{\gamma_i+W}$) for $j = \arg\min_k a_k$, where $k = 1, 2, 3$.

Using this argument, we see that only under a particular set of conditions, i.e. depending on the relationship between the a_j's and the desired γ_i, should the user split up its transmissions into multiple slots. Otherwise, this relaxed problem once again has discrete solution of this lone user.

β_{ij} assigned over M slots

We wish to now examine the optimal values of β_{ij} if user i were to transmit over all M slots. The problem statement would then be to determine

$$\min \sum_{j=1}^{M} \frac{a_j+\beta_{ij}}{1-a_j-\beta_{ij}} \tag{4.66}$$

$$\text{s.t.} \qquad \sum_{j=1}^{M} \frac{\beta_{ij}}{1-\beta_{ij}} \tag{4.67}$$

Let β_{ij}^* denote the optimal value of β_{ij} in slot j. Again using minimization techniques with the Lagrange multiplier λ to handle the equality constraint (4.67), we observe that

$$\beta_{ik}^* = 1 - \frac{a_k}{a_1} + \frac{a_k \beta_{i1}^*}{a_1} \qquad k = 2, 3, \ldots M \tag{4.68}$$

Substitute these results into the equality constraint and solve for β_{i1} and subsequently for each β_{ik} $k = 2, 3, \ldots M$. By induction we are able to show that for a general M

$$\beta_{ik}^* = \frac{(\gamma_i + (M-1)W) \prod_{l \neq k} a_l}{(\gamma_i + MW) \prod_{l \neq k} a_l} - WC \tag{4.69}$$

where C is

$$C = \frac{\left(a_k \sum_{l_1=1, l_1 \neq k} a_{l_1} \sum_{l_2 = l_1+1, l_2 \neq k} a_{l_2} \cdots \sum_{l_{(M-2)} = l_{(M-3)}+1, l_{(M-2)} \neq k} a_{l_{(M-2)}} \right)}{(\gamma_i + MW) \prod_{l \neq k} a_l} \tag{4.70}$$

and $k = 1, 2, \ldots M$. Once again, $\beta_{ik} > 0$ for all k if and only if

$$\gamma_i > W \left(a_m \sum_{l \neq m}^{M} \frac{1}{a_l} - (M-1) \right) \tag{4.71}$$

where $m = \arg\max_j a_j$. Here too we observe that $0 < \gamma_i < W$, thus $(M-1) \leq a_m \sum_{l \neq m}^{M} \frac{1}{a_l} < M$. Since a_m is the maximum a_j, we see that each $\frac{a_m}{a_k} \geq 1$ for all $k \neq m$. Therefore, if each $1 \leq \frac{a_m}{a_k} < 2$, $(k \neq m)$ then user i should divide its transmission over all M slots. Otherwise if for all $k \neq m$, $\frac{a_m}{a_k} \geq 2$ and $\frac{a_{m_2}}{a_l} < 2$ ($m_2 = \arg\max_{j \neq m} a_j$ and $l \neq m, m_2$) and $\gamma_i > W \left(a_{m_2} \sum_{l \neq m, l \neq m_2} \frac{1}{a_l} - (M-2) \right)$, then user i should transmit over $M-1$ slots, i.e. all slots $k \neq m$. We then use a recursive argument to determine how many and which slots the user will transmit over.

4.3 Observation concerning the Relaxed Assignment Policy and Single-Rate Users

In this section, we discuss the implications of the relaxed assignment policy on single-rate users. From the previous chapter, we note that discrete assignments reduce power consumption when compared to standard CDMA signaling in both single and multi-rate systems. In this section, we would like to utilize the freedom of multiple slot transmission to improve on the performance of the discrete assignment policies for single-rate systems. For such a single-rate system, we begin by comparing the power performance of standard CDMA versus a discrete assignment scheme where N, the number of users, is divisible by M, the number of slots. Explicitly, we prove the following theorem:

Theorem 5:

In a discrete-assignment, slotted CDMA system, single-rate users cumulatively consume less average power over the frame than do the identical users operating in the standard, unslotted CDMA system.

Proof:

The average power consumed by these N identical users in the standard CDMA system is

$$\bar{P}_{CDMA} = \frac{\eta_o W}{M} \sum_{j=1}^{M} \frac{\sum_{i=1}^{N} \frac{\frac{\gamma}{M}}{\frac{\gamma}{M}+W}}{1 - \sum_{i=1}^{N} \frac{\frac{\gamma}{M}}{\frac{\gamma}{M}+W}} \tag{4.72}$$

$$= \frac{\eta_o W}{M} \sum_{j=1}^{M} \frac{N \frac{\frac{\gamma}{M}}{\frac{\gamma}{M}+W}}{1 - N \frac{\frac{\gamma}{M}}{\frac{\gamma}{M}+W}} \tag{4.73}$$

$$= \eta_o W \frac{N \frac{\frac{\gamma}{M}}{\frac{\gamma}{M}+W}}{1 - N \frac{\frac{\gamma}{M}}{\frac{\gamma}{M}+W}} \tag{4.74}$$

If the users are discretely assigned to the M slots, then each slot has at least $K = \lfloor \frac{N}{M} \rfloor$ users assigned to it. The remaining $R_N = N - NK$ users are then assigned to any one of the M slots. The average power consumed by all users in such discrete assignment schemes is therefore

$$\bar{P}_D = \frac{\eta_o W}{M} \left(\sum_{j=1}^{M-R_N} \frac{\sum_{i=1}^{K} \frac{\gamma}{\gamma+W}}{1 - \sum_{i=1}^{K} \frac{\gamma}{\gamma+W}} + \sum_{j=1}^{R} {}_N \frac{\sum_{i=1}^{K+1} \frac{\gamma}{\gamma+W}}{1 - \sum_{i=1}^{K+1} \frac{\gamma}{\gamma+W}} \right) \tag{4.75}$$

We wish to show that $\bar{P}_{CDMA} > \bar{P}_D$. Simplifying the expressions above, we note that

$\bar{P}_{CDMA} > \bar{P}_D$ if

$$\gamma\left[K(MK(1-M)) - R_N(1-R_N)\right] > W\left[R_N(1-R_N) + MK(1-M)\right] \tag{4.76}$$

Now observe if $R_N = 0$ (N is divisible by M), then we can further reduce (4.76)

$$\begin{aligned} \gamma K(MK(1-M)) &> WMK(1-M) && (4.77)\\ \gamma K^2 &< WK && (4.78)\\ \gamma K &< W && (4.79) \end{aligned}$$

