RADIO RESOURCE MANAGEMENT IN POWER CONTROLLED CDMA SYSTEMS

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ABSTRACT OF THE DISSERTATION

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With an increasing demand for wireless communication services, better radio resource management is needed to improve system performance. Previous work decomposed the resource management problem into separate power control, base station assignment and channel assignment problems. In this thesis, we study resource management problems in power controlled Code Division Multiple Access (CDMA) systems. We propose an integrated approach to power control, base station assignment and call admission.

In a power controlled system, all users adjust their transmitter powers to maintain feasible connections. We formulate the minimum transmitted power (MTP) problem which minimizes the total transmitted power subject to meeting users' link quality requirements, as measured by the carrier to interference ratio (CIR). We find an iterative distributed algorithm, the minimum power assignment algorithm (MPA), that converges to a global optimal solution for any feasible instance of the MTP problem. In the presence of maximum power constraints, the constrained MPA algorithm is shown always to converge to a unique fixed point. When there is no feasible solution, the infeasibility can be detected at the fixed point solution which lies on the boundary

of the maximum power set. In this case, some users will transmit at maximum power without achieving the desired CIR. Both MPA and constrained MPA are shown to converge both synchronously as well as totally asynchronously.

The rate of convergence of a generalization of the constrained MPA algorithm is also studied. This generalization includes both a fixed assignment of users to base stations as well as a form of soft handoff assignment. It is shown that iterative constrained power control will converge to a unique fixed point at a geometric rate. This is important for the maintenance of good service under rapid variations in channel quality and user mobility. Simulation results also indicate the rate of convergence of the constrained MPA algorithm is fairly practical compared to current CDMA standard.

During the call set up stage, call admission control is required to improve system performance due to limited radio resources. In this work, a weighted combination of call blocking and dropping probabilities is used as a system performance measure. We consider two different approaches to call admission control. The Transmitter Power Call Admission Control (TPCAC) scheme protects existing users by blocking new calls when existing users can not maintain the target CIR. In the Received Power Call Admission Control (RPCAC) algorithm, each base station reserves capacity for handoff traffic by blocking new calls when the total received power at that base exceeds a threshold. The performance of proposed schemes are evaluated by simulation of a one dimensional system. It is found that the RPCAC scheme can provide significantly improved performance.

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Dedication

This document is dedicated to my wife, Ming Chun Chiang. Without her patient and support for those countless overnight works. I can never finish my research works. A special thank to my parents for encouraging me and financially supporting me for all theses years. A wish from them becomes the biggest benefit in my life. A joyful thank is to my son, Jasper. It is worth to delay my study by having him with me for three years.

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Chapter 1

Introduction

With the increasing demand for wireless communication services and more stringent service quality requirements, new transmission technologies and improved radio resource management, including base station assignment, channel assignment, power control and handoff, are required for wireless communication systems. In this chapter, we will explain the basic ideas of cellular radio systems. This introduction will lead to the resource management problems that we will investigate in this thesis. This chapter also contains a literature review and concludes with the objectives of the thesis.

1.1 Cellular Communication Systems

The objective of a cellular communication system is to provide communication services anytime and anywhere and also to maintain connection quality regardless of user motion. The mobility of the users in cellular systems causes new problems that do not occur in wired telephone systems. The changes in radio link quality with user motion make the control problems of radio resource management (RRM) more complicated.

In cellular communication systems, there are sets of mobiles, base stations and mobile switching centers (MSC). As depicted in Figure 1.1, an MSC which has wire connections to all its base stations, links the wireless communication networks to fixed land telephone networks. The base stations, working as an intermediate system, have radio link connections to mobiles. The main purpose of the base station is to process the received signal and to transfer information and control messages between the mobiles and the MSC. Due to user motion, the link quality will vary with time. How to manage the resources to support every user's quality requirement

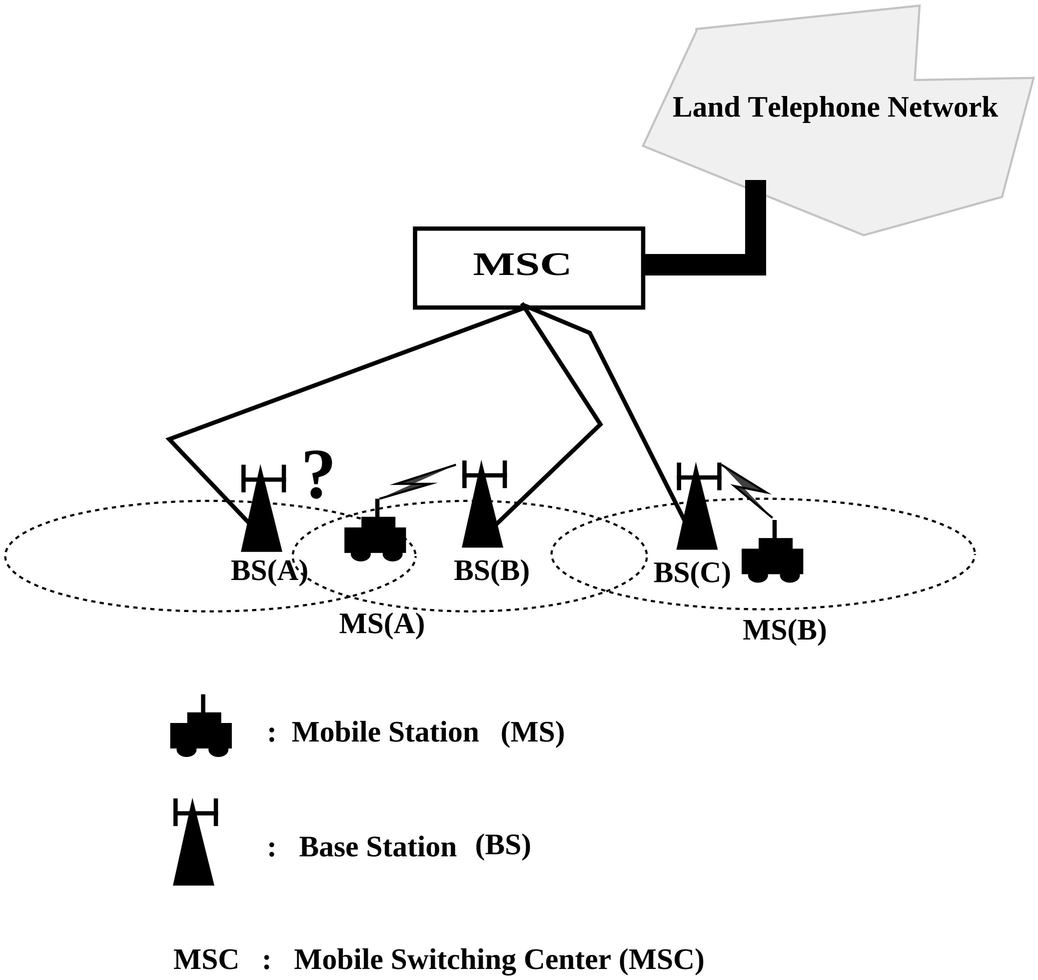


Figure 1.1: The Cellular Communication Systems

is the major issue for the RRM.

When the number of users increases, a more careful resource assignment should be considered. In general, the resources in the uplink (mobile to base station) include base stations, channels and transmitter powers. The system assigns a base station, channel and required transmitted power to a user. However, restricted radio bandwidth limits the number of users that can have acceptable connections.

A user's access to a cellular communication system can be divided into two stages, the call set up stage and the maintenance stage. As shown in Figure 1.2, the call set up stage occurs when there a user tries to obtain the resources necessary for a radio link connection. The

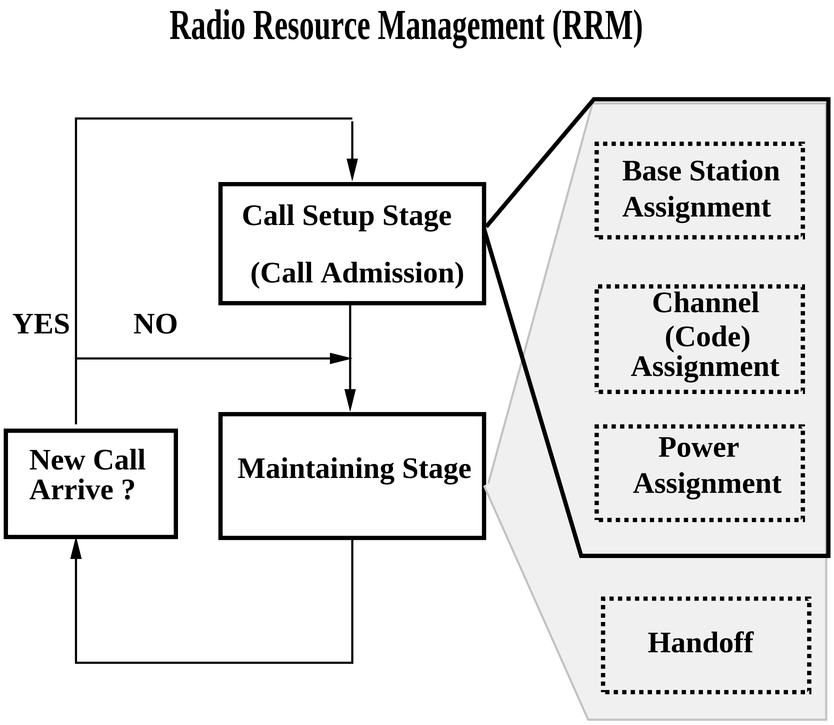


Figure 1.2: Two Stages Radio Resource Management

maintenance stage occurs when there is no new user coming but due to the user motion each user will change his share of resources.

The call admission control that incorporates base station assignment, channel assignment and power control is required for the call set up management. We believe that dropping a call in progress is more serious than blocking a new call. Due to the user mobility, new call might be rejected, even when the system has available resources, to reduce the call dropping. The decision of whether to admit a call should be based on the joint consideration of base station assignment, channel (code) assignment and power assignment.

A dynamic resource management is required in the maintenance stage to maintain link quality. Due to the user mobility, the link quality is not fixed and will vary with time. The protection of existing users during the call set up stage can not be guaranteed to avoid the degradation in link quality. In order to maintain the link quality, a continuous dynamic power control is required to compensate the changes caused by propagation loss and fading. It is important that a power control algorithm can effectively converge to a desirable solution when a fixed radio channel permits such a solution. A reassignment of base station and channel may be needed when the link quality is bad or to optimize the use of radio resources.

As discussed so far, we see that the user mobility complicates the control problems of RRM in wireless systems. We would like to further discuss radio access and resources of different transmission technologies.

1.1.1 Radio Access

In cellular systems, there exist different transmission technologies. Each method provides services in a distinct way. We will discuss the transmission technologies that have been implemented or proposed for the North American market.

The current cellular telephone system, known as Advanced Mobile Phone Services (AMPS), uses Frequency Division Multiple Access (FDMA) for transmission. In AMPS, the frequency spectrum is divided into channels, each with 30 kHz bandwidth. Because of co-channel interference, nearby users will use different frequency bands for transmission.

With the increasing demand for wireless services, digital transmission techniques will be introduced in second generation wireless systems. In particular, both Time Division Multiple Access (TDMA) and Code Division Multiple Access (CDMA) have been proposed to replace or coexist with the current FDMA system.

The TDMA system, known as IS-54-B [1], is a hybrid of FDMA and TDMA techniques. The 30 kHz band, used as one channel in AMPS, will be further divided into six time slots. Those users who share the same spectrum will transmit at different time slots. With the promise of additional capacity and better quality, CDMA, standardized as IS-95 [2], is used in second generation cellular communications and will be the one of the candidates for the future personal communication services (PCS) network [3]. With CDMA, each signal consists of a different pseudorandom binary sequence that provides individual privacy. Unlike TDMA/AMPS systems, a large number of CDMA signals share the same frequency spectrum at the same time.

At the receiver, the signals are separated by using a correlator, which accepts only signal energy from the selected binary sequence and despreads its spectrum. The other users' signals, whose codes do not match contribute only a form of noise and represent a self-interference generated by the system.

1.1.2 Radio Resource Management

In cellular communication systems, the resource management includes base station assignment, channel assignment, power control and handoff. We will discuss these individual resource management tasks for different transmission technologies.

Base Station Assignment

For base station assignment, it is intuitive to think that a base station should be chosen such that less transmitted power is required in order to reduce the interference seen by other users. In fact, reduced transmitter power favors not only the interference but also the power consumption.

Currently, for all systems, a base station will be assigned to a user when the user receives the strongest pilot signal from this base station [4, 5]. If no fading exists, a user always connects to the nearest base station. This assignment minimizes the transmitted power needed to overcome propagation losses. In this work, we will propose a minimum power assignment scheme in which a base station will be chosen when the required transmitted power to achieve the quality requirement is minimum.

Channel Assignment

As mentioned, in the FDMA (AMPS) system, the channel is represented by a frequency band. In the TDMA system, the channel is represented by a time slot. In FDMA and TDMA systems, there are two kinds of channel assignment. One is fixed channel allocation (FCA) where each base station has a fixed subset of channels at its disposal. The other is dynamic channel allocation (DCA) where channels can be used in all base stations as long as the co-channel interference is not too high.

In CDMA systems, the channel is represented by a code. Due to the fact that users share the same frequency spectrum at all time, each user needs its own code to distinguish the signals from others. As described in the IS-95 system, the code assignments are different in the downlink and the uplink. In the downlink (base station to mobile), the information is encoded using a convolutional code and interleaved over 20 ms. The resulting sequence is then coded using a row of the 64×64 Walsh matrix which will be embedded in every code word. The Walsh function has a property that every row is orthogonal to every other row. Since each user use different Walsh function, the orthogonal property will be preserved. In the uplink, signals from different users are not distinguished by the Walsh function but by a user-address long code. The Walsh function is used as 64-ary orthogonal modulation and is determined by the information being transmitted. Even though users' signals received at the same base station are not orthogonal, due to the low correlation between the user-address long codes, different users can still be differentiated after despreading.

In the IS-95 system, channel (code) assignment is not based on link quality measures. As mentioned, in the downlink, the Walsh code is assigned without considering the link quality. A code will be assigned as long as there are available Walsh codes. In the uplink, the user-address long code can be either the terminal's electronic serial number or a predefined private code. In other words, in CDMA systems, the code assignment or reassignment does not relate to link quality measures. In this work, we won't consider the code assignment as a resource control to improve system performance.

Power Control

When we consider the interference limited systems such as the FDMA/TDMA systems using DCA and the CDMA systems, transmitted power is regulated to provide each user an acceptable connection by limiting the interference seen by other users. In the mobile propagation environment, this is achieved by providing dynamic power control for both the mobile station and base station transmitters.

With limited power control for FDMA and TDMA systems using FCA [1,6], the cell planning for the FCA scheme is based on the worst case situation where users at the cell boundary can still have acceptable signal quality. More precise power control schemes should be considered in the DCA scheme in which a channel can be used in all cells as long as the co-channel interference seen on that channel is not excessive.

Users in a given cell are always on the move. Some users are close to the base station and some are not. In CDMA systems, the power control is required to solve the near-far effect. The close users have stronger signals and would cause unacceptable interference on the uplink unless they are controlled.

The power control can be based either on signal strength or on link quality measures. In the signal strength based schemes, a user's transmitter power is adjusted to compensate for the attenuation from user to base station. However this signal strength based power control scheme is sensitive to co-channel interference [7]. The quality based power control scheme [8–10] provides greater control over connection quality but needs more information (quality measure) for power adjustment.

Handoff

Signal strength or a link quality measure will be used to decide the handoff procedure. After a call is initiated, the MSC will continuously keep a record of signal or link quality measurement between the user and the nearby base stations. This measurement can be either reported by base station or user.

For signal strength based handoff schemes, the system needs to set the threshold high enough to be guaranteed the connection quality will be above the minimum quality requirement throughout the handoff procedure. Currently, in AMPS as well as in the proposed IS-54-B and IS-95 systems, a user will be handed over to a base station when that user's received signal strength is below some threshold value. Among the candidate base stations or channels, the MSC will assign a base station or a channel for the user. In the IS-95 system, soft handoff is introduced where more than one base can be simultaneously assigned to a user to improve signal quality through selection diversity [11].

In other cases, it may be of interest to change the assigned base of a given user even if the connection quality is still adequate. This may happen when the global interference level would be significantly reduced if the mobile station were assigned to another base station. This can be done by quality based handoff control.

The quality based handoff scheme offers greater control over connection quality. Basically, a user always reassigns to a channel or a base station where there is a better link quality. Due to the fact that power adjustment directly relates to the link quality measure, the quality based handoff scheme can also reduce the waste of transmitted power.

We emphasize here that the handoff is trying to mitigate signal degradation due to user mobility but the results caused by handoff will be different for different transmission technologies. For TDMA/FDMA systems using a DCA scheme, the system has to be careful about the channel reassignment. In TDMA/FDMA systems, the change of channel assignment will also change the interference seen by other users who share the same channel. This is not true for CDMA systems. In a CDMA system, since all users share the same frequency spectrum, any power reduction is always good for all users. As discussed, the channel (code) reassignment doesn't actually change the link quality so the handoff problem becomes how to choose the best base station.