This last statement has to be true for any slot to be able to hold K users in the first place. (Note that we had to switch the sign of the inequality above since $(1-M) < 0$.) Therefore, $\bar{P}_{CDMA} > \bar{P}_D$ for all N and M such that $R_N = 0$. Now for $M > R_N > 0$, then we can simply (4.76) as:

$$\gamma(Kb - c) > W(c + b) \tag{4.80}$$

where $b = MK(1-M)$ and $c = R_N(1-R_N)$. Observe that $b < 0$, $c < 0$, and $b < c$. Further simplifying (4.80), we note that $\bar{P}_{CDMA} > \bar{P}_D$ if

$$\begin{aligned} Kb - c &> \frac{W}{\gamma}(c+b) && (4.81)\\ Kb - c + Kc - Kc &> \frac{W}{\gamma}(c+b) && (4.82)\\ K(b+c) - c(1+K) &> \frac{W}{\gamma}(c+b) && (4.83)\\ & && (4.84) \end{aligned}$$

As noted earlier, $K < \frac{W}{\gamma}$ and $b < c < 0$. This implies that (4.83) is in fact true which in turn gives us the assertion that $\bar{P}_{CDMA} > \bar{P}_D$.

Our aim in this section is to assign these single rate users to slots so that power assignments can be minimized compared to not only the standard CDMA system but also the discrete assignment of the previous chapter. To do this we utilize the freedom of multiple slot assignments in the case of N and M not divisible. We begin this assignment by letting $\lfloor \frac{N}{M} \rfloor = K$ user be discretely assigned to M slots. We now focus on what to do with the remaining $R_N = N - NK$ users. We know already that since we have a single-rate system, the discrete assignment implies that each slot has been "filled" to the same level, $a = K\frac{\gamma}{\gamma+W}$. The arguments in the previous section clearly show that if just one user remains, then the optimal assignment under these constraints is to divide that user's transmission over the entire frame. Now let us consider other potential remainders.

First let us consider the scenario when M is divisible by R_N. In this case we can use the result above and state the following minimum assignment for the R_N users: treat the remaining R_N users as users in an R_N slot system each with rate requirement $R_N\gamma/M$ and thus assign them each their own slot in the new R_N slot system. This means that if a user is assigned its "own" slot in the new system, then it has been effectively given M/R_N slots in the older system. The justification for such an optimal assignment is merely a product of the assignment of M users in an M slot system. Here we just treat the other users transmitting in each slot as channel characteristics that are constant over all M or rather R_N slots. We then compare the power consumption of the multiple slot assignment of the R_N users versus that of the discrete scenario in which the remaining users were simply each given one of the M slots. The result is easily obtained via simple algebra and points to a power savings in the multislot assignment of the relaxed scheme.

Now let us consider the more interesting situation when R_N is not divisible by M. Here we begin by assuming that the remaining users will employ conventional CDMA, i.e. $\beta_{ij} = \frac{\gamma}{\gamma+MW}$, for those user i who are remaining and for all $j = 1, 2, \ldots M$. The average power consumed in this case is

$$\bar{P}_1 = M\frac{Ka + \frac{R_N\gamma}{\gamma+MW}}{1 - Ka - \frac{R_N\gamma}{\gamma+MW}} \tag{4.85}$$

Comparing this to the minimum discrete assignment from Chapter 3 (4.75), we obtain

$$M\frac{Ka + \frac{R_N\gamma}{\gamma+MW}}{1 - Ka - \frac{R_N\gamma}{\gamma+MW}} > (M - R_N)\frac{Ka}{1 - Ka} + R_N\frac{(K+1)a}{1 - (K+1)a} \tag{4.86}$$

The above statement is true for any $a = \frac{\gamma}{\gamma+W}$. Thus the simplistic multi-slot assignment of the remaining users, i.e. assign the remaining users via conventional CDMA, is not minimum to the discrete assignment. So to assign these R_N users in a way that is power conserving to the discrete assignment, we have to look towards alternate multi-slot schemes.

One scheme is to assign V slots in which there are $K+1$ users and S slots ($S = M - V$) in which there are R_N users all performing conventional CDMA. Refer to the V slots as type 1 and the S slots as type 2. Under this assumption, we first determine the optimal β_{ij} assignments for the R_N users. Plugging into (4.3) for this explicit assignment, the total average power consumption over the frame is

$$\bar{P}_2 = \frac{\eta_o W}{M}\left(\frac{V(Ka+\beta_1)}{1-Ka-\beta_1} + \frac{S(Ka+R_N\beta_2)}{1-Ka-R_N\beta_2}\right) \tag{4.87}$$

where $\beta_i = \frac{\delta_i\gamma}{\delta_i\gamma+W}$ and δ_i is the portion of γ transmitted in type i slot by the R_N remaining users. The constraint to the β assignments is that

$$V\frac{\beta_1}{1-\beta_1} + S\frac{\beta_2}{1-\beta_2} = \frac{\gamma}{W} \tag{4.88}$$

Due to the first order conditions stated earlier, it is required that if a user transmits in multiple slots, then the effective bandwidth desired by this user in each of these slots, i.e. β_{ij}, must be such that the ratio $\frac{1-\beta_{ij}}{1-a_j-\beta_{ij}}$ remains constant over the assigned slots. From this we then obtain the additional constraint that

$$\frac{1-\beta_1}{1-Ka-\beta_1} = \frac{1-\beta_2}{1-Ka-R_N\beta_2} \tag{4.89}$$

Solving the above two constraints, we obtain the following optimal levels of β_1 and β_2

$$\beta_1^* = \frac{\frac{\gamma}{W}(Ka-1+R_N)}{(R_N-1)(V+\frac{\gamma}{W})+Ka(M+\frac{\gamma}{W})} \tag{4.90}$$

$$\beta_2^* = \frac{\frac{\gamma}{W}Ka}{V(R_N-1)+Ka(M+\frac{\gamma}{W})} \tag{4.91}$$

which then implies that:

$$\delta_1^* = \frac{Ka-1+R_N}{V(R_N-1)+MKa} \tag{4.92}$$

$$\delta_2^* = \frac{Ka}{V(R_N-1)+MKa} \tag{4.93}$$

Now we would like to compare this scheme to the discrete assignment of Chapter 3. More specifically, we would like to determine when: $\bar{P}_2 < \bar{P}_D$. Where $\bar{P}_2$ and $\bar{P}_D$ are:

$$\bar{P}_2 = \frac{\eta_o W}{M}\left(\frac{V(Ka+\beta_1)}{1-Ka-\beta_1} + \frac{(M-V)(Ka+R_N\beta_2)}{1-Ka-R_N\beta_2}\right) \tag{4.94}$$

$$\bar{P}_D = \frac{\eta_o W}{M}\left((M-R_N)\frac{Ka}{1-Ka} + R_N\frac{(K+1)a}{1-(K+1)a}\right) \tag{4.95}$$