1.2 Literature Review

1.2.1 Power Control

A number of papers addressing the issue of the power control and channel assignment have been published in the past two decades. The signal strength based power control schemes have been discussed in [7, 12]. The transmitter power is adjusted to compensate the attenuation in the connection link. Since the signal strength based scheme doesn't consider the effect of interference, the performance will be sensitive to co-channel interference from users transmitting in the same channel [7]. The idea of carrier to interference ratio (CIR) based power control schemes is to control the transmitted power such that the signal quality can maintain a desired target CIR regardless of the user motion and connection impairments caused by fading. Aein [13] addressed the problem of balanced power control to reach a common CIR in satellite systems. The existence and the uniqueness of a feasible power vector associated with the eigenvalue of the gain matrix are found to be a consequence of the Perron-Frobenius theorem. From Aein's balanced power control to reach the common CIR in satellite systems, Meyerhoff [14] proposed an iterative procedure for determining this unique set of carrier power levels and demonstrated that maximizing a common CIR is equivalent to maximizing the minimum CIR over all users. Nettleton and Alavi [15] applied these results with no background noise to the spread-spectrum system. Nagatsu [7] and Fujii [16] showed the improvement of capacity by simulation. Zander [8, 17, 18] further refined the concepts of Aein, Nettleton and Alavi in the context of minimizing the outage probability of a target CIR and also focused on a distributed implementation to these algorithm and the relationship to dynamic channel allocation. Grandhi, Vijayan and Goodman [9, 19, 20] followed the same approach as Zander in FDMA and TDMA systems. Foschini [10] generalized the result to individual CIR targets with nonzero background noise and also showed a simple distributed power control that converges at a geometric rate. Mitra [21] proved geometric convergence for an asynchronous version of Foschini's algorithm. A new distributed power control scheme was proposed in [22] to maintain the CIR of all active links above the desired CIR at all times. Ariyavistakul [23] showed by simulation that the CIR based power control out performs a signal based power control scheme. Joint power control and base station assignment in the absence of transmitted power constraints is considered in [24, 25] for continuous power adjustments and in [26] for discrete power adjustments. In these works, constraints on the maximum power level are not considered and the assignment of users to base stations is specified by outside means. Under a fixed assignment of users to base stations, Grandhi and Zander [27] examined the maximization of the minimum CIR. In this work, it is found that when there is no feasible solution there exists a fixed point which lies on the boundary of the maximum power constraints. Yates and Huang [28] discuss the convergence problems for integrated power control and base station assignment in the presence of maximum power constraints. A framework for uplink power control and conditions for convergence are provided in [29]. For a system with two base stations, Papavassiliou and Tassiulas [30] consider an algorithm to solve joint channel, base station and power assignment.

1.2.2 Capacity

In a CDMA system, the number of users that each cell can support is limited by the total received interference at each base station and will vary with time. The cell capacity will be used not only by users actually assigned to the base station but also by users assigned to other base stations. This is because in a CDMA system all users share the same frequency spectrum in which a user will see the interference from all other active users.

The capacity of spread spectrum for mobile communication was first studied by Cooper and Nettleton [31] in which reuse factors were calculated as a function of the order of the propagation loss. In [32], the capacity analysis is further extended by adding log-normal fading in the path loss calculation. Viterbi, et al [33] calculated the other cell interference factor with varying standard deviation of log-normal fading. The effect on the system performance of choosing a base station with the least interference among the set of N_c base stations is included. In this work, the same number of users are assigned to each base station and all are spatially and uniformly distributed. The external interference will roughly use $25\% - 35\%$ capacity if all cells are equally loaded. The reduction of other-cell interference in going from using just the nearest cell ($N_c = 1$) to using the better of the nearest pair of cells ($N_c = 2$) is quite dramatic. Viterbi [34] calculated the Erlang capacity by replacing the classical blocking formulation condition to the requirement that the total interference exceeds the background noise by 10 dB. This soft blocking was used to argue that the capacity of the IS-95 system is 20 times that of AMPS. In [35, 36], it is found that the capacity is very sensitive to both propagation conditions and errors in the power control process.

1.2.3 Call Admission

In a wireless cellular network, optimal call admission policies have been found in [37] for systems without handoffs and in [38] for a linear highway system with handoffs. These works model the call admission problem as a Markov decision process. However, the combinatorial explosion of the state space makes this approach infeasible for large systems. An alternative is to find sub-optimal solutions by using heuristic search algorithms [39]. When the system state must include users' positions and transmitter powers, the continuous state space makes state based admission control algorithms intractable. In [40, 41], a power control method to maintain the active links above their required quality while new users are been admitted or blocked is considered. In [42, 43], carrier to interference ratio (CIR) thresholds are used for call admission control. In these works, call admission is based on available capacity and attempts with the protection of existing users. In [18], an admission control based on the outage probability associated with a target CIR is examined. In [44], admission control is formulated as an optimization problem with the objective being to provide acceptable service to as many mobiles as possible. In [45], soft admission control is concerned with the protection of existing users in orthogonal channel systems.

1.3 Scope of the thesis

The objective of this thesis is to increase the understanding of the effect of the RRM by integrating base station assignment, channel assignment, power control and handoff for the uplink communication in CDMA systems. The purpose of the RRM is to set up call connections and to maintain the radio link quality regardless of user motion, fading and changes in co-channel interference. Due to user mobility, it is important that a dynamic power control algorithm can effectively converge to s desirable solution. With limited radio resources, call admission control is required during the call set up stage to improve the system performance.

The thesis is is organized as follows. Chapter 2 introduces the system models, the quality measures for the radio link and the ways to evaluate the system performance. In Chapter 3, we consider the minimization of the total transmitted uplink power subject to maintaining an individual target CIR for each mobile. We show that this problem has special structure and identify synchronous and asynchronous distributed algorithms that find the optimal power vector and base station assignment. In Chapter 4, with constraints on the maximum transmitter power, we integrate power control and base station assignment. We find synchronous and asynchronous distributed algorithms that converge to a fixed point. When there exists a feasible solution, the fixed point is optimal in the sense of minimizing the total transmitted power subject to maintaining an individual target CIR for each mobile. In Chapter 5, the rate of convergence for uplink power control is investigated. We show that no matter the system can support all users the target CIR or not, a iteration power control based on CIR measures can guarantee to at least converges to a fixed point at a geometric rate. In Chapter 6, power based call admission schemes are proposed and evaluated by simulation of a one dimensional system. The discussion of conclusions and future work are included in Chapter 7.

Chapter 2

Models and Performance Measures

2.1 Introduction

In this chapter, we will first discuss quality measures for radio link connections and then we describe the system model used in the thesis. In system models, we focus only on the uplink connection. The performance measures for call admission design will be discussed in the last section.

2.2 Quality Measures

In digital communication systems, the quality measure is based on bit error rate (BER) or E_b/N_0 where E_b is the bit energy and N_0 is the noise power spectrum density. The CIR, γ, on the uplink is defined as

$$\gamma = \frac{\text{user's received power at the base station}}{\text{total interference received at the base station}} \tag{2.1}$$

and can be related to E_b/N_0 in following equation [46].

$$\gamma_i = \frac{E_b r_i}{N_0 \omega} \tag{2.2}$$

where r_i is the information rate of users i and ω is the bandwidth. We can see that the CIR of user i, γ_i, is a function of E_b/N_0 and the transmission rate r_i. With a fixed transmission rate, the CIR can be used as an index for signal quality.

Suppose we assume that each user i has an identical CIR requirement, $\gamma_i = \gamma$. For CIR balancing power control [13], all users assigned to base station k will have equal received power R_k. The CIR measure in terms of the received power is

$$\frac{R_k}{(N_k - 1)R_k + I_k + \eta_k} = \gamma \tag{2.3}$$

where at base station k, N_k is the number of users assigned, I_k is the external interference and η_k is the receiver noise. For a single cell system, $I_k = 0$ and if we assume $\eta_k = 0$ we have

$$N_k = 1 + \frac{1}{\gamma} \tag{2.4}$$

In CDMA systems, the processing gain, ω/r, is usually greater than 100 and the required E_b/N_0 is less than 10 (dB). From Equation (2.2), the required γ is less than 1. It says that in CDMA systems, each cell can support more than one user sharing the same frequency spectrum. For the same reason, the link quality in a CDMA system is less sensitive to the addition of one more user.

2.3 CDMA System Model

We assume N users and M base stations and a common radio channel. Let p_i denote the transmitted power of user i so that $\boldsymbol{p} = [p_1, ..., p_N]^\top$ denotes the power vector of the system. The corresponding received signal power of user i at base k is $h_{ki}p_i$ where h_{ki} denotes the gain for user i to base k. The interference seen by user i at base k is $\sum_{j \neq i} h_{kj}p_j$. We define $a_i = k$ if user i is assigned to base k. A base station assignment is a vector $[a_1, \ldots, a_N]$ that specifies an assigned base for each mobile. The CIR measure can be expressed in term of the transmitted powers

$$\frac{h_{a_i i}p_i}{\sum_{j \neq i} h_{a_i j}p_j + \eta_{a_i}} \geq \gamma_i \tag{2.5}$$

The quantity $\eta_{a_i} = N_0\omega$ describes the receiver noise at the assigned base of user i and γ_i is the required CIR for user i. A user simply treats all other users as the interference seen at the connected base station. This is especially true for a CDMA system where all active users share the same frequency spectrum. Any reduction in transmitted power always benefits all other users in the system.

When user i is assigned to base k, we can write the CIR constraint for user i as

$$p_i \geq \boldsymbol{H}_i^{(k)} \boldsymbol{p} + \delta_i^{(k)} \tag{2.6}$$

where $\delta_i^{(k)} = \gamma_i \eta_k / h_{ki}$ and $\boldsymbol{H}_i^{(k)}$ is a normalized row vector with jth element

$$H_{ij}^{(k)} = \begin{cases} \dfrac{\gamma_i h_{kj}}{h_{ki}} & j \neq i \\ 0 & j = i \end{cases} \tag{2.7}$$

We define $\boldsymbol{G}^{(l)}$, the normalized interference matrix under assignment l, as having i, jth element

$$G_{ij}^{(l)} = H_{ij}^{(a_i(l))} \tag{2.8}$$

Similarly, the normalized noise vector $\boldsymbol{\sigma}^{(l)}$ is defined to have ith component $\sigma_i^{(l)} = \delta_i^{(a_i(l))}$. We embed the set of CIR targets $\{\gamma_i\}$ in the interference matrix $\boldsymbol{G}^{(l)}$. Equation (2.7) can be rewritten in vector form

$$\boldsymbol{p} \geq \boldsymbol{G}^{(l)} \boldsymbol{p} + \boldsymbol{\sigma}^{(l)} \tag{2.9}$$

a power vector $\boldsymbol{p}$ is feasible under assignment l, if for each user i, $p_i \geq \boldsymbol{G}_i^{(l)} \boldsymbol{p} + \boldsymbol{\sigma}_i^{(l)}$, where $\boldsymbol{G}_i^{(l)}$ denotes the ith row of $\boldsymbol{G}^{(l)}$.

2.4 Simulation Model

In this study, we consider a 1-dimensional linear system model. As depicted Figure 2.1, all base stations are spaced uniformly on a ring. Each site has one base station connected to an omni directional antenna covering the street on both sides of the site. At call initiation, a new call chooses a direction, clockwise or counterclockwise, and a speed that remains fixed for the

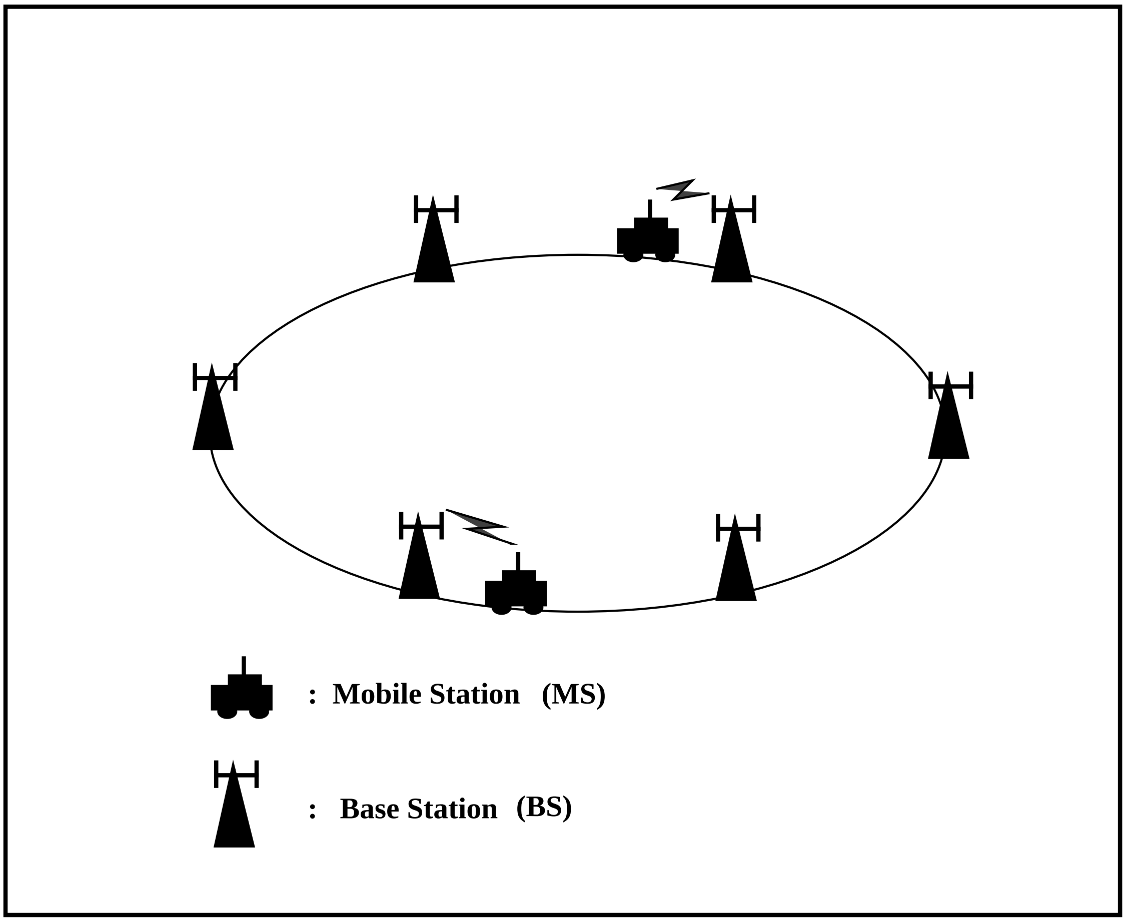


Figure 2.1: The ring model for linear cellular system.

duration of the call. The distance between mobile and base station is measured in linear scale.

Two types of simulation experiments are described in this thesis. First, we repeatedly place stationary mobiles on the ring and examine the rate of convergence of the power control algorithm by recording as a function of the number of iterations n, the fraction of mobiles that are guaranteed to be within 0.1 dB of the eventual CIR at the fixed point. Second, we examine the performance of the power control algorithm in a simulation of mobile users arriving and departing on the ring of base stations. In this case, call blocking and call dropping that results from user mobility define the system performance.

2.5 Teletraffic Model

In CDMA systems, we can view the system with M base stations as one isolated system with finite resources. The capacity of each base station is limited by the total interference received at each base station and depends on the number assigned to each base station and the position of each user in the system. For FDMA/TDMA using an FCA scheme, the performance analysis can be assumed Poisson call arrivals and exponential call holding time. If we assume cell holding time is also exponentially distributed, the system can be modeled as a Markovian queuing system. In a CDMA system, it is intractable to model the system with a finite number of states, especially if we consider a power controlled cellular system. In this case, the system state must include users' locations and transmitter powers. This continuous state space makes a state based formulation intractable. Therefore, to obtain the performance of a call admission strategy, extensive simulation studies have to be made.

In this study, we assume that new call requests arrive as a Poisson process of rate λ. Each new call is equally likely to occur anywhere on the ring and the duration of each call is exponential distributed with time $1/\mu$. We will derive all our results as a function of the traffic load, ρ, which is defined as

$$\rho = \frac{\lambda}{\mu M} \tag{2.10}$$

where the traffic load, ρ, is defined as Erlang per cell.

2.6 Propagation Models

Without line of sight communication, the radio signals from user to base station can be modeled as Rayleigh fading [47, 48] that is because the base station receives multipath from the desired signal. The phase of the incoming signal changes rapidly with the change of the users positions. For fast moving users, power control can not respond quickly enough to adapt to the changes caused by Rayleigh fading [49]. In this study, we assume that the Rayleigh fading has been averaged out in power measurements The received power of a mobile depends only on propagation loss and lognormal shadow fading. For each base station, the uplink gain included an order $\alpha = 4$ propagation loss and a position dependent lognormal shadow fading component. That is, expressed in dB, the uplink gain h_{ki} of user i to base k at a distance d_{ki} is

$$10 \log h_{ki} = -10\alpha \log d_{ki} + S_k(d_{ki}) \tag{2.11}$$

The shadow fading component $S_k(d_{ki})$ was taken to be a normal random variable with mean 0 and standard deviation $\eta = 8$ dB. The shadow fading processes $S_k(d)$ and $S_{k'}(d')$ for distinct base stations k and k' were assumed to be independent. The correlation over distance of the shadow fading was described by the autocorrelation function

$$\rho(d) = E[S_j(d')S_j(d'+d)] = \eta^2 e^{-|d|/d_0} \tag{2.12}$$

with a correlation distance of $d_0 = 45$ meters. Each mobile i aimed for a CIR target of $\gamma_i = 1/20$ that reflected the processing gain associated with a CDMA system. From Equation (2.2), for the IS-95 system, $r_i = 9.6$ $kbps$ and $\omega = 1.228$ MHz. We have

$$\frac{1}{20} = \frac{E_b}{N_0} \frac{9.6 \times 10^3}{1.228 \times 10^6}$$

The corresponding $E_b/N_0 = 8$ dB.

2.7 Performance Measures

In general, the call blocking probability P_b and call dropping probability P_d will be used as performance measures. In CDMA systems, the system capacity is not limited by the number of codes available at each base station but instead is limited by the interference seen at each base station. Call dropping may occur when the system admits too many users at the same base station or when a user moves from his serving cell to another cell but can not find the resources to maintain the link quality. As a result, we have to consider both the call blocking and the call dropping for system performance. We use a weighted combination of call blocking and call dropping probabilities as a cost function measuring system performance.

$$P_b + C_d P_d \tag{2.13}$$

The parameter C_d is the relative cost of dropping an existing user. We have $C_d > 1$ to make sure call admission control can lower the chance of call dropping. In this study, a quality based dropping policy will be used to drop the ongoing users. Let γ_{drop} denote the dropping CIR or the minimum quality requirement for radio link. An ongoing user will be dropped from the system if the link quality falls below the γ_{drop}. In general, we have $\gamma_{drop} < \gamma$.

The system needs call admission control to make call dropping a rare event. With this interference limited system, we will implement call admission control algorithms based on user's transmitted power and received power at each base station.

Chapter 3

Integrated Power Control and Base Station Assignment

In cellular wireless communication systems, transmitted power is regulated to provide each user an acceptable connection while limiting the interference seen by other users. Previous work has focused on maximizing the minimum carrier to interference ratio (CIR) or attaining a common CIR over all radio links. However, previous work has assumed the assignment of mobiles to base stations is known and fixed. In this work, we integrate power control and base station assignment. In the context of a CDMA system, we consider the minimization of the total transmitted uplink power subject to maintaining an individual target CIR for each mobile. This minimization occurs over the set of power vectors and base station assignments. In the absence of maximum power constraints, we show that this problem has special structure and identify synchronous and asynchronous distributed algorithms that find the optimal power vector and base station assignment.

3.1 Introduction

There has been significant work on power control in mobile wireless communications systems. In this problem, there is a set of mobile users that wish to communicate with a set of base stations. These users share a common channel and one user's transmission causes interference for all other users. As a result, transmitted power is regulated both to provide each user an acceptable connection and to limit the interference seen by other users.

Previous work has assumed the assignment of users to base stations is fixed or specified by outside means. In this work, we wish to determine which base station should be responsible for receiving a user's signal. In particular, we consider the combined problem of regulating transmitter powers and assigning users to base stations.

The goal of power control and base station assignment is to provide each user an acceptable connection that meets each individual CIR requirement. As a result, we will formulate the Minimum Transmitted Power (MTP) problem in which the total transmitted uplink power is minimized subject to maintaining an individual target CIR for each mobile. This minimization occurs over the set of power vectors and base station assignments.

We emphasize that the objective of power minimization is in fact secondary to that of finding a feasible solution to the MTP problem. The benefit of an algorithm that solves the MTP problem is that it always will find a power vector and base station assignment that provides acceptable connections for all users as long as such a feasible solution exists. Of course, among all feasible solutions, the solution that minimizes total transmitted power would be desirable, particularly when users have limited battery power.

We will describe synchronous and asynchronous versions of a simple algorithm called Minimum Power Assignment (MPA) that iteratively solves the MTP problem. At each iteration of the MPA algorithm, each user selects a base station and transmitter power for which minimum power is needed to maintain an acceptable CIR under the assumption that all other users keep their powers fixed. The intuition behind MPA is simple. A user that reduces its transmitted power while maintaining an acceptable connection does all other users a favor. Similarly, a user that increases its transmitted power so that its CIR exceeds its target causes all other users unnecessary interference.

Since the MPA algorithm assigns users to base stations in response to the interfering signals of other users, the MPA algorithm can be viewed as a handoff algorithm that adapts to the location of each user in the system. One also can consider the base station reassignments specified by the MPA algorithm as defining expanding and contracting cells as in [50]. Alternatively, the MPA algorithm can be viewed a form of the soft handoff [51] selection diversity mechanism of the IS-95 system in which each user is in permanent soft handoff with all base stations.

In this work, we will assume there exists a feasible solution to the MTP problem. When a feasible solution does not exist, the primary issue becomes how to maximize the number of acceptable connections, a very different optimization problem. In future work, we hope to use the framework of the MTP problem to consider this problem.

3.2 The MTP Problem

In the call maintenance stage, no new call will try to get access from the system. Minimizing the use of resources is the major concern at this stage. We wish to solve the Minimum Transmitted Power (MTP) problem:

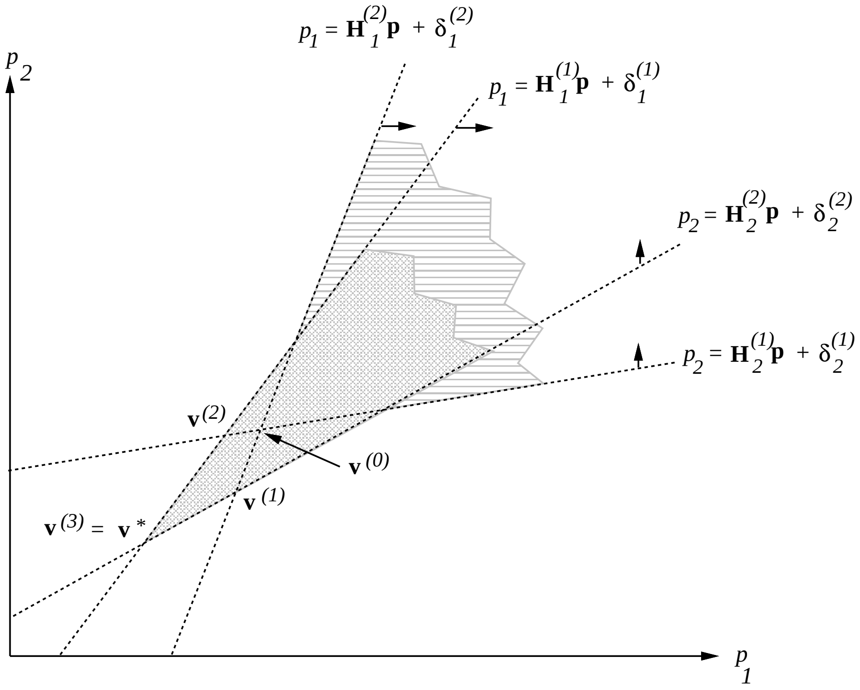
$$\begin{array}{lll} \text{minimize} & \sum_i p_i & \\ \text{subject to} & & \\ h_{a_i i} p_i \geq \gamma_i \left(\sum_{j \neq i} h_{a_i j} p_j + \sigma_{a_i} \right) & & 1 \leq i \leq N \\ p_i \geq 0 & & 1 \leq i \leq N \\ a_i \in \{1, \ldots, M\} & & 1 \leq i \leq N \end{array} \tag{3.1}$$

The first set of constraints specifies that each user i has an acceptable connection that attains an individual target CIR γ_i. The second constraint simply demands non-negativity of the power vector. The quantity σ_{a_i} describes the receiver noise at the assigned base of user i. Finally, we also specify that the assignment vector must take on integer values. In short, our problem is to minimize the total transmitted power subject to maintaining an acceptable connection for each user.

According to Equation (2.9), the set of feasible power vectors under assignment l is

$$P^{(l)} = \left\{ \boldsymbol{p} \geq 0 | \boldsymbol{p} \geq \boldsymbol{G}^{(l)} \boldsymbol{p} + \boldsymbol{\sigma}^{(l)} \right\} \tag{3.2}$$

The set $P^{(l)}$ describes a cone of feasible powers in that if $\boldsymbol{p} \in P^{(l)}$ then $\alpha \boldsymbol{p} \in P^{(l)}$ for all $\alpha \geq 1$. Each cone $P^{(l)}$ is specified by a normalized interference matrix $\boldsymbol{G}^{(l)}$ and normalized noise vector $\boldsymbol{\sigma}^{(l)}$. The nonnegative noise vector $\boldsymbol{\sigma}^{(l)}$ displaces these cones from the origin. A depiction of



This figure depicts the set of feasible power vectors for a system of two users and two base stations. Rays labeled $p_i = \boldsymbol{H}_i^{(k)}\boldsymbol{p}+\delta_i^{(k)}$ describe the minimum power user i needs to communicate with base k as a function of the transmitted power of the other user. Four base station assignments are possible corresponding to the four cones with labeled vertices $\boldsymbol{v}^{(0)}, \ldots, \boldsymbol{v}^{(3)} = \boldsymbol{v}^*$. The union of shaded regions depicts the nonconvex set $\cup_l P^{(l)}$.

Figure 3.1: The feasible region of an instance of MTP

these cones is provided in Figure 3.1. Finally, we mention that we could view the MTP problem as the minimization of total transmitted power over the set of feasible power vectors $\cup_l P^{(l)}$. However, as depicted in Figure 3.1, $\cup_l P^{(l)}$ is typically not a convex set so that the standard approaches to minimizing a linear function over a convex set are not directly applicable.

For a fixed assignment, [8, 9, 13, 15, 17, 19] have examined forms of the following maxmin CIR problem [1]:

$$\begin{aligned} &\text{maximize} && \gamma \\ &\text{subject to} && \boldsymbol{p} \geq \gamma\,(\boldsymbol{G}\boldsymbol{p} + \boldsymbol{\sigma}) \\ & && \boldsymbol{p} \geq 0 \end{aligned} \tag{3.3}$$

[1] Note that $\boldsymbol{G}$ in (3.3) uses a somewhat different indexing scheme than that used in this work. In particular, G_{kj} is the normalized uplink gain from mobile j to the assigned base station of mobile k.

For fixed γ, the constraint set of the maxmin CIR problem defines a cone of feasible power vectors. In [17], Zander addresses the interference limited case of $\boldsymbol{\sigma} = 0$. Grandhi [20] and Foschini [10] have identified iterative algorithms that solve the subproblem of finding a feasible power vector $\boldsymbol{p}$ given a fixed assignment and common CIR target γ. In [21], Mitra proves geometric convergence for an asynchronous version of Foschini's algorithm. These methods find the power vector $\boldsymbol{p} = \boldsymbol{v}$, the vertex of the cone of feasible powers, satisfying $\boldsymbol{v} = \gamma\,(\boldsymbol{G}\boldsymbol{v} + \boldsymbol{\sigma})$.

In the context of the MTP problem, we embed the set of CIR targets $\{\gamma_i\}$ in the interference matrix $\boldsymbol{G}^{(l)}$ so that the vertex of $P^{(l)}$ is a power vector $\boldsymbol{v}^{(l)}$ satisfying

$$\boldsymbol{v}^{(l)} = \boldsymbol{G}^{(l)}\boldsymbol{v}^{(l)} + \boldsymbol{\sigma}^{(l)} \tag{3.4}$$

We will assume that $\boldsymbol{G}^{(l)}$ is nondegenerate, implying $\boldsymbol{v}^{(l)}$ is the unique solution to (3.4). The vertex $\boldsymbol{v}^{(l)}$ has the following special property. We can construct an example of degeneracy for $\boldsymbol{G}^{(l)}$ where two base stations are co-located and thus all users have identical uplink gains to both base stations. In this study, we won't consider this specially case and assume that $\boldsymbol{G}^{(l)}$ is nondegenerate.

Lemma 3.1 *For all $\boldsymbol{p} \in P^{(l)}$, $\boldsymbol{p} \geq \boldsymbol{v}^{(l)}$.*

- **Proof:** Given $\boldsymbol{p} \in P^{(l)}$, let $\hat{\boldsymbol{p}} = \boldsymbol{v}^{(l)} + \alpha(\boldsymbol{p} - \boldsymbol{v}^{(l)})$. Since $\boldsymbol{v}^{(l)} = \boldsymbol{G}^{(l)}\boldsymbol{v}^{(l)} + \boldsymbol{\sigma}^{(l)}$ and $\boldsymbol{p} \geq \boldsymbol{G}^{(l)}\boldsymbol{p} + \boldsymbol{\sigma}^{(l)}$,

$$\begin{aligned} \hat{\boldsymbol{p}} - \left(\boldsymbol{G}^{(l)}\hat{\boldsymbol{p}} + \boldsymbol{\sigma}^{(l)}\right) &= \alpha\left(\boldsymbol{p} - \boldsymbol{G}^{(l)}\boldsymbol{p} - \boldsymbol{\sigma}^{(l)}\right) && (3.5) \\ &\geq 0 && (3.6) \end{aligned}$$

Hence, $\hat{\boldsymbol{p}} \in P^{(l)}$ for all nonnegative α. Now suppose that $p_i < \boldsymbol{v}_i^{(l)}$ for some i. In this case, we can choose α such that for some i, $\hat{p}_i = 0$ and $\hat{p}_j \geq 0$ for all $j \neq i$. For this choice of α,

$$0 = \hat{p}_i < \boldsymbol{G}_i^{(l)}\hat{\boldsymbol{p}} + \boldsymbol{\sigma}^{(l)} \tag{3.7}$$

which contradicts $\hat{\boldsymbol{p}} \in P^{(l)}$.

We see that for a fixed assignment l, total transmitted power is minimized by choosing $\boldsymbol{p} = \boldsymbol{v}^{(l)}$. Moreover, the minimization of total power over all possible assignments can be reduced to finding the minimum power vertex $\boldsymbol{v}^{(l)}$ among all vertices.

Theorem 3.1 *There exists a vertex $\boldsymbol{v}^*$ such that $\boldsymbol{v}^* \leq \boldsymbol{v}^{(l)}$ for all l.*

• **Proof:** Among all feasible assignments l, let l_i denote the assignment that minimizes $v_i^{(l)}$, the ith component of vertex $\boldsymbol{v}^{(l)}$. By definition, $v_i^{(l_i)} \leq v_i^{(l)}$ for all l and

$$v_i^{(l_i)} = \boldsymbol{G}_i^{(l_i)} \boldsymbol{v}^{(l_i)} + \sigma_i^{(l_i)} \tag{3.8}$$

where $\boldsymbol{G}_i^{(l_i)}$ denotes the ith row of $\boldsymbol{G}^{(l_i)}$. Consider $\boldsymbol{p} = [v_1^{(l_1)} \cdots v_N^{(l_N)}]^\top$. We observe that $\boldsymbol{p} \leq \boldsymbol{v}^{(l_i)}$ for all i, and that

$$\begin{aligned} p_i = v_i^{(l_i)} &= \boldsymbol{G}_i^{(l_i)} \boldsymbol{v}^{(l_i)} + \sigma_i^{(l_i)} && (3.9) \\ &\geq \boldsymbol{G}_i^{(l_i)} \boldsymbol{p} + \sigma_i^{(l_i)} && (3.10) \end{aligned}$$

That is, $\boldsymbol{p}$ is feasible with respect to the ith constraint of cone $P^{(l_i)}$. Let $\boldsymbol{G}^*$ denote the interference matrix with ith row $\boldsymbol{G}_i^* = \boldsymbol{G}_i^{(l_i)}$. Let $\boldsymbol{\sigma}^*$ denote the normalized noise vector with ith element $\sigma_i^* = \sigma_i^{(l_i)}$. The pair $\boldsymbol{G}^*$, $\boldsymbol{\sigma}^*$ describes the feasible cone P^* with a vertex $\boldsymbol{v}^*$. We have observed that $\boldsymbol{p} \in P^*$, so Lemma 3.1 implies $\boldsymbol{p} \geq \boldsymbol{v}^*$. By construction $\boldsymbol{p} \leq \boldsymbol{v}^{(l)}$ for all l. Hence $\boldsymbol{v}^* = \boldsymbol{p} \leq \boldsymbol{v}^{(l)}$ for all l.

Theorem 3.2 *Over all feasible power vectors $\boldsymbol{p}$ and base assignments l, vertex $\boldsymbol{v}^*$ and its corresponding base station assignment solves the MTP problem.*

• **Proof:** Consider any feasible power vector $\boldsymbol{p}$. Since $\boldsymbol{p} \in P^{(l)}$ for some l, Lemma 3.1 and Theorem 3.1 imply $\boldsymbol{p} \geq \boldsymbol{v}^{(l)} \geq \boldsymbol{v}^*$. Hence, $\sum_i p_i^{(l)} \geq \sum_i v_i^*$.

Note that the power vector $\boldsymbol{v}^*$ is unique; however, $\boldsymbol{v}^*$ may be the vertex of more than one cone. In this case, there would be multiple base station assignments for which $\boldsymbol{v}^*$ is the vertex power vector. A trivial example occurs when two base stations share the same location in space.

In addition, we observe that if the objective function of the MTP problem were not the total transmitted power but rather $f(\boldsymbol{p})$, a nondecreasing function[2] of the power vector $\boldsymbol{p}$, then $\boldsymbol{v}^*$ would still be the optimal solution. However, we have not resolved how to solve for $\boldsymbol{v}^*$ nor how to find a corresponding optimal assignment for arbitrary $f(\boldsymbol{p})$.