After some extensive algebraic book-keeping, we realize that we have to show:

$$\frac{-\gamma(W+\gamma)f_1(V)}{(W-K\gamma)(W-(K-1)\gamma)f_2(V)} < 0 \tag{4.96}$$

where

$$\begin{aligned} f_1(V) &= \left(R\gamma^2 K(\gamma(1-K-R-MK)+W(1-R))\right) && (4.97)\\ & \qquad V\left((R-1)(W+\gamma)(W+\gamma(1-K))(K\gamma+W(R-1))\right) && (4.98)\\ f_2(V) &= \left(\gamma K(\gamma^2(1-K-R)+\gamma W(1-R-KM+M)+MW^2)\right) && (4.99)\\ & \qquad V\left(W(W+\gamma)(R-1)(W-(K-1)\gamma)\right) && (4.100) \end{aligned}$$

Let V_1 be the solution to $f_1(V)=0$ and V_2 be the solution to $f_2(V)=0$. Thus, (4.96) is true if $V > \max(V_1, V_2)$ or if $V < \min(V_1, V_2)$. Examining V_1 and V_2, we can show that for particular values of R and K, there exist both V_1 and V_2 in the interval $[0, R]$ that satisfy the inequality above. Thus we can assign users in this manner and achieve a reduction in average power consumption. In fact, such a result can be shown even if during the S slots, $R < R_N$ users were allowed to employ conventional CDMA.

Since there are many free floating variables in this discussion, let us examine this potential savings for a particular system with three slots and N user of which $N\lfloor\frac{N}{3}\rfloor$ are discretely assigned, leaving a remainder of R_N. Note that if $R_N = 1$, we have already shown that the remaining user should be ideally assigned to each of the three slots and given a rate requirement $\gamma/3$ in each slot of the frame. Now if $R_N = 2$, we would like to know if assigning each R_N to a slot and requiring the two to share the third slot is

1. minimum to standard, unslotted CDMA?

2. minimum to discrete assignments of Chapter 2?

To do this comparison effectively, we must first determine the optimal rate assignments per slot for the R_N users when we perform the multislot assignment stated above. Since the users are identical, one can assume that the portion of γ that each user transmits

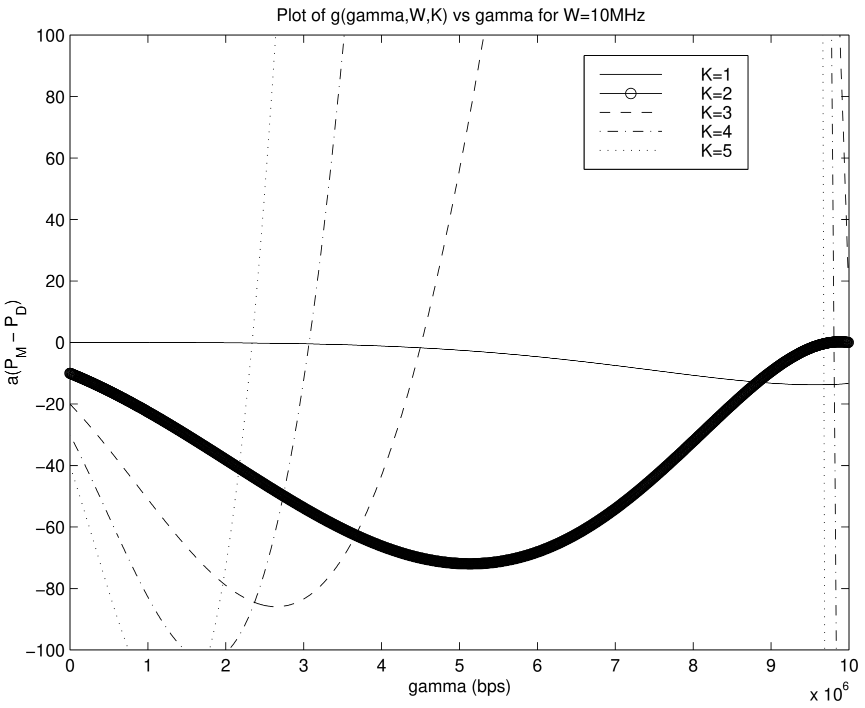

Figure 4.1: Plot of $g(\gamma)$ when $W = 10MHz$ for different values of K. Note that for $K = 1$, $g(\gamma) < 0$ for $0 \leq \gamma \leq W$.

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in the "unshared" slots are the same as is the amount of γ sent by each user during the "shared" slot, i.e. type 2. By "unshared" we refer to the two slots in which only one of the remaining users is assigned (type 1). Conversely, the shared slot is when both R_N transmit. Due to the first order conditions stated earlier, we know that

$$\frac{Ka+\beta_1}{1-Ka+\beta_1} = \frac{Ka+2\beta_2}{1-Ka+2\beta_2} \tag{4.101}$$

where $a = \frac{\gamma}{\gamma+W}$, $\beta_1 = \frac{\delta_1\gamma}{\delta_1\gamma+W}$ and δ_1 is the portion of γ that each R_N transmits in its "unshared" slot. Similarly, $\beta_2 = \frac{\delta_2\gamma}{\delta_2\gamma+W}$ and δ_2 is the portion of γ that both users transmit during the shared slot. Also it is required that

$$\frac{\beta_1}{1-\beta_1} + \frac{\beta_2}{1-\beta_2} = \frac{\gamma}{W} \tag{4.102}$$

Solving the two equations above we see that

$$\beta_1 = \frac{(1+Ka)\,a}{1+2Ka-Ka^2} \tag{4.103}$$

$$\beta_2 = \frac{Ka^2}{1+2Ka-Ka^2-a} \tag{4.104}$$

Since we have already shown that $\bar{P}_D < \bar{P}_{CDMA}$, then one can easily deduce that this multislot assignment is minimum to standard CDMA if it is minimum to the discrete case. The average power consumed by this multislot scheme is

$$\bar{P}_M = \frac{\eta_o W}{3}\left(\frac{a(2+3Ka-Ka^2)}{1+2Ka+Ka^3-2a-4Ka^2} + \frac{a(1+4Ka-Ka^2-a)}{1+2Ka+a^2-5Ka^2-2a+Ka^3}\right) \tag{4.105}$$

After some careful algebra, we are able to show that $\bar{P}_M < \bar{P}_D$ for $\gamma K < W$. For clarity, we show this result here in graphical terms. Simplifying our expression, we get that $\bar{P}_M < \bar{P}_D$, if and only if $g(\gamma, W, K) < 0$, where g is a polynomial in γ, W, and K. Figure 4.1 plots $g(\gamma)$ for values of γ in a system with W = 10MHz. Each curve in the graph represents a different value of K. Comparing each curve, we see that $g(\gamma, W, K) < 0$ for $\gamma < W/K$. Thus we see that the multislot access scheme proposed here is in fact minimum to both the discrete assignment as well as the standard CDMA assignment schemes.

4.4 Performance Analysis

Here we present some comparisons that are similiar to those in Chapter 3 but in this case we attempt to employ some of the additional power savings techniques discovered in this chapter. The results presented here are for a single-rate system. We first assign $\lfloor \frac{N}{M} \rfloor$ users to each slot. Here we utilized the techniques of the previous section to allow multislot access to the remaining R_N users to improve power performance. The results presented are for 10MHz system with three slots and each user demanding rate 24Kbps at an SIR of 6db. The resulting performance of this system for N = 4 to N = 160 is presented as the top graph in Figure 4.2. By P_{MOD} we refer to the average power consumed in the modified assignment case which allows multislot access to the R_N users. We see that the performance of the discrete scheme for these identical users (from Chapter 2) is extremely close to that of the same scheme with multislot access. The bottom of Figure 4.2 extends the number of system users until the schemes become infeasible. In fact, it is noted that both schemes become infeasible at N = 205. The exact ratio between these two power levels (P_{MOD}/P_{ZZ}) is shown in Figure 4.3. Note

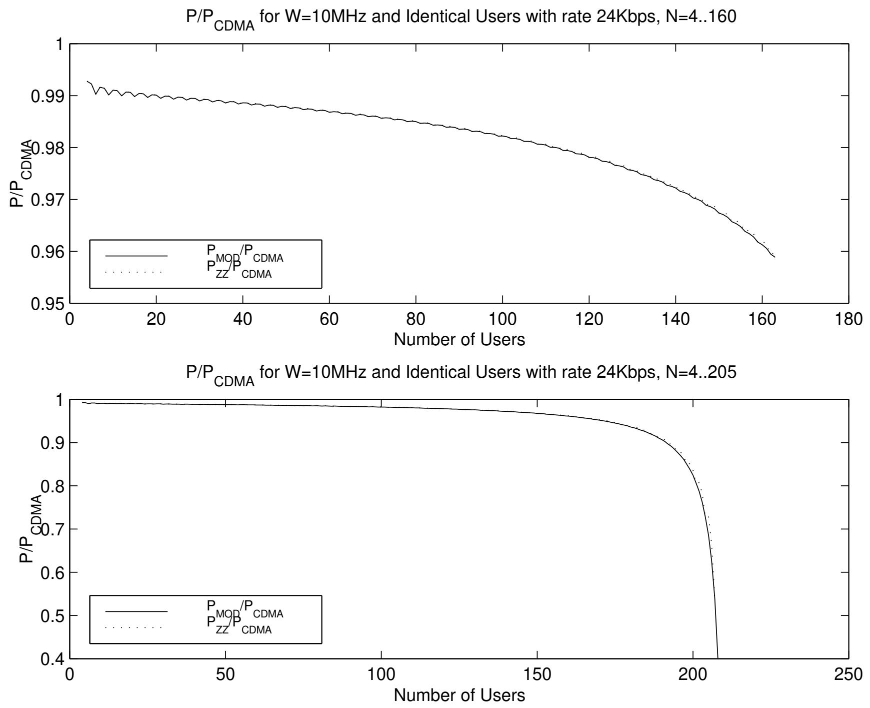

Figure 4.2: Relative performance of zig-zag and the modified zig-zag with multislot access for a single-rate system. Top: for $N = 4$ to $N = 160$. Bottom: for $N = 4$ to $N = 250$.

that at every third user, the modified, relaxed assignment and the zig-zag scheme are identical, thus leading to the cyclic occurrence of the ratio 1. On the other hand, as the system load grows, there are instances in which one can find power reductions down to .75 by implementing the multislot access for the remaining R_N users. Discrete assignment of multiple slots thus implies power savings that are comparable to the modified policy.

Also, in this analysis section we would like make a point about the relative merits of slotted assignments versus that of standard CDMA for single rate users. We note in the graphs in both Chapters 3 and 4 that the performance benefits of the slotted schemes for single rate users are substantial in heavy loaded systems. Note, however, that though these relative merits can be measured as low as .2, the system tends to perform comparable to standard CDMA (a ratio within .9) for the most part. The power reductions begin only after a certain point and the roll-off then happens fairly quickly. Thus, the true benefits of the slotted scheme in a single rate system (when

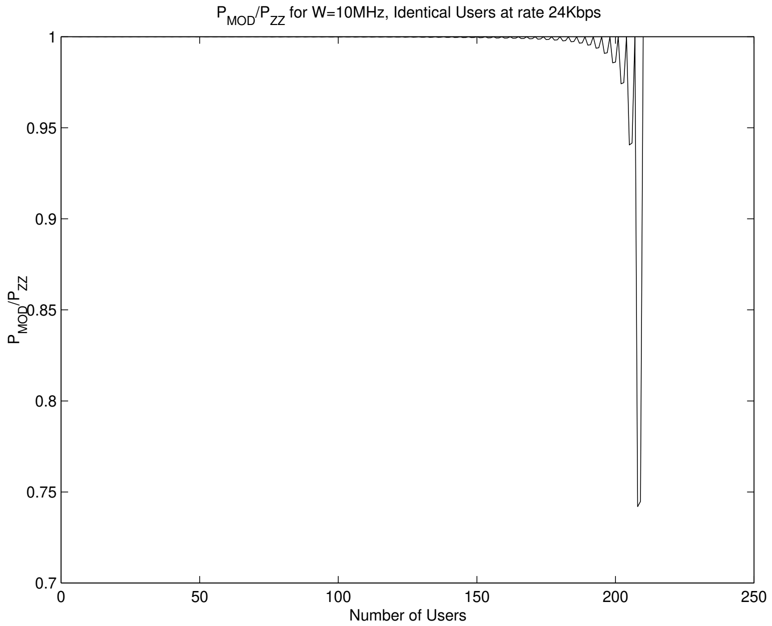

Figure 4.3: Ratio of the Power of the modified scheme to the Power of the discrete zig-zag scheme.