3.3 Iterative Solutions to MTP

We now describe the iterative MPA (Minimum Power Assignment) algorithm that converges to the optimal assignment and power vector $\boldsymbol{v}^*$. To simplify the description of MPA, we define

$$M_i^{(k)}(\boldsymbol{p}) = \boldsymbol{H}_i^{(k)}\boldsymbol{p} + \delta_i^{(k)} \tag{3.11}$$

$$M_i(\boldsymbol{p}) = \min_k M_i^{(k)}(\boldsymbol{p}) \tag{3.12}$$

$$A_i(\boldsymbol{p}) = \arg\min_k M_i^{(k)}(\boldsymbol{p}) \tag{3.13}$$

From Equation (2.7), $M_i^{(k)}(\boldsymbol{p})$ equals the minimum power needed by user i to transmit to base station k with CIR γ_i if each user $j \neq i$ keeps its power p_j constant. Hence, $M_i(\boldsymbol{p})$ equals the minimum power user i needs to transmit to a base station with CIR γ_i and $A_i(\boldsymbol{p})$ would be the corresponding base station. Given a power vector $\boldsymbol{p}(n)$ and assignment $\boldsymbol{k}(n)$ at time n, an MPA iteration finds a new power vector $\boldsymbol{p}(n+1)$, and assignment vector $\boldsymbol{k}(n+1)$ described by

$$\boldsymbol{p}(n+1) = \boldsymbol{T}(\boldsymbol{p}(n)) \tag{3.14}$$

$$\boldsymbol{k}(n+1) = \boldsymbol{A}(\boldsymbol{p}(n)) \tag{3.15}$$

---

[2] $f(\boldsymbol{p})$ is nondecreasing in the sense that $\boldsymbol{p} \leq \boldsymbol{p}'$ implies $f(\boldsymbol{p}) \leq f(\boldsymbol{p}')$.

where

$$\boldsymbol{T}(\boldsymbol{p}) = [M_1(\boldsymbol{p}), \ldots, M_N(\boldsymbol{p})]^\top \tag{3.16}$$

$$\boldsymbol{A}(\boldsymbol{p}) = [A_1(\boldsymbol{p}), \cdots, A_N(\boldsymbol{p})]^\top \tag{3.17}$$

Starting from an initial power vector $\boldsymbol{p}$, n iterations of the MPA algorithm produce the power vector $\boldsymbol{T}^n(\boldsymbol{p})$ and the base station assignment $\boldsymbol{k}(n) = \boldsymbol{A}(\boldsymbol{p}(n-1))$.

In the enumeration of base station assignments, there exists an assignment l_n such that $a_i(l_n) = k_i(n)$ for all i. Effectively, at step n, the users' base station choices define a new assignment l_n and the new power vector can be written

$$\boldsymbol{p}(n) = \boldsymbol{G}^{(l_n)}\boldsymbol{p}(n-1) + \boldsymbol{\sigma}^{(l_n)} \tag{3.18}$$

General proof of the convergence of MPA relies on proving convergence in two special cases, as shown in the following sequence of lemmas.

Lemma 3.2 *For power vectors $\boldsymbol{p}$ and $\boldsymbol{p}'$, if $\boldsymbol{p} \leq \boldsymbol{p}'$, then $\boldsymbol{T}(\boldsymbol{p}) \leq \boldsymbol{T}(\boldsymbol{p}')$.*

• **Proof:** Equation (3.11) implies that for all i and k, $M_i^{(k)}(\boldsymbol{p}) \leq M_i^{(k)}(\boldsymbol{p}')$. Hence, from Equation (3.12), $M_i(\boldsymbol{p})$, the ith component of $\boldsymbol{T}(\boldsymbol{p})$, satisfies $M_i(\boldsymbol{p}) \leq M_i(\boldsymbol{p}')$.

Lemma 3.3 *If $\boldsymbol{p}$ is a feasible power vector, then $\boldsymbol{p} \geq \boldsymbol{T}(\boldsymbol{p})$ and $\boldsymbol{T}(\boldsymbol{p})$ is a feasible power vector.*

• **Proof:** Let $\boldsymbol{p}' = \boldsymbol{T}(\boldsymbol{p})$. Since $\boldsymbol{p}$ is feasible, $\boldsymbol{p} \in P^{(l)}$ for some l. Hence, for all i,

$$p_i' = M_i(\boldsymbol{p}) \tag{3.19}$$

$$\leq M_i^{(a_i(l))}(\boldsymbol{p}) \tag{3.20}$$

$$\leq p_i \tag{3.21}$$

Hence $\boldsymbol{p}' \leq \boldsymbol{p}$. To verify feasibility of $\boldsymbol{p}'$, let l' denote the assignment corresponding to $\boldsymbol{A}(\boldsymbol{p})$. In this case, from Equation (3.18)

$$
\begin{aligned}
p_i' &= \boldsymbol{G}_i^{(l')}\boldsymbol{p} + \boldsymbol{\sigma}^{(l')} && (3.22)\\
&\geq \boldsymbol{G}^{(l')}\boldsymbol{p}' + \boldsymbol{\sigma}^{(l')} && (3.23)
\end{aligned}
$$

Hence, $\boldsymbol{p}' \in P^{(l')}$ and $\boldsymbol{T}(\boldsymbol{p})$ is feasible.

Lemma 3.4 *The power vector $\boldsymbol{v}^*$ is the unique fixed point of the synchronous MPA algorithm.*

• **Proof:** By Lemma 3.3, since $\boldsymbol{v}^*$ is feasible, $\boldsymbol{T}(\boldsymbol{v}^*) \leq \boldsymbol{v}^*$ and $\boldsymbol{T}(\boldsymbol{v}^*)$ is also feasible. Since $\boldsymbol{T}(\boldsymbol{v}^*)$ is feasible, Lemma 3.1 and Theorem 3.1 imply $\boldsymbol{T}(\boldsymbol{v}^*) \geq \boldsymbol{v}^*$. Hence, $\boldsymbol{v}^* = \boldsymbol{T}(\boldsymbol{v}^*)$. To verify uniqueness of the fixed point $\boldsymbol{v}^*$, suppose there exists some other fixed point $\hat{\boldsymbol{v}} \neq \boldsymbol{v}^*$. Since $\hat{\boldsymbol{v}}$ is feasible, $\hat{\boldsymbol{v}} \geq \boldsymbol{v}^*$. Let $\alpha = \max_i(\hat{v}_i/v_i^*)$. Since $\hat{\boldsymbol{v}}$ and $\boldsymbol{v}^*$ are distinct, $\alpha > 1$, $\alpha\boldsymbol{v}^* \geq \hat{\boldsymbol{v}}$, and for some j, $\alpha v_j^* = \hat{v}_j$. In this case, since $\hat{\boldsymbol{v}}$ is a fixed point,

$$
\begin{aligned}
\hat{v}_j &= \min_k \boldsymbol{H}_j^{(k)}\hat{\boldsymbol{v}} + \delta_j^{(k)} && (3.24)\\
&\leq \min_k \boldsymbol{H}_j^{(k)}\alpha\boldsymbol{v}^* + \delta_j^{(k)} && (3.25)\\
&< \alpha\left(\min_k \boldsymbol{H}_j^{(k)}\boldsymbol{v}^* + \delta_j^{(k)}\right) && (3.26)\\
&= \alpha v_j^* && (3.27)
\end{aligned}
$$

which is a contradiction.

Lemma 3.5 *Starting from a feasible power vector $\boldsymbol{p}$, the MPA algorithm produces $\boldsymbol{T}^n(\boldsymbol{p})$, a decreasing sequence of feasible power vectors that converges to $\boldsymbol{v}^*$.*

• **Proof:** Let $\boldsymbol{p}(n) = \boldsymbol{T}^n(\boldsymbol{p})$. Since $\boldsymbol{p}(0)$ is feasible, Lemma 3.3 implies that $\boldsymbol{p}(0) \geq \boldsymbol{p}(1)$ and that $\boldsymbol{p}(1)$ is feasible. Now suppose $\boldsymbol{p}(0) \geq \boldsymbol{p}(1) \geq \cdots \geq \boldsymbol{p}(n)$. In this case, Lemma 3.3 implies $\boldsymbol{p}(n+1) \leq \boldsymbol{p}(n)$ and $\boldsymbol{p}(n+1)$ is feasible. Hence the sequence of $\boldsymbol{p}(n)$ is decreasing and always feasible. By Lemma 3.1 and Theorem 3.1, $\boldsymbol{p}(n) \geq \boldsymbol{v}^*$ for all n. Hence, the sequence of $\boldsymbol{p}(n)$ is bounded below by $\boldsymbol{v}^*$ and so must converge to the unique fixed point $\boldsymbol{v}^*$.

Lemma 3.6 *Starting from $\boldsymbol{z}$, the all zero vector, $\boldsymbol{T}^n(\boldsymbol{z})$ is an increasing sequence of power vectors that converges to $\boldsymbol{v}^*$.*

- **Proof:** *Lemma 3.6* Let $\boldsymbol{z}(n) = \boldsymbol{T}^n(\boldsymbol{z})$. First, we note that $\boldsymbol{z}(1) = \boldsymbol{T}(\boldsymbol{z}) \geq \boldsymbol{z}$. Assuming $\boldsymbol{z}(0) \leq \boldsymbol{z}(1) \leq \cdots \leq \boldsymbol{z}(n)$, Lemma 3.2 implies

$$\boldsymbol{z}(n+1) = \boldsymbol{T}(\boldsymbol{z}(n)) \geq \boldsymbol{T}(\boldsymbol{z}(n-1)) = \boldsymbol{z}(n) \tag{3.28}$$

Hence, the sequence of power vectors is nondecreasing. Second, since $\boldsymbol{z} \leq \boldsymbol{v}^*$, assuming $\boldsymbol{z}(n) \leq \boldsymbol{v}^*$ implies

$$\boldsymbol{z}(n+1) = \boldsymbol{T}(\boldsymbol{z}(n)) \leq \boldsymbol{T}(\boldsymbol{v}^*) = \boldsymbol{v}^* \tag{3.29}$$

That is, the increasing sequence of power vectors $\boldsymbol{z}(n)$ is bounded above by $\boldsymbol{v}^*$ and so must converge to the unique fixed point $\boldsymbol{v}^*$.

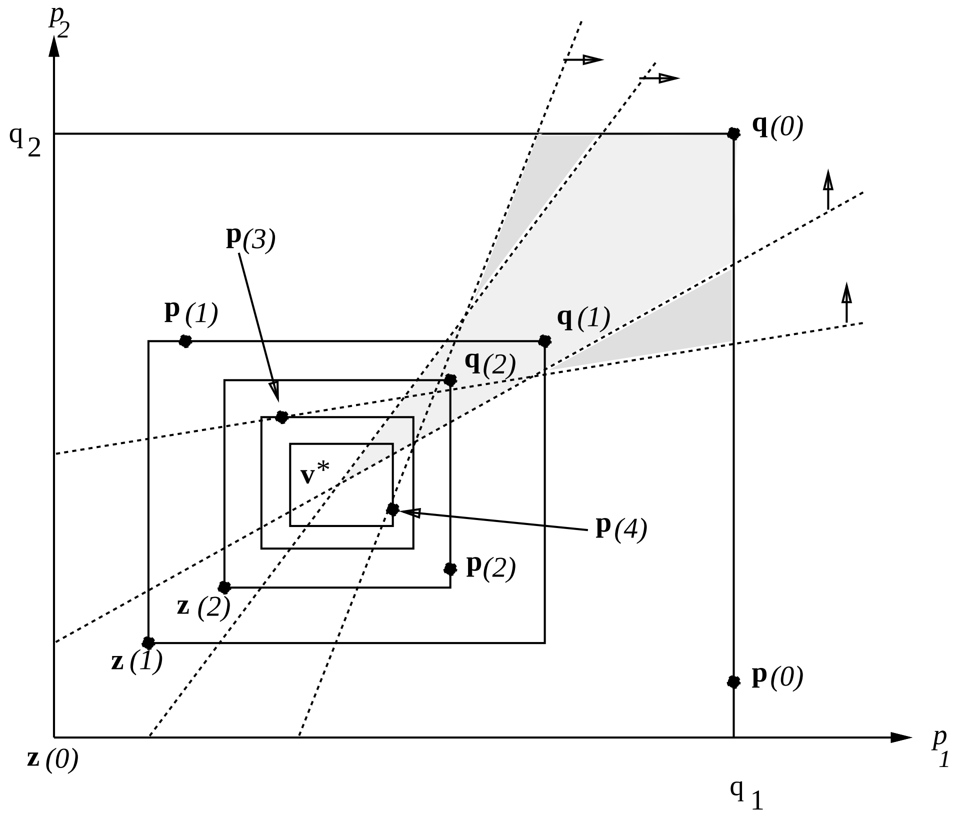
Theorem 3.3 *From any initial power vector $\boldsymbol{p}$, the MPA algorithm produces a sequences of power vectors $\boldsymbol{T}^n(\boldsymbol{p})$ that converges to $\boldsymbol{v}^*$.*

- **Proof:** Since $\sigma_i^{(l)} > 0$ for all l, $v_i^* > 0$ for all i. Hence, there exists $\beta > 0$ such that $\boldsymbol{q} = \beta\boldsymbol{v}^* \geq \boldsymbol{p}$. Let $\boldsymbol{z}$ denote the all zero vector so that $\boldsymbol{z} \leq \boldsymbol{p} \leq \boldsymbol{q}$. Lemma 3.2 implies

$$\boldsymbol{T}^n(\boldsymbol{z}) \leq \boldsymbol{T}^n(\boldsymbol{p}) \leq \boldsymbol{T}^n(\boldsymbol{q}) \tag{3.30}$$

Since $\boldsymbol{q}$ and $\boldsymbol{z}$ meet the requirements of Lemmas 3.5 and 3.6 respectively for initial power vectors, $\lim_{n\to\infty} \boldsymbol{T}^n(\boldsymbol{z}) = \lim_{n\to\infty} \boldsymbol{T}^n(\boldsymbol{q}) = \boldsymbol{v}^*$. By Equation (3.30), we must have $\lim_{n\to\infty} \boldsymbol{T}^n(\boldsymbol{p}) = \boldsymbol{v}^*$. MPA is fairly practical in that the iteration step can be written as

$$p_i(n) = \min_k \frac{\gamma_i \left(R_k(n-1) - h_{ki} p_i(n-1)\right)}{h_{ki}} \tag{3.31}$$



In this example, there are two base stations and two mobiles with powers p_1 and p_2. Each pair of rays, one marked by $\rightarrow$ and one by $\uparrow$, forms the cone corresponding to a base station assignment. The cone with vertex $\boldsymbol{v}^*$ is shaded. With respect to a particular assignment, rays marked by $\rightarrow$ indicate constraints that p_1 must exceed while rays marked by $\uparrow$ indicate constraints that p_2 must exceed. The initial power vector is $\boldsymbol{p}(0)$. Subsequent power vectors produced by MPA are denoted $\boldsymbol{p}(n)$. Following the proof of Theorem 3.3, $\boldsymbol{z}(n) = \boldsymbol{T}^n(\boldsymbol{z})$ and $\boldsymbol{q}(n) = \boldsymbol{T}^n(\boldsymbol{q})$ constrain $\boldsymbol{p}(n)$ to the box $\boldsymbol{z}(n) \leq \boldsymbol{p}(n) \leq \boldsymbol{q}(n)$.

Figure 3.2: Convergence of the MPA Algorithm

where $R_k(n-1) = \sum_j h_{kj}p_j(n-1) + \sigma_k$ is the total received power (signal plus noise) at base k using power vector $\boldsymbol{p}(n-1)$. The base station assignment $k_i(n)$ is simply the minimizing argument k of Equation (3.31). Although the minimization in Equation (3.31) occurs over all base stations k, in practice it would be necessary only to consider nearby base stations as a result of signal attenuation. Effectively, a user needs only to know the total received power and its uplink gain h_{ki} at each nearby base station. If the uplink and downlink face the same attenuation, then user i can deduce the uplink gain h_{ki} from a pilot tone from base station k. Furthermore, it would also be possible for the downlink communication channel to provide each user $R_k(n)$ for all nearby base stations k.