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factoring its implementational complexity) lies only within a particular range of system loading.

Chapter 5

Repeated Transmissions

This final chapter addresses the slotted CDMA system with a slightly different set of assumptions. Like the relaxed assignment policy, this allocation scheme also allows for system users to access more than one slot in the time frame. The transmission in each slot, however, constitutes the same sequence of bits or packets. Thus for each assigned slot, the user transmits the same information bits. What varies in each repeated transmission is the amount of energy the user expends in transmitting the sequence of bits, i.e. the power per slot. The receiver's role in decoding the packets is relegated over the entire frame. For each slot received at the end-station, the receiver weights the received waveform. It then linearly combines the received, weighted waveforms from each slot and then detects the transmitted sequence of bits, i.e packets. The end requirement would then be that the final weighted waveform meet the minimum SIR requirement of that user's data stream.

The intial incentive for such a formulation of the slotted scheme came from its parallel to the results presented in [25] and [26]. In these works, the authors determined the optimal code sequences for users in a synchronous CDMA system operating with linear receivers. What we show here is that the analysis performed in calculating the optimal code sequences can be utilized to compute optimal power assignments in this repeated transmission model of the slotted CDMA system. To do this effectively, we begin in section 5.1 by summarizing the results from [25] and [26] in regard to optimal code sequences. Then in section 5.2, the particularities of this repeated transmission systems are laid out. In addition, we then use the results of section 5.1 to propose the optimal power assignments for this type of a slotted scheme. Due to time constraints, we were unable to implement any of the results in this chapter. Thus the discussion presented in this chapter are purely theoretical.

5.1 Summary of Optimal Code Sequence Calculation

Previous work, [25]-[26], has presented results for a problem that is fairly to the proposed repeated transmissions. The most obvious difference in the two is the time-scale of the problem. In the previous literature, the authors determine the optimal code sequences for users in a synchronous CDMA system using linear receivers. The optimal code sequences are at the chip-level, whereas the problem we are looking at attempts to find the optimal power levels at the slot level. Before looking at these differences, let us first determine the optimal codes for a synchronous CDMA system using a linear receiver as in [26].

Consider the uplink of a synchronous spread-spectrum system in which there are N simultaneous users with identical rate requirements. Let the processing gain of the system be L so that the signature sequences $\mathbf{s_1}, \mathbf{s_2}, \ldots \mathbf{s_N}$ will each be chosen from the unit sphere in $\Re^L$. The information transmitted by each user is modeled as zero-mean random variables, $X_1, X_2, \ldots X_N$. Let the variance of these random variables represent the power at which user i is received, i.e. p_i. Also assume the presence of ambient white Gaussian noise with one-sided power spectral density, η_o. Let us denote the noise for all the users in vector notation as $\mathbf{W} \sim N(0, \eta_o \mathbf{I})$. The received signal at the receiver, $\mathbf{Y}$, is then

$$\mathbf{Y} = \sum_{i=1}^{N} \mathbf{s_i} X_i + \mathbf{W} \tag{5.1}$$

Now we will decode this received signal for user i using a linear receiver denoted by $\mathbf{c}_i$ which is also a vector in $\Re^L$. The linear receiver effectively scales each chip, i.e. each element of the vector $\mathbf{Y}$, with the corresponding element of $\mathbf{c}_i$. The receiver then sums these weighted chips over the L chips to yield an initial decision statistic. The resulting statistic for user i would then be

$$\mathbf{c}_i^T \mathbf{s}_i X_i + \sum_{j \neq i} \mathbf{c}_j^T \mathbf{s}_j X_j + \mathbf{c}_i^T \mathbf{W} \tag{5.2}$$

The power of the signal component (i.e. the first term above), is then $(\mathbf{c}_i^T\mathbf{s}_i)^2p_i$. Similarly the total interference power is $\sum_{j\neq i}(\mathbf{c}_j^T\mathbf{s}_j)^2p_j$ and the power of the noise is $\eta_o\mathbf{c}_i^T\mathbf{c}_i$. The signal to interference ratio of user i, SIR_i is then

$$SIR_i = \frac{(\mathbf{c}_i^T\mathbf{s}_i)^2p_i}{\sum_{j\neq i}(\mathbf{c}_j^T\mathbf{s}_j)^2p_j + \eta_o\mathbf{c}_i^T\mathbf{c}_i} \tag{5.3}$$

It is well known that the Minimum-Mean-Square-Error (MMSE) receiver is one that maximizes the SIR of each user. A derivation for this statement is found in [26]-[27]. The choice of the MMSE receiver implies that the linear coefficients, $\mathbf{c_i}$ should be chosen such that

$$\mathbf{c}_i = \mathbf{Z}_i^{-1}\mathbf{s}_i \tag{5.4}$$

where

$$\mathbf{Z}_i = \mathbf{S}_i\mathbf{D}_i\mathbf{S}_i^T + \eta_o\mathbf{I} \tag{5.5}$$

$$\mathbf{S}_i = [\mathbf{s}_1 \ldots, \mathbf{s}_{i-1}, \mathbf{s}_{i+1}, \ldots, \mathbf{s_N}] \tag{5.6}$$

$$\mathbf{D_i} = \mathrm{diag}[p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_N] \tag{5.7}$$

The SIR of each user for MMSE detection is then found by replacing (5.4) in (5.3)

$$SIR_i = \mathbf{s}_i^T\mathbf{Z}_i^{-1}\mathbf{s}_ip_i \tag{5.8}$$

The authors of [26] then determine the optimal sequences $\mathbf{s}_1, \mathbf{s}_2, \ldots \mathbf{s}_N$ for a synchronous CDMA system equipped with an MMSE receiver. The optimality of these codes are such that they both maximize the system capacity and can minimize the component-wise power selection $p_1, p_2, \ldots p_N$. The proof of this is detailed in [26] and is not presented here. These optimal codes are determined to satisfy $\mathbf{S}\mathbf{S}^T = \frac{N}{L}\mathbf{I}$ where $\mathbf{S} = [\mathbf{s}_1\ \mathbf{s}_2 \ldots \mathbf{s}_N]$. Such $\mathbf{S} = [\mathbf{s}_1\ \mathbf{s}_2 \ldots \mathbf{s}_N]$ are referred to as Welch Bound Equality (WBE) codes [28], [25], [29]. Thus using WBE codes leads to optimal capacity as well as optimal power performance. In fact the minimum power allocation for each user when using WBE codes is equal to

$$p_i = \frac{\eta_o\gamma L}{L(1+\gamma) - N\gamma} \qquad \forall i \tag{5.9}$$

where γ is the desired minimum SIR for each user.