The convergence of the MPA algorithm to the optimal power vector $\boldsymbol{v}^*$ is graphically depicted in Figure 3.2. We observe that for an initial power vector $\boldsymbol{p} \geq \boldsymbol{v}^*$, the convergence occurs at least as quickly as the convergence of $\boldsymbol{T}^n(\boldsymbol{q})$. Starting from an initial power vector $\boldsymbol{q}(0) = \beta \boldsymbol{v}^*$, it is readily shown that the resulting sequence of power vectors $\boldsymbol{q}(n)$ satisfies $\boldsymbol{q}(n+1) \leq \boldsymbol{G}^* \boldsymbol{q}(n) + \boldsymbol{\sigma}^*$. From the Perron-Frobenius Theorem [52, 53], for a nonnegative matrix $\boldsymbol{G}$ the iteration

$$\boldsymbol{p}(n+1) = \boldsymbol{G}\boldsymbol{p}(n) + \boldsymbol{\sigma} \tag{3.32}$$

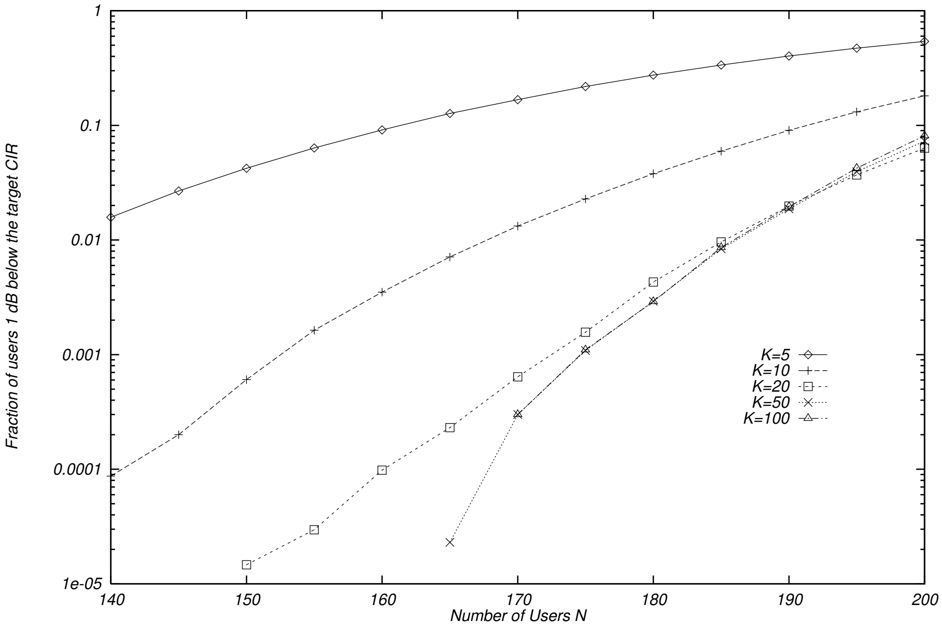
converges to a unique fixed point iff the largest eigenvalue of $\boldsymbol{G}$ is less than unity. In this case, the iteration (3.32) is a contraction mapping that converges at a geometric rate. Hence, if there exists a feasible power vector $\boldsymbol{p}$ for the MTP problem, then the interference matrix $\boldsymbol{G}^*$, corresponding to the optimal solution, must have largest eigenvalue less than unity. This implies that starting from a feasible solution $\boldsymbol{p}$, geometric convergence to $\boldsymbol{v}^*$ does occur.

From other starting points, however, the rate of convergence is not as well understood. In particular, one can construct examples in which from an infeasible power vector $\boldsymbol{p}$, the MPA algorithm chooses an assignment l_n such that the iteration at step n does not constitute a contraction mapping, thus complicating rate of convergence issues.

To provide some indication of the MPA rate of convergence, a simulation experiment was performed. We considered a system of ten base stations uniformly spaced every 2000 meters on a ring. For each base station, the uplink gain included an order $\alpha = 4$ propagation loss and a position dependent log normal shadow fading component. That is, expressed in dB, the uplink gain h_{ki} of user i to base k at a distance d_{ki} is

$$10 \log h_{ki} = -10\alpha \log d_{ki} + S_k(d_{ki}) \tag{3.33}$$

The shadow fading component $S_k(d_{ki})$ was taken to be a normal random variable with mean 0 and standard deviation $\eta = 8$ dB. The shadow fading processes $S_k(d)$ and $S_{k'}(d')$ for distinct



As a function of the number of iterations K and the number of mobiles N, this plot records the fraction of stationary mobiles with CIR 1 dB below the target CIR over many repeated trials.

Figure 3.3: Convergence of the MPA algorithm

base stations k and k' were assumed to be independent. The correlation over distance of the shadow fading was described by the autocorrelation function

$$\rho(d) = E[S_j(d')S_j(d'+d)] = \eta^2 e^{-|d|/d_0} \tag{3.34}$$

with a correlation distance of $d_0 = 45$ meters.

Each mobile was required to achieve a CIR target of $\gamma = 1/20$ that reflected the processing gain associated with a CDMA system. Note that with $\gamma = 1/20$, a single base station in isolation would be able to support 20 mobiles. For the 10 base station system, we considered a number of mobiles N ranging from 140 to 200. For each N, we repeatedly placed N mobiles randomly on the ring. We defined a mobile to have unacceptable CIR if it was not within 1 dB of its target CIR. For each placement of the mobile users, we started the MPA algorithm with initial power zero and recorded the number of mobiles with unacceptable CIR as a function of

K, the number of synchronous MPA iterations. In Figure 3.3, we graph the fraction of mobiles with unacceptable CIR over a large number of trials for $K = 5, 10, 20, 50, 100$. We see that 20 iterations are nearly as effective as 100 iterations and that for small N, 10 or fewer iterations is often sufficient.

Careful inspection of Figure 3.3 shows that for $N > 190$, 20 iterations leads to fewer users with unacceptable CIR than 50 or 100 iterations. This arises when the common CIR target γ is infeasible. In this case, the MPA iteration converges to a base station assignment l and for large n, the corresponding power control iteration becomes

$$\boldsymbol{p}(n) = \boldsymbol{G}^{(l)}\boldsymbol{p}(n-1) + \boldsymbol{\sigma}^{(l)} \tag{3.35}$$

Moreover, the power vector $\boldsymbol{p}(n)$ increases without bound while each user's CIR converges to the maximum attainable CIR, as noted in [18]. When the maximum attainable CIR is more than 1 dB below the target CIR, all users eventually end up with unacceptable CIR. However, in this situation, we have observed that starting from zero power, a modest number of iterations, such as $K = 20$, often leaves many of the users within 1 dB of the target CIR. Hence, when the number of users N is large and it becomes more likely that the maximum attainable CIR is more than 1 dB below the target CIR, a modest number of iterations tends to provide more users an acceptable CIR.

We note that this experiment assumed that all mobiles had perfect information about radio channel. That is, each mobile i knew at every step n both the uplink gains h_{ki} and the total received powers $R_k(n)$ at all base stations in order to execute the MPA using (3.31). This suggests that the limiting factor in the effectiveness of the MPA algorithm will be the ability of the mobile to collect timely, accurate information about the channel conditions.

3.4 The Asynchronous MPA (AMPA) Algorithm

In this section, we examine an asynchronous version of the MPA algorithm using the framework for totally asynchronous algorithms of Bertsekas and Tsitsiklis [52]. The AMPA algorithm allows some users to perform power adjustments faster and execute more iterations than others. In addition, the AMPA algorithm allows users to perform these updates while using outdated information on the interference caused by other users. Let $p_i(t)$ denote the transmitted power of user i at time t so that the power vector at time t is $\boldsymbol{p}(t) = [p_1(t), \ldots, p_N(t)]^\top$.

We assume that user i may not have access to the most recent values of the components of $\boldsymbol{p}(t)$. This occurs when user i has outdated information about the received power at certain bases. At time t, let $\tau_j^i(t)$ denote the most recent time for which p_j is known to user i. Note that $0 \leq \tau_j^i(t) \leq t$. For user i at time t, $p_j(\tau_j^i(t))$ denotes the most recently known value of $p_j(t)$. Thus, at time t, user i may update its transmitter power and base station assignment using the power vector

$$\boldsymbol{p}(\tau^i(t)) = \left[p_1(\tau_1^i(t)), p_2(\tau_2^i(t)), ..., p_N(\tau_N^i(t))\right]^\top \tag{3.36}$$

We assume a set of times $T = \{0, 1, 2, ...\}$ at which one or more components $p_i(t)$ of $\boldsymbol{p}(t)$ are updated. Let T^i be the set of times at which $p_i(t)$ is updated. At times $t \notin T^i$, $p_i(t)$ is left unchanged. Hence, the AMPA algorithm can be written as

$$p_i(t+1) = \begin{cases} M_i(\boldsymbol{p}(\tau^i(t))) & t \in T^i \\ p_i(t) & t \notin T^i \end{cases} \tag{3.37}$$

$$k_i(t+1) = \begin{cases} A_i(\boldsymbol{p}(\tau^i(t))) & t \in T^i \\ k_i(t) & t \notin T^i \end{cases} \tag{3.38}$$

We assume the sets T^i are infinite and given any time t_0, there exists t_1 such that $\tau_j^i(t) \geq t_0$ for all $t \geq t_1$. Convergence of the AMPA algorithm will be proven by the Asynchronous Convergence Theorem as taken from [52]. We note that x and $f(x)$ in the statement of Theorem 3.4

represent the power vector $\boldsymbol{p}$ and iteration function $\boldsymbol{T}(\boldsymbol{p})$ in the context of this work.

Theorem 3.4 *(Asynchronous Convergence Theorem) If there is a sequence of nonempty sets $\{X(n)\}$ with $X(n+1) \subset X(n)$ for all n satisfying the following two conditions:*

1. *Synchronous Convergence Condition: For all n and $x \in X(n)$, $f(x) \in X(n+1)$. If $\{y^n\}$ is a sequence such that $y^n \in X(n)$ for all n, then every limit point of $\{y^n\}$ is a fixed point of f.*

2. *Box Condition: For every n, there exists sets $X_i(n) \in X_i$ such that $X(n) = X_1(n) \times X_2(n) \times \cdots \times X_N(n)$.*

and the initial solution estimate $x(0)$ belongs to the set $X(0)$, then every limit point of $\{x(t)\}$ is a fixed point of f.

Theorem 3.5 *From any initial power vector $\boldsymbol{p}$, the AMPA algorithm converges to $\boldsymbol{v}^*$.*

• **Proof:** Given an initial power vector $\boldsymbol{p}$, we can choose $\beta > 0$ so that $\boldsymbol{q} = \beta \boldsymbol{v}^* \geq \boldsymbol{p}$. Using $\boldsymbol{z}$ to denote the all zero vector, we define

$$X(n) = \{\boldsymbol{p} | \boldsymbol{T}^n(\boldsymbol{z}) \leq \boldsymbol{p} \leq \boldsymbol{T}^n(\boldsymbol{q})\} \tag{3.39}$$

For all n, the set $X(n)$ satisfies the box condition. By Lemmas 3.5 and 3.6, $X(n+1) \subset X(n)$ for all n. From the proof of Theorem 3.3, $\lim_{n\to\infty} \boldsymbol{T}^n(\boldsymbol{z}) = \lim_{n\to\infty} \boldsymbol{T}^n(\boldsymbol{q}) = \boldsymbol{v}^*$. Hence any sequence $\{y^n\}$ such that $y^n \in X(n)$ for all n must converge to $\boldsymbol{v}^*$. Since the initial power vector $\boldsymbol{p}$ satisfies $\boldsymbol{p} \in X(0)$, the asynchronous convergence theorem implies the AMPA algorithm will converge to $\boldsymbol{v}^*$.

The implementation of AMPA defined by Equations (3.37) and (3.38) is not the most practical since it requires each user to notify every other user of its transmitted power. Just as in Equation (3.31) for MPA, the AMPA iteration can be expressed in terms of the total received power (signal plus noise) at each base. In this approach, each base station k uses the downlink channel to inform the users of $R_k(t)$, the total received power at base k at time t. Of course, $R_k(t)$ would not be continuously updated and users may use outdated values of $R_k(t)$ to perform updates. These delays would permit a user i at time t to know $R_k(\hat{\tau}_k^i(t))$, the received power at base k most recently communicated to user i. At time $t \in T^i$, updates of AMPA can be written

$$p_i(t+1) = \min_k \frac{\gamma_i \left(R_k(\hat{\tau}_k^i(t)) - h_{ki}p_i(t)\right)}{h_{ki}} \tag{3.40}$$

where

$$R_k(\hat{\tau}_k^i(t)) = \sum_i h_{ki}p_i(\hat{\tau}_k^i(t)) + \sigma_k \tag{3.41}$$

However, at base k, $R_k(\hat{\tau}_k^i(t))$ is calculated using $p_j(\hat{\tau}_k^i(t))$ while at base k', $R_{k'}(\hat{\tau}_{k'}^i(t))$ is calculated using $p_j(\hat{\tau}_{k'}^i(t))$. In the iteration of Equation (3.40), user i effectively uses p_j at time $\hat{\tau}_k^i(t)$ from base k while also using p_j at time $\hat{\tau}_{k'}^i(t)$ from base k'. The framework of the asynchronous convergence theorem assumes that updates by user i are based on $p_j(\tau_j^i(t))$, a single delayed value of $p_j(t)$. However, we know that the synchronous version of Equation (3.40) is equivalent to the MPA algorithm. An extension to the asynchronous convergence theorem[3] will verify that the asynchronous iteration defined by Equation (3.40) converges.

3.5 Discussion

We believe that MPA can appropriately handle new calls and call completions. In particular, upon a call completion, a user powers down. If all users had acceptable connections before the call completion, then the remaining users will continue to have acceptable connections since the call completion reduces interference. That is, the power vector associated with the remaining users stays feasible. Continued execution of MPA by the remaining users leads to subsequent power reductions and perhaps reassignments. A new call arrival can be initialized with zero power. During call setup, the new user participates in the MPA algorithm. If the call can be carried, MPA will find the optimal power vector and a corresponding base station assignment.

[3] See Problem 2.2 in Chapter 6 of [52].

However, if it is not possible to provide all users an acceptable CIR, then measures external to the MPA algorithm must be taken to block the new call. This subject merits further attention.

The AMPA algorithm is a flexible totally asynchronous version of MPA. Each user is permitted to adjust its transmitted power and base station assignment using possibly outdated estimates of the interference power at the base stations without waiting for all other users to perform updates. In addition, we note that a user update that includes a base station reassignment will require coordination between the user and the base stations and would take significantly longer than a simple power adjustment. Consequently, the asynchronous algorithm permits users to continue to adjust powers while other users perform more time consuming base station reassignments. Just as for the synchronous case, the rate of convergence of the AMPA algorithm needs further investigation.

Chapter 4

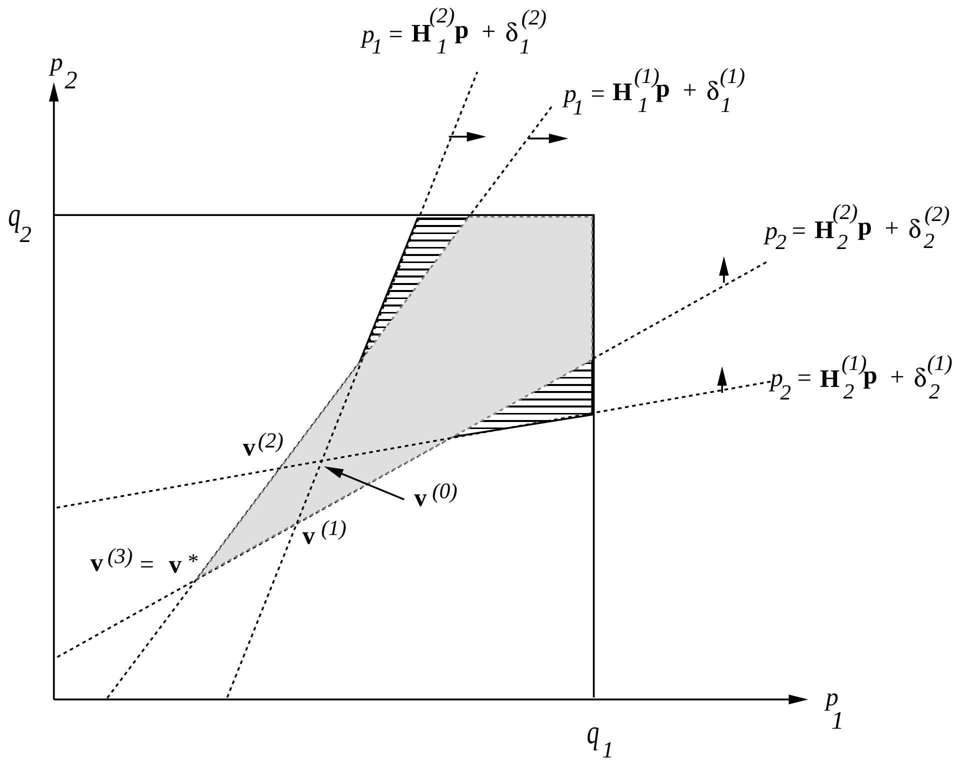
Constrained Power Control and Base Station Assignment

In this chapter, with constraints on the maximum transmitter power, we integrate power control and base station assignment. We find synchronous and asynchronous distributed algorithms that converge to a fixed point. When there exists a feasible solution, the fixed point is optimal in the sense of minimizing the total transmitted power subject to maintaining an individual target CIR for each mobile. The performance of the iterative algorithm is evaluated by simulation of a one dimensional system.

4.1 Introduction

As discussed in Chapter 3, the convergence properties of MPA algorithm are limited by the requirement of a feasible solution and that maximum power constraints are ignored. In practice, the maximum power limits are real and should be incorporated. In this work, the objective of providing each user an acceptable connection remains the same. However, this work considers the joint optimization of the users' transmitted powers and base station assignments in the presence of maximum power constraints.

We formulate the Constrained Minimum Transmitted Power (CMTP) problem in which the total transmitted uplink power is minimized subject to a minimum CIR and maximum transmitted power constraints for each individual user. We will describe synchronous and asynchronous versions of a simple iterative algorithm called Constrained Minimum Power Assignment (CMPA). If there exists a feasible solution, the CMPA algorithm iteratively solves the CMTP problem. If there is no feasible solution, then the CMPA algorithm will converge to a fixed point from which infeasibility can be detected.



This figure depicts the set of feasible power vectors for a system of two users and two base stations. Rays labeled $p_i = \boldsymbol{H}_i^{(k)}\boldsymbol{p}+\delta_i^{(k)}$ describe the minimum power user i needs to communicate with base k as a function of the transmitted power of the other user. Four base station assignments are possible corresponding to the four cones with labeled vertices $\boldsymbol{v}^{(0)}, \ldots, \boldsymbol{v}^{(3)} = \boldsymbol{v}^*$. The union of shaded regions depicts the nonconvex set $\cup_l P^{(l)}$.

Figure 4.1: The feasible region of an instance of CMTP

A tradeoff between the system performance and the complexity of the implementation will be discussed. A simple strategy is to assign users to the base station with the strongest pilot tone. In the absence of fading, this strategy assigns users to the closest base station. In comparison to this simple approach, we shall see that a base station assignment based on the minimization of transmitter power provides significant improvement in call blocking and call dropping. This improvement, however, is at the expense of many more base station reassignments. If the system performance on the radio link is the major concern for system design, the CMPA is a good candidate for base station assignment. If, however, the signaling costs of handoffs dominate the system costs, then perhaps CMPA is not such a wise choice.

4.2 The CMTP Problem

In the call maintenance stage, we wish to minimize the use of use of resources and provide all users the feasible connection. In the presence of maximum power constraints, we want to solve the Constrained Minimum Transmitted Power (CMTP) problem:

$$\begin{array}{lll} \text{minimize} & \sum_i p_i & \\ \text{subject to} & h_{a_i i} p_i \geq \gamma_i \left(\sum_{j \neq i} h_{a_i j} p_j + \nu_{a_i} \right) & i \in \{1, \ldots, N\} \\ & 0 \leq p_i \leq q_i & i \in \{1, \ldots, N\} \\ & a_i \in \{1, \ldots, M\} & i \in \{1, \ldots, N\} \end{array}$$

The CMTP constraint set specifies that each user i has an acceptable connection meeting an individual target CIR γ_i, that the non-negative power vector $\boldsymbol{p}$ will be bounded above by a maximum power vector $\boldsymbol{q}$, and that the assignment vector must take on integer values.

If user i is assigned to base k, then the CIR constraint for user i can be written

$$q_i \geq p_i \geq \boldsymbol{H}_i^{(k)} \boldsymbol{p} + \delta_i^{(k)} \tag{4.1}$$

where $\delta_i^{(k)} = \gamma_i \nu_k / h_{ki}$ and $\boldsymbol{H}_i^{(k)}$, as defined in Equation (2.7), is a normalized row vector with jth element.

Since each assignment is specified by an assignment vector $(a_1, \ldots, a_N)$, the number of possible assignment vectors is M^N. We can number the assignments $l = 1, 2, \ldots$ and associate assignment l with the assignment vector $(a_1(l), \ldots, a_N(l))$.

We define $\boldsymbol{G}^{(l)}$, the normalized interference matrix under assignment l, as having i, jth element

$$G_{ij}^{(l)} = H_{ij}^{(a_i(l))} \tag{4.2}$$

Similarly, the normalized noise vector $\boldsymbol{\sigma}^{(l)}$ is defined to have ith component $\sigma_i^{(l)} = \delta_i^{(a_i(l))}$. We say that a power vector $\boldsymbol{p}$ is feasible under assignment l, if for each user i, $\boldsymbol{G}_i^{(l)} \boldsymbol{p} + \sigma_i^{(l)} \leq p_i \leq$

q_i, where $\boldsymbol{G}_i^{(l)}$ denotes the ith row of $\boldsymbol{G}^{(l)}$. That is, the set of feasible power vectors under assignment l is

$$P^{(l)} = \left\{\boldsymbol{p} \geq 0 | \boldsymbol{G}^{(l)}\boldsymbol{p} + \boldsymbol{\sigma}^{(l)} \leq \boldsymbol{p} \leq \boldsymbol{q}\right\} \tag{4.3}$$

Thus, we say that a power vector $\boldsymbol{p}$ is feasible if $\boldsymbol{p} \in \cup_l P^{(l)}$.

Note that changing the assigned base station of user i simply corresponds to changing the row i constraints of $P^{(l)}$. Each polytope $P^{(l)}$ is specified by a normalized interference matrix $\boldsymbol{G}^{(l)}$ and normalized noise vector $\boldsymbol{\sigma}^{(l)}$. A sketch of these polytopes is provided in Figure 4.1. Finally, we mention that we could view the CMTP problem as the minimization of total transmitted power over the set of feasible power vectors $\cup_l P^{(l)}$. However, as depicted in Figure 4.1, $\cup_l P^{(l)}$ is typically not a convex set so that the standard approaches to minimizing a linear function over a convex set are not directly applicable.

In the context of the CMTP problem, we embed the set of CIR targets $\{\gamma_i\}$ in the interference matrix $\boldsymbol{G}^{(l)}$. When $P^{(l)}$ is nonempty and nondegenerate, we define the power vector $\boldsymbol{v}^{(l)}$ as the unique solution to

$$\boldsymbol{v}^{(l)} = \boldsymbol{G}^{(l)}\boldsymbol{v}^{(l)} + \boldsymbol{\sigma}^{(l)} \tag{4.4}$$

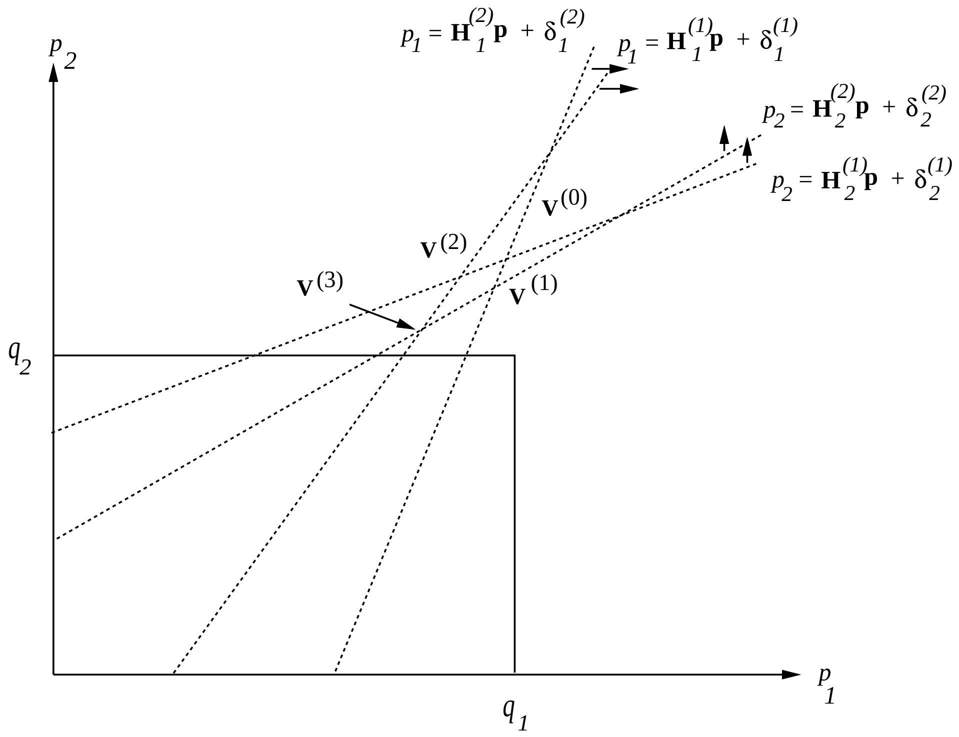
we will call $\boldsymbol{v}^{(l)}$ the minimum power vertex of $P^{(l)}$.

Lemma 4.1 *For all $\boldsymbol{p} \in P^{(l)}$, $\boldsymbol{p} \geq \boldsymbol{v}^{(l)}$.*

Proof: Lemma 4.1 Suppose there exists $\boldsymbol{p} \in P^{(l)}$ such that $p_i < v_i^{(l)}$ for some i. Let $\hat{\boldsymbol{p}} = \boldsymbol{v}^{(l)} + \alpha(\boldsymbol{p} - \boldsymbol{v}^{(l)})$. Since $\boldsymbol{v}^{(l)} = \boldsymbol{G}^{(l)}\boldsymbol{v}^{(l)} + \boldsymbol{\sigma}^{(l)}$ and $\boldsymbol{p} \geq \boldsymbol{G}^{(l)}\boldsymbol{p} + \boldsymbol{\sigma}^{(l)}$, we have for all $\alpha \geq 0$,

$$\begin{aligned} \hat{\boldsymbol{p}} - \left(\boldsymbol{G}^{(l)}\hat{\boldsymbol{p}} + \boldsymbol{\sigma}^{(l)}\right) &= \alpha\left(\boldsymbol{p} - \boldsymbol{G}^{(l)}\boldsymbol{p} - \boldsymbol{\sigma}^{(l)}\right) \\ &\geq 0 \end{aligned}$$

However, since there exists i such that $p_i < v_i^{(l)}$, we can choose $\alpha \geq 0$ such that for some i, $\hat{p}_i = 0$ but $\hat{p}_j \geq 0$ for all $j \neq i$. For this choice of α, $0 = \hat{p}_i < \boldsymbol{G}_i^{(l)}\hat{\boldsymbol{p}} + \boldsymbol{\sigma}^{(l)}$ which is a contradiction. □



This figure depicts the set of infeasible power vectors for a system of two users and two base stations. Rays labeled $p_i = \gamma_i \boldsymbol{H}_i^{(k)} \boldsymbol{p} + = \gamma_i \delta_i^{(k)}$ describe the minimum power user i needs to communicate with base k as a function of the transmitted power of the other user. Four base station assignments are possible corresponding to the four cones with labeled vertices $\boldsymbol{v}^{(0)}, \ldots, \boldsymbol{v}^{(3)} = \boldsymbol{v}^*$. Without maximum power constraints, the vertex $\boldsymbol{v}^{(3)}$ would be the optimal vertex $\boldsymbol{v}^*$.

Figure 4.2: An infeasible instance of CMTP

Lemma 4.1 implies that for a fixed assignment l, total transmitted power is minimized by $\boldsymbol{p} = \boldsymbol{v}^{(l)}$. Furthermore, the minimization of total power over all possible assignments can be reduced to finding the minimum power vertex $\boldsymbol{v}^{(l)}$ among all vertices. As the proof of the next claim shows, we can find the optimal vertex by construction.

Theorem 4.1 *If the CMTP problem is feasible, then there exists a minimum power vertex $\boldsymbol{v}^*$ such that $\boldsymbol{v}^* \leq \boldsymbol{v}^{(l)}$ for all l.*

Proof: Among all feasible assignments l, let l_i denote the assignment that minimizes $v_i^{(l)}$, the ith component of minimum power vertex $\boldsymbol{v}^{(l)}$. By definition, $v_i^{(l_i)} \leq v_i^{(l)}$ for all l and

$$v_i^{(l_i)} = \boldsymbol{G}_i^{(l_i)} \boldsymbol{v}^{(l_i)} + \sigma_i^{(l_i)} \tag{4.5}$$

where $\boldsymbol{G}_i^{(l_i)}$ denotes the ith row of $\boldsymbol{G}^{(l_i)}$. Consider $\boldsymbol{p} = [v_1^{(l_1)} \cdots v_N^{(l_N)}]^\top$. Since $\boldsymbol{p} \le \boldsymbol{v}^{(l_i)}$ for all i,

$$\begin{aligned} p_i = v_i^{(l_i)} &= \boldsymbol{G}_i^{(l_i)} \boldsymbol{v}^{(l_i)} + \sigma_i^{(l_i)} && (4.6)\\ &\ge \boldsymbol{G}_i^{(l_i)} \boldsymbol{p} + \sigma_i^{(l_i)} && (4.7) \end{aligned}$$

That is, $\boldsymbol{p}$ is feasible with respect to the ith constraint of polytope $P^{(l_i)}$. Let $\boldsymbol{G}^*$ denote the interference matrix with ith row $\boldsymbol{G}_i^* = \boldsymbol{G}_i^{(l_i)}$. Let $\boldsymbol{\sigma}^*$ denote the normalized noise vector with ith element $\sigma_i^* = \sigma_i^{(l_i)}$. The pair $\boldsymbol{G}^*$, $\boldsymbol{\sigma}^*$ describes a feasible polytope P^* with vertex $\boldsymbol{v}^*$. We have observed that $\boldsymbol{p} \in P^*$, so Lemma 4.1 implies $\boldsymbol{p} \ge \boldsymbol{v}^*$. By construction $\boldsymbol{p} \le \boldsymbol{v}^{(l)}$ for all l. Hence $\boldsymbol{p} \le \boldsymbol{v}^*$ so that $\boldsymbol{v}^* \le \boldsymbol{v}^{(l)}$ for all l. □

Theorem 4.2 *The minimum power vertex $\boldsymbol{v}^*$ and its corresponding base station assignment solves the CMTP problem.*

Proof: Consider any feasible power vector $\boldsymbol{p}$. Since $\boldsymbol{p} \in P^{(l)}$ for some l, Lemma 4.1 and Theorem 4.1 imply $\boldsymbol{p} \ge \boldsymbol{v}^{(l)} \ge \boldsymbol{v}^*$. Hence, $\sum_i p_i^{(l)} \ge \sum_i v_i^*$. □

Feasible and infeasible instances of CMTP are depicted in Figures 4.1 and 4.2 respectively. Although the power vector $\boldsymbol{v}^*$ is unique, $\boldsymbol{v}^*$ may be the minimum power vertex of more than one polytope. In this case, there would be multiple base station assignments for which $\boldsymbol{v}^*$ is the minimum vertex power vector. A trivial example occurs when two base stations share the same location in space.

We observe that if the objective function of the CMTP problem were not the total transmitted power but rather $f(\boldsymbol{p})$, a nondecreasing function[1] of the power vector $\boldsymbol{p}$, then $\boldsymbol{v}^*$ remains as the optimal solution. However, since there are M^N vertices, we have not identified an efficient

[1] $f(\boldsymbol{p})$ is nondecreasing in the sense that $\boldsymbol{p} \le \boldsymbol{p}'$ implies $f(\boldsymbol{p}) \le f(\boldsymbol{p}')$.

CMPA
$n := 0$;
`repeat` {
    $n := n + 1$;
    $\boldsymbol{k}(n) = \boldsymbol{A}(\boldsymbol{p}(n-1))$;
    $\boldsymbol{p}(n) = \boldsymbol{T}(\boldsymbol{p}(n-1))$;
    }
`until` $\boldsymbol{p}(n) = \boldsymbol{p}(n-1)$;

The CMPA Algorithm: Starting with an initial power vector $\boldsymbol{p}(0)$, $\boldsymbol{p}(n)$ denotes the power vector and $\boldsymbol{k}(n)$ the corresponding base station assignment after iteration n.

Figure 4.3: The CMPA Algorithm

procedure to find $\boldsymbol{v}^*$, when there exists a feasible solution, nor how to identify if the CMTP problem is infeasible.

4.3 Iterative Solutions to CMTP

In this section, we describe the iterative Constrained Minimum Power Assignment (CMPA) algorithm. We will show that CMPA always converges to a fixed point. Moreover, if the CMTP has a feasible solution, the CMPA fixed point will be the optimal power vector $\boldsymbol{v}^*$. In the CMPA algorithm, each user i adjusts its transmitter power to attain an individual target CIR γ_i subject to the maximum power constraint $p_i \leq q_i$. For this purpose, we define

$$M_i^{(k)}(\boldsymbol{p}) = \boldsymbol{H}_i^{(k)}\boldsymbol{p} + \delta_i^{(k)} \tag{4.8}$$

$$M_i(\boldsymbol{p}) = \min\{\min_k M_i^{(k)}(\boldsymbol{p}), q_i\} \tag{4.9}$$

$$A_i(\boldsymbol{p}) = \arg\min_k M_i^{(k)}(\boldsymbol{p}) \tag{4.10}$$

We note that $M_i^{(k)}(\boldsymbol{p})$ equals the power needed by user i to transmit to base station k with CIR γ_i, assuming that all other users keep their powers fixed. When there exists a base station k such that $M_i^{(k)}(\boldsymbol{p}) \leq q_i$, then $M_i(\boldsymbol{p})$ equals the minimum power needed by user i to transmit to a base station with CIR γ_i and $A_i(\boldsymbol{p})$ would be the corresponding base station. Otherwise, when no such base exists, $M_i(\boldsymbol{p}) = q_i$, the maximum possible power of user i, while $A_i(\boldsymbol{p})$ is the base station assignment at which user i maximizes its CIR.

Given a power vector $\boldsymbol{p}$, a CMPA iteration finds a new power vector $\boldsymbol{T}(\boldsymbol{p})$ and assignment vector $\boldsymbol{A}(\boldsymbol{p})$ described by

$$\begin{aligned} \boldsymbol{T}(\boldsymbol{p}) &= (M_1(\boldsymbol{p}), ..., M_N(\boldsymbol{p})) \\ \boldsymbol{A}(\boldsymbol{p}) &= (A_1(\boldsymbol{p}), ..., A_N(\boldsymbol{p})) \end{aligned}$$

Starting from an initial power vector $\boldsymbol{p}$, n iterations of the CMPA algorithm produce the power vector $\boldsymbol{T}^n(\boldsymbol{p})$. The CMPA algorithm is presented in Figure 4.3.