A second implication of the use of WBE codes lies in the receiver structure. These optimal codes were derived for a system operating with an MMSE receiver. However, applying the minimum power allocation above to each user, we see that the linear coefficients for the MMSE receiver are

$$
\begin{aligned}
\mathbf{c}_i &= \mathbf{Z}_i^{-1}\mathbf{s}_i && (5.10)\\
&= \left(\eta_o\mathbf{I} + \sum_{j\neq i} p_j \mathbf{s}_j^T\mathbf{s}_j\right)^{-1}\mathbf{s}_i && (5.11)\\
&= \left(\eta_o\mathbf{I} + p\mathbf{s}_i^T\mathbf{s}_i + p\mathbf{S}\mathbf{S}^T\right)^{-1}\mathbf{s}_i && (5.12)\\
&= \left(\eta_o\mathbf{I} + p\mathbf{s}_i^T\mathbf{s}_i + \frac{pN}{L}\mathbf{I}\right)\mathbf{s}_i && (5.13)\\
&= K\mathbf{s}_i && (5.14)
\end{aligned}
$$

where K is a constant. Utilizing the optimal sequences, the MMSE filter is therefore a scaled matched filter receiver for the same system.

5.2 Repeated Transmissions Slotted System

Having gleaned the above results from previous work done on optimal sequence selection for synchronous CDMA systems, we now revisit our repeated transmission formulation. In it, we assume that each user has a certain maximum power limit that it will expend for transmission over a frame. We also make the assumption that each user will transmit identical packets in each assigned slot. The variation in the transmitted signal in each assigned slot will be in the amount of power the user spends on transmission, i.e. there will be a variation in the power-per-slot. Note that this power-per-slot may equal zero during certain slots.

Since the same bits are transmitted in each slot, the detection of each bit is performed by combining these measurements in each slot over the entire frame. First the receiver of user i takes the received signal in each slot and filters it thru a standard linear receiver as in (5.2). This linear filter of user i, denoted by $\mathbf{c}_i$, is the same for each slot over the frame. The resulting statistic, similar to (5.2), is then scaled on a per-slot basis. These scaled statistics are summed over all the slots for each user. Finally, the resulting summed statistic is then used as the measurement for detecting the sequence of bits transmitted by the desired user. The details of this reception process and its implications are presented below.

As stated earlier each user i transmits the same bits in each slot. The identical signals are however transmitted at different power levels. In slot j user i transmits its bits with power $\alpha_{ij}^2 p_i$. Note that p_i is the maximum power user i will dissipate over a frame. Thus we have the requirement

$$\sum_{j=1}^{M} \alpha_{ij}^2 p_i \leq p_i \tag{5.15}$$

To achieve average power $\alpha_{ij}^2 p_i$ in each slot, the user can scale its unit energy signature waveform to $\alpha_{ij}\sqrt{p_i}$. The transmitted signal of a bit of user i in slot j then looks like $\alpha_{ij}\sqrt{p_i} X_i \mathbf{s}_i$, where $\mathbf{s}_i$ is user i's signature sequence and X_i is the transmitted symbol (bit).

As before a linear receiver is used for decoding each user's symbol. Without a loss of generality, we will now look at the decoding process described here for the first bit transmitted by user i. The result of this "filtering" for user i's first bit in slot j is (using (5.2))

$$\alpha_{ij} X_{i1} \mathbf{c}_i^T \mathbf{s}_i + \sum_{k \neq i} \alpha_{kj} \mathbf{c}_i^T \tilde{\mathbf{s}}_k + \mathbf{c}_i^T \mathbf{w} \tag{5.16}$$

Here X_{il} represents the lth distinct bit transmitted by user i. Also note that since we are dealing with a multirate system, $\tilde{\mathbf{s}}_k$ above ($\forall\ k \neq i$) is the "effective" transmitted sequences of the other users relative to the symbol duration of user i. For example, if user $k \neq i$ has a rate greater than that of user i (but less than twice the rate of any user i) and the symbol duration of user k is L_k, then

$$\tilde{\mathbf{s}}_k = \left[X_{k1} s_{k1} \quad X_{k1} s_{k2} \ldots X_{k1} s_{kL_k} \quad X_{k2} s_{k1} \quad X_{k2} s_{k2} \ldots X_{k2} s_{k(L_i - L_k)} \right] \tag{5.17}$$

Note that $s_{k1}, s_{k2}, \ldots s_{kL_k}$ is the signature sequence of user k. If, on the other hand, user k has rate less than user i:

$$\tilde{\mathbf{s}}_k = [X_{k1} s_{k1} \quad X_{k1} s_{k2} \ldots X_{k1} s_{kL_i}] \tag{5.18}$$

Now, as a part of the decoding for this proposed slotted system, we will scale the output of the linear receiver of user i in slot j with a factor β_{ij}. Thus for the result in (5.16), we get:

$$\beta_{ij}\alpha_{ij}X_{i1}\mathbf{c}_i^T\mathbf{s}_i + \sum_{k\neq i}\beta_{ij}\alpha_{kj}\mathbf{c}_i^T\tilde{\mathbf{s}}_k + \beta_{ij}\mathbf{c}_i^T\mathbf{w} \tag{5.19}$$