The minimization over the base stations k in Equation (4.9), can also be expressed as a minimization over the assignments l. In particular, if $a_i(l) = k$, then $M_i^{(k)}(\boldsymbol{p}) = \boldsymbol{G}_i^{(l)}\boldsymbol{p} + \sigma_i^{(l)}$. The CMPA iteration of Equation (4.9) can be written

$$p_i(n) = \min\left\{ \left(\min_l \boldsymbol{G}_i^{(l)}\boldsymbol{p}(n-1) + \sigma_i^{(l)} \right), q_i \right\} \tag{4.11}$$

Consequently, a fixed point $\bar{\boldsymbol{p}}$ of the CMPA algorithm must satisfy the following necessary condition.

$$\bar{p}_i = \min\left\{ \left(\min_l \boldsymbol{G}_i^{(l)}\bar{\boldsymbol{p}} + \sigma_i^{(l)} \right), q_i \right\} \qquad (1 \le i \le N) \tag{4.12}$$

From Figure 4.3, we see that the next base station assignment $\boldsymbol{k}(n)$ and power vector $\boldsymbol{p}(n)$ are specified by the previous power vector $\boldsymbol{p}(n-1)$. Note that it is possible to express CMPA just as an iteration of the power vector without specification of the base station assignment at every step. In this case, the assignment $\boldsymbol{A}(\bar{\boldsymbol{p}})$ corresponding to the fixed point $\bar{\boldsymbol{p}}$ could be specified at termination. However, we prefer to include the base station assignment in the CMPA algorithm to emphasize that in the context of a dynamic system, CMPA is intended for both power control and base station assignment in a system with mobile users.

We will prove that CMPA converges to a unique power vector $\bar{\boldsymbol{p}}$. Then we will show that $\bar{\boldsymbol{p}} = \boldsymbol{v}^*$ when the CMTP problem is feasible. The proof of the convergence of CMPA relies on proving convergence in two special cases, as shown in the following sequence of lemmas.

Lemma 4.2 *If $\boldsymbol{p} \leq \boldsymbol{p}'$, then $\boldsymbol{T}(\boldsymbol{p}) \leq \boldsymbol{T}(\boldsymbol{p}')$.*

Proof: Lemma 4.2 Equation (4.8) implies that for all i and k, $M_i^{(k)}(\boldsymbol{p}) \leq M_i^{(k)}(\boldsymbol{p}')$. Hence, from Equation (4.9), $M_i(\boldsymbol{p})$, the ith component of $\boldsymbol{T}(\boldsymbol{p})$, satisfies $M_i(\boldsymbol{p}) \leq M_i(\boldsymbol{p}')$. $\square$

Lemma 4.3 *Starting from $\boldsymbol{z}$, the all zero vector, the CMPA algorithm produces a monotone increasing sequence of power vectors $\boldsymbol{z}(n) = \boldsymbol{T}^n(\boldsymbol{z})$ that converges to a fixed point $\boldsymbol{z}^*$.*

Proof: Lemma 4.3 Let $\boldsymbol{z}(n) = \boldsymbol{T}^n(\boldsymbol{z})$. We observe $\boldsymbol{z}(1) = \boldsymbol{T}(\boldsymbol{z}) \geq \boldsymbol{z}$. Suppose $\boldsymbol{z} \leq \boldsymbol{z}(1) \leq \ldots \leq \boldsymbol{z}(n)$, Lemma 4.2 implies

$$\boldsymbol{z}(n+1) = \boldsymbol{T}(\boldsymbol{z}(n)) \geq \boldsymbol{T}(\boldsymbol{z}(n-1)) = \boldsymbol{z}(n)$$

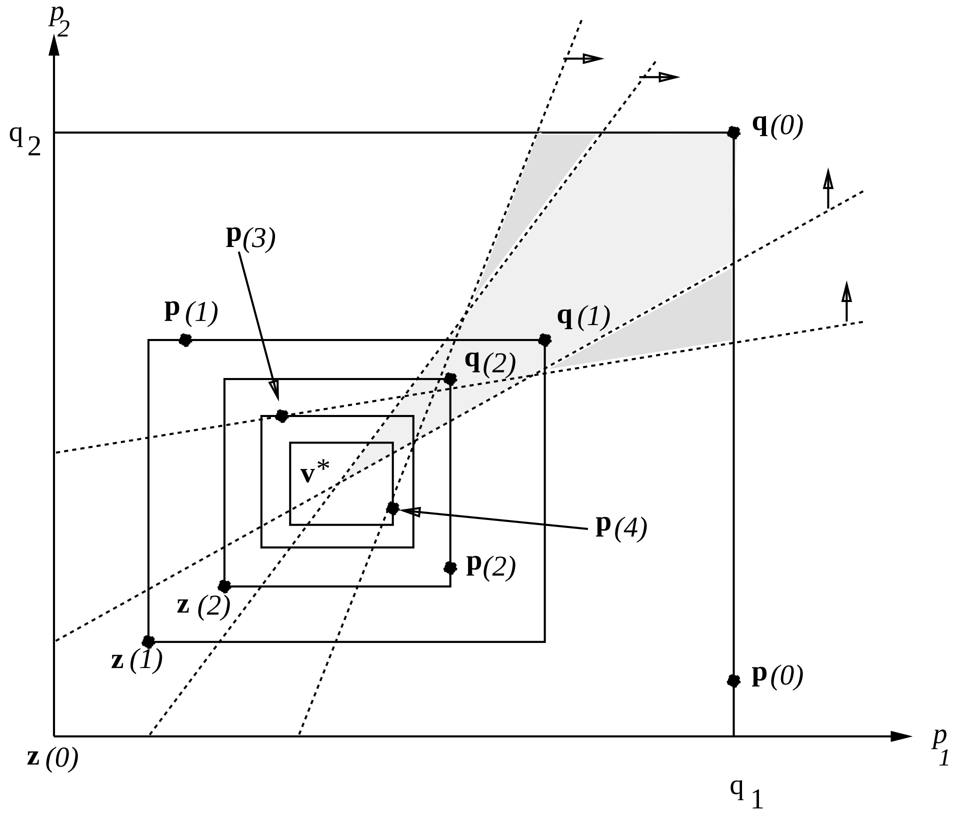
Hence the sequence of $\boldsymbol{z}(n)$ is nondecreasing. Note that for any power vector $\boldsymbol{p}$, $\boldsymbol{T}(\boldsymbol{p}) \leq \boldsymbol{q}$. This implies $\boldsymbol{z}(n) \leq \boldsymbol{q}$ for all n. That is, the monotone increasing sequence of power vectors $\boldsymbol{z}(n)$ is bounded above by $\boldsymbol{q}$ and so must converge. $\square$

Lemma 4.4 *Starting from $\boldsymbol{q}$, the maximum power vector, the CMPA algorithm produces a monotone decreasing sequence of power vectors $\boldsymbol{q}(n) = \boldsymbol{T}^n(\boldsymbol{q})$ that converges to a fixed point $\boldsymbol{q}^*$.*

Proof: Lemma 4.4 Let $\boldsymbol{q}(n) = \boldsymbol{T}^n(\boldsymbol{q})$. Since for any $\boldsymbol{p}$, $\boldsymbol{T}(\boldsymbol{p}) \leq \boldsymbol{q}$, we must have $\boldsymbol{T}(\boldsymbol{q}) \leq \boldsymbol{q}$. Suppose $\boldsymbol{q} \geq \boldsymbol{q}(1) \geq \ldots \geq \boldsymbol{q}(n)$, then Lemma 4.2 implies

$$\boldsymbol{q}(n+1) = \boldsymbol{T}(\boldsymbol{q}(n)) \leq \boldsymbol{T}(\boldsymbol{q}(n-1)) = \boldsymbol{q}(n) \tag{4.13}$$

Hence the sequence of $\boldsymbol{q}(n)$ is decreasing. For any power vector $\boldsymbol{p}$, $\boldsymbol{T}(\boldsymbol{p}) \geq 0$. Hence, $\boldsymbol{q}(n) \geq \boldsymbol{z}$ for all n. That is, the monotone decreasing sequence of power vectors $\boldsymbol{q}(n)$ is bounded below by $\boldsymbol{z}$ and so must converge. $\square$



In this example, there are two base stations and two mobiles with powers p_1 and p_2. Each pair of rays, one marked by $\rightarrow$ and one by $\uparrow$, forms the polytope corresponding to a base station assignment. The polytope with minimum power vertex $\boldsymbol{v}^*$ is shaded. With respect to a particular assignment, rays marked by $\rightarrow$ indicate constraints that p_1 must exceed while rays marked by $\uparrow$ indicate constraints that p_2 must exceed. The initial power vector is $\boldsymbol{p}(0)$. Subsequent power vectors produced by CMPA are denoted $\boldsymbol{p}(n)$. Following the proof of Theorem 4.3, $\boldsymbol{z}(n) = \boldsymbol{T}^n(\boldsymbol{z})$ and $\boldsymbol{q}(n) = \boldsymbol{T}^n(\boldsymbol{q})$ constrain $\boldsymbol{p}(n)$ to the box $\boldsymbol{z}(n) \leq \boldsymbol{p}(n) \leq \boldsymbol{q}(n)$.

Figure 4.4: Convergence of the CMPA Algorithm

Theorem 4.3 *There exists a fixed point $\bar{\boldsymbol{p}}$ such that for any initial power vector $\boldsymbol{p}$, the CMPA algorithm will converge to $\bar{\boldsymbol{p}}$.*

Proof: From Lemmas 4.3 and 4.4, let fixed points $\boldsymbol{z}^*$ and $\boldsymbol{q}^*$ have assignment vectors $\boldsymbol{A}(\boldsymbol{z}^*)$ and $\boldsymbol{A}(\boldsymbol{q}^*)$ with corresponding assignments l_z and l_q respectively. We will first show that $\boldsymbol{z}^* = \boldsymbol{q}^*$. Since $\boldsymbol{z} \leq \boldsymbol{q}$, Lemma 4.2 implies $\boldsymbol{T}^n(\boldsymbol{z}) \leq \boldsymbol{T}^n(\boldsymbol{q})$ for all n. Hence, $\boldsymbol{z}^* \leq \boldsymbol{q}^*$. Suppose there exists some j such that $z_j^* < q_j^*$. Let $\hat{\boldsymbol{p}} = \boldsymbol{z}^* + \alpha(\boldsymbol{z}^* - \boldsymbol{q}^*)$. We can choose $\alpha \geq 0$ so that there exists i with the property that $\hat{p}_i = 0$ but $\hat{p}_j \geq 0$ for all $j \neq i$. For such an i, we must have $z_i^* < q_i^* \leq q_i$. Applying the necessary condition (4.12) to the fixed point $\boldsymbol{z}^*$ yields $z_i^* = \boldsymbol{G}_i^{(l_z)}\boldsymbol{z}^* + \sigma_i^{(l_z)}$. In addition, since $\boldsymbol{q}^*$ is a fixed point, the necessary condition (4.12) requires $q_i^* \leq \boldsymbol{G}_i^{(l_z)}\boldsymbol{q}^* + \sigma_i^{(l_z)}$. Since $\alpha \geq 0$,

$$\begin{aligned} \hat{\boldsymbol{p}}_i - (\boldsymbol{G}_i^{(l_z)}\hat{\boldsymbol{p}} + \sigma_i^{(l_z)}) &= -\alpha(q_i^* - \boldsymbol{G}_i^{(l_z)}\boldsymbol{q}^* - \sigma_i^{(l_z)}) \\ &\geq 0 \end{aligned}$$

However, we have defined α such that

$$\hat{\boldsymbol{p}}_i = 0 < G_i^{(l_z)}\hat{\boldsymbol{p}} + \sigma_i^{(l_z)} \tag{4.14}$$

which is a contradiction. Hence, there exists $\bar{\boldsymbol{p}}$ such that $\boldsymbol{z}^* = \boldsymbol{q}^* = \bar{\boldsymbol{p}}$.

Consider an arbitrary initial power vector $\boldsymbol{p}$. Note that after one CMPA iteration, the power vector $\boldsymbol{T}(\boldsymbol{p})$ satisfies $0 \leq \boldsymbol{T}(\boldsymbol{p}) \leq \boldsymbol{q}$. Hence, we can assume the initial power vector $\boldsymbol{p}$ satisfies $\boldsymbol{z} \leq \boldsymbol{p} \leq \boldsymbol{q}$. Lemma 4.2 implies

$$\boldsymbol{T}^n(\boldsymbol{z}) \leq \boldsymbol{T}^n(\boldsymbol{p}) \leq \boldsymbol{T}^n(\boldsymbol{q}) \tag{4.15}$$

Since $\lim_{n\to\infty} \boldsymbol{T}^n(\boldsymbol{z}) = \lim_{n\to\infty} \boldsymbol{T}^n(\boldsymbol{q}) = \bar{\boldsymbol{p}}$, (4.15) implies $\lim_{n\to\infty} \boldsymbol{T}^n(\boldsymbol{p}) = \bar{\boldsymbol{p}}$. $\square$

We have shown that for any initial power vector, the CMPA algorithm converges to a unique fixed point $\bar{\boldsymbol{p}}$. When the CMTP problem is infeasible, the fixed point $\bar{\boldsymbol{p}}$ has the property that user i either attains its desired CIR γ_i or is transmitting at maximum power q_i. Now we will show that when the CMTP problem is feasible, the CMPA fixed point $\bar{\boldsymbol{p}}$ is the optimal solution $\boldsymbol{v}^*$ to the CMTP problem.

Lemma 4.5 *If $\boldsymbol{p}$ is a feasible power vector, then $\boldsymbol{T}(\boldsymbol{p})$ is also a feasible power vector such that $\boldsymbol{T}(\boldsymbol{p}) \leq \boldsymbol{p}$.*

Proof: Lemma 4.5 Given $\boldsymbol{p}$ is feasible, there exists l_0 such that $\boldsymbol{p} \in P^{(l_0)}$. Let $\hat{\boldsymbol{p}} = \boldsymbol{T}(\boldsymbol{p})$. From Equation (4.11), we have

$$\begin{aligned} \hat{p}_i &\leq \boldsymbol{G_i}^{(l_0)}\boldsymbol{p} + \sigma_i^{(l_0)} \\ &\leq p_i \end{aligned}$$

Hence $\hat{\boldsymbol{p}} \leq \boldsymbol{p}$. Moreover, there exists an assignment l_1, corresponding to assignment vector $\boldsymbol{A}(\boldsymbol{p})$, such that $\hat{\boldsymbol{p}} = \boldsymbol{G}^{(l_1)}\boldsymbol{p} + \boldsymbol{\sigma}^{(l_1)}$. Since $\boldsymbol{p} \geq \hat{\boldsymbol{p}}$, we have $\hat{\boldsymbol{p}} \geq \boldsymbol{G}^{(l_1)}\hat{\boldsymbol{p}} + \boldsymbol{\sigma}^{(l_1)}$. That is, $\hat{\boldsymbol{p}} \in P^{(l_1)}$. □

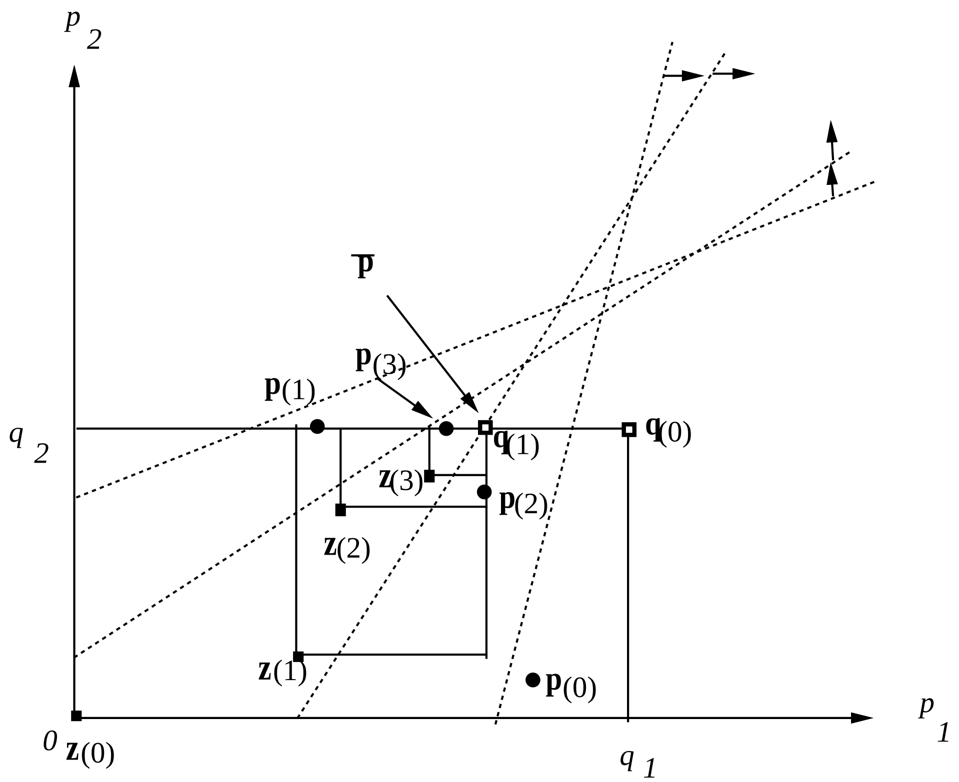
Theorem 4.4 *If the CMTP problem is feasible, then $\bar{\boldsymbol{p}} = \boldsymbol{v}^*$.*

Proof: All that is needed is to show that $\boldsymbol{T}(\boldsymbol{v}^*) = \boldsymbol{v}^*$. Since $\boldsymbol{v}^*$ is feasible, Lemma 4.5 implies $\boldsymbol{T}(\boldsymbol{v}^*) \leq \boldsymbol{v}^*$ and that $\boldsymbol{T}(\boldsymbol{v}^*)$ is feasible. Since $\boldsymbol{T}(\boldsymbol{v}^*)$ is feasible, Theorem 4.1 implies $\boldsymbol{T}(\boldsymbol{v}^*) \geq \boldsymbol{v}^*$. Hence $\boldsymbol{T}(\boldsymbol{v}^*) = \boldsymbol{v}^*$. □

Note that the CMPA iteration can be implemented as

$$p_i(n) = \min\left\{\left(\min_k \frac{\gamma_i\left(R_k(\boldsymbol{p}(n-1)) - h_{ki}p_i(n-1)\right)}{h_{ki}}, q_i\right)\right\} \tag{4.16}$$

where $R_k(\boldsymbol{p}) = \sum_i h_{ki}p_i + \sigma_k$ is the total received power (signal plus noise) at base k using power vector $\boldsymbol{p}$. That is, whether user i can communicate with base station k at CIR γ_i may be determined from the total received power at base k, the transmitted power p_i of user i and the uplink gain h_{ki} to base k. Although the minimization of $M_i^{(k)}(\boldsymbol{p})$ in Equation (4.9) occurs



In this example, there is no feasible solution. The fixed point $\bar{\boldsymbol{p}}$ is on the boundary of the polytope formed by the constraint on the individual maximum power level. Rays marked by $\rightarrow$ indicate constraints that p_1 must exceed while rays marked by $\uparrow$ indicate constraints that p_2 must exceed. The initial power vector is $\boldsymbol{p}(0)$. Subsequent power vectors produced by CMPA are denoted $\boldsymbol{p}(n)$.