Once we have obtained these statistics of user i's first bit in each slot j, we sum these measurements over all j to get:

$$\beta_i^T\alpha_i\left(X_{il}\mathbf{c}_i^T\mathbf{s}_i\right) + \sum_{k\neq i}\beta_i^T\alpha_k\left(\mathbf{c}_i^T\tilde{\mathbf{s}}_k\right) + \beta_i^T\left[\mathbf{c}_i^T\mathbf{w}_1 \;\; \mathbf{c}_i^T\mathbf{w}_2\ldots\mathbf{c}_i^T\mathbf{w}_M\right]^T \tag{5.20}$$

where $\mathbf{w}_j$ is the noise vector in slot j, $\beta_{\mathbf{i}}^{\mathbf{T}} = [\beta_{i1}\ldots\beta_{iM}]^T$ and $\alpha_{\mathbf{i}} = [\alpha_{i1}\ldots\alpha_{iM}]^T$. Let us call $X_i^* = X_{i1}\mathbf{c}_i^T\mathbf{s}_i$ and $\tilde{X}_j = \mathbf{c}_i^T\tilde{\mathbf{s}}_k$. Let $\tilde{\mathbf{w}} = \left[\mathbf{c}_i^T\mathbf{w}_1 \;\; \mathbf{c}_i^T\mathbf{w}_2\ldots\mathbf{c}_i^T\mathbf{w}_M\right]^T$. Note that X_i^* and $\tilde{X}_j$ are zero-mean independent random variables with finite second moments. Also observe that $\tilde{\mathbf{w}}$ is a zero-mean Gaussian random vector. Simplifying (5.20), we get:

$$\beta_i^T\alpha_i X_i^* + \sum_{k\neq i}\beta_i^T\alpha_k\tilde{X}_k + \beta_i^T\tilde{\mathbf{w}} \tag{5.21}$$

This is measurement is exactly as (5.2) with $\beta_i \sim \mathbf{c}_i$, $\alpha_i \sim \mathbf{s}_i$, $X_i^* \sim X_i$, $\tilde{X}_j \sim X_j$, and finally $\tilde{\mathbf{w}} \sim \mathbf{w}$. Therefore, we can follow the arguments that the authors presented in [26], and compute the optimal β_i as

$$\beta_i = \mathcal{Z}_i^{-1}\alpha_i \tag{5.22}$$

Here $\mathcal{Z}_i = \mathbf{A}_i\mathbf{P}_i\mathbf{A}_i^T + \sigma^2\mathbf{I}$. Note that

$$\begin{aligned}
\mathbf{A}_i &= [\alpha_1 \;\; \alpha_2\ldots\alpha_{i-1} \;\; \alpha_{i+1}\ldots\alpha_N] && (5.23)\\
\mathbf{P}_i &= \mathrm{diag}\left[\mathrm{E}[(\tilde{X}_1)^2] \;\; \mathrm{E}[(\tilde{X}_2)^2]\ldots\mathrm{E}[(\tilde{X}_{i-1})^2] \;\; \mathrm{E}[(\tilde{X}_{i+1})^2]\ldots\mathrm{E}[(\tilde{X}_N)^2]\right] && (5.24)\\
\sigma^2\mathbf{I} &= \mathrm{E}[(\tilde{\mathbf{w}})^2] && (5.25)
\end{aligned}$$

where $\alpha_k = [\alpha_{k1} \;\; \alpha_{k2}\ldots\alpha_{kM}]^T$. Once again using the similarity between (5.2) and (5.21), we can deduce that the optimal α_i's (for both maximum capacity and minimum power consumption) must satisfy the WBE code requirement: $\mathbf{A}\mathbf{A}^T = \frac{N}{M}\mathbf{I}$, where $\mathbf{A} = [\alpha_1 \;\; \alpha_2\ldots\alpha_N]$.

Planning the transmissions and the reception in the sequence described above therefore converts this particular slotted system into a time-scaled version of the optimal sequence computation in [26].

chapterConclusion

The recent attention given to alternate CDMA-based transmission schemes has attempted to provide performance benefits for future systems. The study in this thesis attempted to quantify any power benefits of one such system that relies on time-separation of multirate CDMA signals. To do this, we constructed a time-frame atop the standard variable-processing gain CDMA system. This time frame was composed of M different slots that were at the least capable of supporting the most demanding user rate. Within each slot, each system user was assigned a transmission rate which coupled with its transmission rates in the other $M-1$ slots allowed for the user to meets its minimum rate requirement over the frame. With a given rate, each user in each slot achieved that particular rate via variable-processing gain multirate CDMA signaling. The thesis work then attempted to discern those user-to-slot allocation schemes that reduced power consumption, or equivalently increased system capacity.

More specifically, the work here focused primarily on two explicit types of channel or slot allocation schemes. In the first, users were given one of M orthogonal channels. In the second multi-channel access was allowed among the users, i.e. multiple slot assignments. The problem with both of these schemes lies in the complexity in finding an optimal solution. In the discrete case, the lowest power consumption of any of the assignments studied was the load-balancing which clearly was too complicated for practical implementation. In the relaxed assignment policy, we were unable to pinpoint one succinct power-minimizing heuristic. Our study of these two policies therefore had to focus on finding power-reducing techniques, not necessarily the optimal power reduction method.

For the discrete assignment policy, we devised several schedulers. The pairwise load-balancing scheduler was too complicated for implementation in practical systems while certain random schedulers had inferior performance compared to standard CDMA. We did, however, implement the zig-zag scheduler for the discrete assignment policy. The performance of such a slotted system produced varied results depending on the user rate requirements. For a single rate system, this zig-zag scheduler was in essence the optimal discrete scheduler for this system. The slotted system operating on single rate users produced power savings that improved with system loading. For lightly loaded conditions, the power reduction due to time-slotting was fairly marginal when compared to the complexity introduced by the actual implementation of this slotting scheme. The benefits were more consequential under heavier user demands. Not only was average power consumption reduced but system capacity was simultaneously improved via the few additional users supported in the identical rate system. Though the single-rate system indicate performance improvements, the measured gains were countered by inherent complexity in slotting a CDMA system.

Implementing slotted transmission with multirate requirements, on the other hand, produced substantial power reduction and capacity gains. Whereas the single rate slotted system could support additional users in the order of the number of slots, the multirate version could support many times more (as per the results presented here). As we saw at the end of Chapter 3, the *sub-optimal* zig-zag scheme alone reduced power consumption by more than a factor of ten and supported significantly more users when compared to standard CDMA.

The nature of the problem formulation for the relaxed assignment policy implied that we could not develop a scheduler for the multirate system. With some analysis, we were able to devise a scheduler for the single-rate system that improves the power performance when compared to the discrete assignment case. The savings in moving from discrete to multislot access for the technique presented here was not ultimately substantial. Though the formulation and analysis of the second of these schemes has left room for potentially many more allocation schemes, those studied here did not offer more than a 25 percent additional reduction in power when compared to the results of the discrete allocation scheme. In fact, such reduction was seen only at high system loading. Also, in this study one observed that conventional CDMA, for both systems with multirate and single rate users, is a suboptimal design. This came about both via theoretical work on simple systems and experimental results from much larger and more complex systems.

In conclusion, we saw here that as long as single-rate systems remain lightly loaded their performance does not really suffer in a substantial way using CDMA signaling. However, under heavy load, average power measurements differ significantly between the standard and the slotted systems. Additionally, the introduction of multirate traffic stream brings the limitations of conventional CDMA drastically closer. As proposed by the work done here, the simple answer to such limitations is to introduce time (or frequency) based separation of users. Under heavy system load or multirate users, there are benefits to be gained in such a system at the expense of increasing infrastructure complexity.

Since we have obtained significant results for slotting multirate users, this problem begs further analysis. In the performance measures presented here, we implemented just the zig-zag scheme. Other, perhaps more optimal, schedulers may produce even more substantial results. Furthermore, we approached our analysis using received power measurements, not transmit power levels. This variation needs to be examined as well since transmit power reductions are of particular interest in wireless systems. For such a formulation, we would have to incorporate each user's maximum transmit power level constraint when calculating the transmission schedule.

In addition to the discrete and relaxed assignment policies, this thesis also proposed an alternate slotted CDMA system in which users repeat their transmission during each assigned slot. The analysis of this system was done completely in theoretical terms. Optimal power levels were computed for such a system in terms of WBE in a procedure that very closely resembles the computation of optimal sequence codes for a CDMA system. Clearly one future course for this work is to formalize the repeated transmission scheme with further analysis and performance measures.

Appendix 1

Recall the the Lagrangian for the relaxed assignment problem was

$$\mathbf{l}(\beta^*) = \nabla f(\beta^*) + \lambda^T \nabla \mathbf{g}(\beta^*) + \mu + \nu = 0 \tag{.26}$$

Under standard CDMA transmissions, we noted that $\mu = \nu = 0$. The Lagrangian at the CDMA extrema (which we will now denote as β^*_{CDMA}) is therefore

$$\mathbf{l}(\beta^*_{CDMA}) = \nabla f(\beta^*_{CDMA}) + \lambda^T \nabla \mathbf{g}(\beta^*_{CDMA}) \tag{.27}$$

The multirate CDMA operating point is a relative minimum of the relaxed assignment problem if the Hessian of the Lagrangian in (.26) evaluated at β^*_{CDMA} is positive semidefinite on the subspace tangent to the constraint set. In other words, if

$$\mathbf{L}(\beta^*_{CDMA}) = \mathbf{F}(\beta^*_{CDMA}) + \lambda^T \mathbf{G}(\beta^*_{CDMA}) \tag{.28}$$

(where $\mathbf{L}(\beta^*)$ is the Hessian of the Lagrangian) is positive semidefinite on

$$M = \{\mathbf{y} | \nabla \mathbf{g}(\beta^*_{CDMA})\mathbf{y} = \mathbf{0}\} \tag{.29}$$

Because of the convexity of the cost function $f(\beta)$, we already know that $\forall \sum_{j=1}^{M} \beta_{ij} < 1$, the Hessian of $f(\beta)$ is positive definite. Thus for the Hessian of $\mathbf{l}(\beta)$ to be positive semidefinite, we have to provide conclusive proof of the positive semi-definiteness of $\lambda^T \mathbf{G}(\beta^*_{CDMA})$. To do this, let us first compute $\lambda^T \mathbf{G}(\beta^*) = \nabla(\lambda^T \nabla \mathbf{g}(\beta^*_{CDMA})$:

$$\lambda^T \mathbf{G}(\beta^*) = \begin{bmatrix} c_{11} & \frac{\frac{\partial \lambda_1}{\partial \beta_{21}}}{(1-\beta_{11})^2} & \cdots & \frac{\frac{\partial \lambda_1}{\partial \beta_{NM}}}{(1-\beta_{11})^2} \\ \frac{\frac{\partial \lambda_2}{\partial \beta_{11}}}{(1-\beta_{21})^2} & c_{21} & \cdots \frac{\frac{\partial \lambda_2}{\partial \beta_{NM}}}{(1-\beta_{21})^2} & \\ \vdots & & & \\ \frac{\frac{\partial \lambda_N}{\partial \beta_{11}}}{(1-\beta_{NM})^2} & \frac{\frac{\partial \lambda_N}{\partial \beta_{21}}}{(1-\beta_{NM})^2} & \cdots & c_{NM} \end{bmatrix} \tag{.30}$$

where c_{ij} is:

$$c_{ij} = \frac{\frac{\partial \lambda_i}{\partial \beta_{ij}}(1-\beta_{ij})^2 + 2\lambda_i(1-\beta_{ij})}{(1-\beta_{ij})^4} \tag{.31}$$

For the CDMA operating point we know that: $\beta_{ij} = \frac{\frac{\gamma_i}{M}}{\frac{\gamma_i}{M}+W}$ for all i and j. Furthermore, $\lambda_i = \frac{(1-\beta_{ij})^2}{(1-\sum_{k=1}^{N}\beta_{kj})^2}$. Thus:

$$\frac{\partial \lambda_i}{\partial \beta_{ij}} = \frac{2(1-\beta_{ij})\left[(1-\beta_{ij})-(1-\sum_{k=1}^{N}\beta_{kj})\right]}{(1-\sum_{k=1}^{N}\beta_{kj})^3} \quad \forall i,j \tag{.32}$$

$$\frac{\partial \lambda_i}{\partial \beta_{nj}} = \frac{2(1-\beta_{ij})^2}{(1-\sum_{k=1}^{N}\beta_{kj})^3} \quad \forall n \neq i \tag{.33}$$

$$\tag{.34}$$

Plugging these values into the matrix above, we get:

$$\lambda^T \mathbf{G}(\beta^*_{CDMA}) = \begin{bmatrix} a & a & a & \dots & a \\ a & a & a & \dots & a \\ \vdots & & & & \\ a & a & a & \dots & a \end{bmatrix} \tag{.35}$$

where $a = \frac{2}{(1-\sum_{k=1}^{N}\beta_{kj})^3}$. For any non-zero vector $\mathbf{x} \in \Re^{NM}$, it is now trivial to show that $\mathbf{x}^T(\lambda^T\mathbf{G}(\beta^*_{CDMA}))\mathbf{x} = a\left(\sum_{i=1}^{NM} x_i\right)^2 > 0$. With the positive semidefiniteness of $\lambda^T\mathbf{G}(\beta^*_{CDMA})$, we know that $\mathbf{L}(\beta^*_{CDMA})$ is also positive semi-definite. The result therefore is that we can now characterize the multirate CDMA operating point as a local minimum of the relaxed assignment problem.

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