Figure 4.5: Convergence of the CMPA Algorithm

over all base stations k, in practice it would be necessary only to consider nearby base stations as a result of signal attenuation. Effectively, a user needs only to know the total received power and its uplink gain h_{ki} at each nearby base station. If the uplink and downlink face the same attenuation, then user i can deduce the uplink gain h_{ki} from a pilot tone from base station k. Furthermore, it would also be possible for the downlink communication channel to provide each user $R_k(\boldsymbol{p}(n))$ for all nearby base stations k.

The convergence of the CMPA algorithm to the fixed point $\bar{\boldsymbol{p}}$ is depicted in Figures 4.4 and 4.5. Starting from an initial feasible power vector $\boldsymbol{p}(0) = \boldsymbol{p}$, it is readily shown that the resulting sequence of power vectors $\boldsymbol{p}(n)$ satisfies $\boldsymbol{p}(n+1) \leq \boldsymbol{G}^*\boldsymbol{p}(n) + \boldsymbol{\sigma}^*$. In [10], it is verified that an iteration of the form $\boldsymbol{p}(n+1) = \boldsymbol{G}\boldsymbol{p}(n) + \boldsymbol{\sigma}$ is a contraction mapping that converges at a geometric rate when the largest eigenvalue of G is less than unity. Moreover, if there exists a feasible power vector $\boldsymbol{p}$ for the CMTP problem, then the interference matrix $\boldsymbol{G}^*$ must have largest eigenvalue less than unity. Hence, starting from a feasible solution $\boldsymbol{p}$, geometric convergence to $\boldsymbol{v}^* = \bar{\boldsymbol{p}}$ does occur. However, when there is no feasible solution or when the CMPA algorithm starts from an infeasible power vector, the rate of convergence is not as well understood. In particular, one can construct examples in which a feasible solution exists but the CMPA iteration at step n does not constitute a contraction mapping.

4.4 The Asynchronous CMPA Algorithm

In this section, we examine an asynchronous version of the CMPA algorithm using the framework for totally asynchronous algorithms of Bertsekas and Tsitsiklis [52]. The asynchronous CMPA algorithm allows some users to perform power adjustments faster and execute more iterations than others. In addition, the asynchronous CMPA algorithm allows users to perform these updates while using outdated information on the interference caused by other users. Let $p_i(t)$ denote the transmitted power of user i at time t so that the power vector at time t is $\boldsymbol{p}(t) = (p_1(t), \ldots, p_N(t))$.

We assume that user i may not have access to the most recent values of the components of

$\boldsymbol{p}(t)$. This occurs when user i has outdated information about the received power at certain bases. At time t, let $\tau_j^i(t)$ denote the most recent time for which p_j is known to user i. Note that $0 \leq \tau_j^i(t) \leq t$. For user i at time t, $p_j(\tau_j^i(t))$ denotes the most recently known value of $p_j(t)$. Thus, at time t, user i may update its transmitter power and base station assignment using the power vector

$$\boldsymbol{p}(\tau^i(t)) = \left[p_1(\tau_1^i(t)), p_2(\tau_2^i(t)), ..., p_N(\tau_N^i(t))\right]^\top \tag{4.17}$$

We assume a set of times $T = \{0, 1, 2, ...\}$ at which one or more components $p_i(t)$ of $\boldsymbol{p}(t)$ are updated. Let T^i be the set of times at which $p_i(t)$ is updated. At times $t \notin T^i$, $p_i(t)$ is left unchanged. Hence, the asynchronous CMPA algorithm can be written as

$$p_i(t+1) = \begin{cases} M_i(\boldsymbol{p}(\tau^i(t))) & t \in T^i \\ p_i(t) & t \notin T^i \end{cases} \tag{4.18}$$

$$k_i(t+1) = \begin{cases} A_i(\boldsymbol{p}(\tau^i(t))) & t \in T^i \\ k_i(t) & t \notin T^i \end{cases} \tag{4.19}$$

We assume the sets T^i are infinite and given any time t_0, there exists t_1 such that $\tau_j^i(t) \geq t_0$ for all $t \geq t_1$. Convergence of the asynchronous CMPA algorithm will be proven by the Asynchronous Convergence Theorem as taken from [52]. We note that x and $f(x)$ in the statement of Theorem 4.5 represent the power vector $\boldsymbol{p}$ and iteration function $\boldsymbol{T}(\boldsymbol{p})$ in the context of this work.

Theorem 4.5 *(Asynchronous Convergence Theorem) If there is a sequence of nonempty sets* $\{X(n)\}$ *with* $X(n+1) \subset X(n)$ *for all* n *satisfying the following two conditions:*

1. *Synchronous Convergence Condition: For all* n *and* $x \in X(n)$, $f(x) \in X(n+1)$. *If* $\{y^n\}$ *is a sequence such that* $y^n \in X(n)$ *for all* n, *then every limit point of* $\{y^n\}$ *is a fixed point of* f.

2. *Box Condition: For every n, there exists sets $X_i(n) \in X_i$ such that $X(n) = X_1(n) \times X_2(n) \times \cdots \times X_N(n)$.*

and the initial solution estimate $x(0)$ belongs to the set $X(0)$, then every limit point of $\{x(t)\}$ is a fixed point of f.

Theorem 4.6 *From any initial power vector $\boldsymbol{p}$, $0 \leq \boldsymbol{p} \leq \boldsymbol{q}$, the asynchronous CMPA algorithm converges to $\bar{\boldsymbol{p}}$.*

Proof: Let $\boldsymbol{z}$ to denote the all zero vector. We define

$$X(n) = \{\boldsymbol{p} | \boldsymbol{T}^n(\boldsymbol{z}) \leq \boldsymbol{p} \leq \boldsymbol{T}^n(\boldsymbol{q})\} \tag{4.20}$$

For all n, the set $X(n)$ satisfies the box condition. By Lemmas 4.3 and 4.4, $X(n+1) \subset X(n)$ for all n. From the proof of Theorem 4.3, $\lim_{n\to\infty} \boldsymbol{T}^n(\boldsymbol{z}) = \lim_{n\to\infty} \boldsymbol{T}^n(\boldsymbol{q}) = \bar{p}$. Hence any sequence $\{y^n\}$ such that $y^n \in X(n)$ for all n must converge to $\bar{\boldsymbol{p}}$. Since the initial power vector $\boldsymbol{p}$ satisfies $\boldsymbol{z} \leq \boldsymbol{p} \leq \boldsymbol{q}$, $\boldsymbol{p} \in X(0)$. Thus, the asynchronous convergence theorem implies the asynchronous CMPA algorithm will converge to $\bar{\boldsymbol{p}}$. □

The implementation of the asynchronous CMPA defined by (4.18) and (4.19) is not the most practical since it requires each user to notify every other user of its transmitted power. Just as in the iteration (4.16) for CMPA, the asynchronous CMPA can be expressed in terms of the total received power (signal plus noise) at each base. In this approach, each base station k uses the downlink channel to inform the users of $R_k(t)$, the total received power at base k at time t. Of course, $R_k(t)$ would not be continuously updated and users may use outdated values of $R_k(t)$ to perform updates. These delays would permit a user i at time t to know $R_k(\hat{\tau}_k^i(t))$, the received power at base k most recently communicated to user i. At time $t \in T^i$, updates of asynchronous CMPA can be written

$$p_i(t+1) = \min\left\{\min_k \frac{\gamma_i\left(R_k(\hat{\tau}_k^i(t)) - h_{ki}p_i(t)\right)}{h_{ki}}, q_i\right\} \tag{4.21}$$

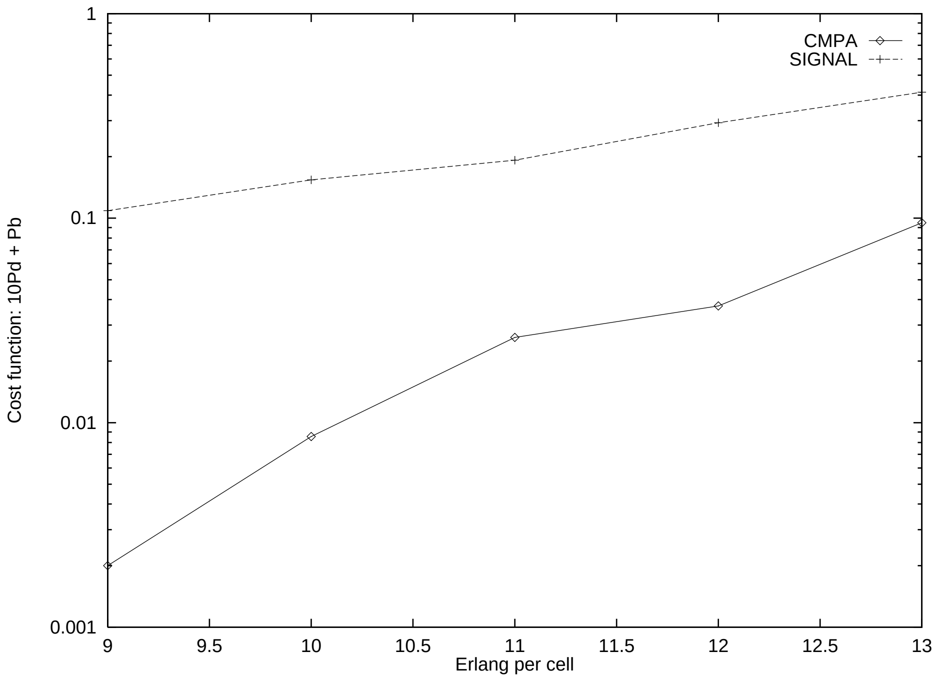


Figure 4.6: Weighted call blocking and dropping for CMPA vs. signal strength based assignment scheme.

where

$$R_k(\hat{\tau}_k^i(t)) = \sum_i h_{ki} p_i(\hat{\tau}_k^i(t)) + \sigma_k \tag{4.22}$$

However, at base k, $R_k(\hat{\tau}_k^i(t))$ is calculated using $p_j(\hat{\tau}_k^i(t))$ while at base k', $R_{k'}(\hat{\tau}_{k'}^i(t))$ is calculated using $p_j(\hat{\tau}_{k'}^i(t))$. In the iteration (4.21), user i effectively uses p_j at time $\hat{\tau}_k^i(t)$ from base k while also using p_j at time $\hat{\tau}_{k'}^i(t)$ from base k'. The framework of the asynchronous convergence theorem assumes that updates by user i are based on $p_j(\tau_j^i(t))$, a single delayed value of $p_j(t)$. However, we know that the synchronous version of (4.21) is equivalent to the CMPA algorithm. An extension to the asynchronous convergence theorem[2] will verify that the asynchronous iteration (4.21) will converge to $\bar{\boldsymbol{p}}$.

[2] See Problem 2.2 in Chapter 6 of [52].

4.5 Simulation Results

We now examine the performance of CMPA for a system with mobile users. In particular, we wish to identify whether the combined base station assignment and power control algorithm of CMPA provides any significant benefits over a simpler strategy in which the power control still aims for the target CIR but the base station assignment is based on the received signal strength of a base station pilot tone. Let P_b and P_d denote the fraction of mobile calls that are blocked and dropped respectively during a simulation. We define a cost function, $10P_d + P_b$ in which we assume that dropping a call is 10 times worse than blocking a new call. Blocking of new call attempts occurs when the required transmitted power of the new call exceeds the maximum power and call dropping occurs if at any time a user has CIR 3 dB lower than the target CIR.

Figure 4.6 shows the simulation results of system performance (cost function) for the pilot signal strength based assignment scheme and the CMPA scheme. It is clear that the CMPA perform much better than the pilot signal strength based assignment scheme. When the traffic load increases, the extent of the improvement will be reduced. This appears to occur because when user density is high in all cells, a user will be most likely assigned to the base station with the highest link gain.

Base station reassignments will occur when the required transmitted power can be reduced by reassigning to a new base station. As shown in Figure 4.7, 6 to 8 times the number of reassignments are required for the CMPA algorithm over the signal strength based assignment. This increase in reassignments, whether implemented by a traditional hard handoff or by a soft handoff [51] in the IS-95 system, constitutes an increase in implementation complexity.

4.6 Discussion

When it is possible to provide N acceptable connections, the CMPA algorithm will find the minimum power solution and a corresponding base station assignment. When there is no feasible solution, the CMPA algorithm will still converge to a fixed point, from which infeasibility can

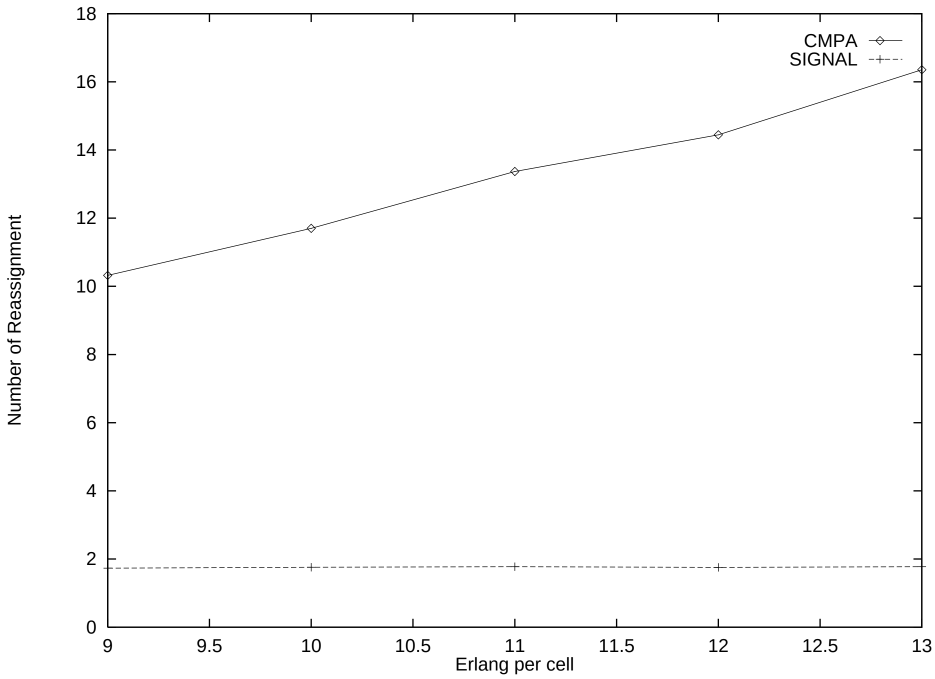


Figure 4.7: Average number of base station reassignments per call for CMPA vs. signal strength based assignment scheme

be deduced. It is possible to use the CMPA algorithm in which power is minimized over base station and channel assignment. It is important to keep in mind that in this context, CMPA is merely an heuristic for the combined problem of power, base station and channel assignment. The theorems proven in the paper only apply to single channel system.

We believe call arrivals and departures can be incorporated into the CMPA framework. In particular, upon a call completion, a user powers down. Those users that had acceptable connections before the call completion will continue to have acceptable connection since the call completion reduces interference. Continued execution of CMPA by the remaining users leads to subsequent power reductions and perhaps reassignments. A new call arrival can be initialized with zero power. During call setup, the new user participates in the CMPA algorithm. If the call can be carried, CMPA will find the optimal power vector and a corresponding base station assignment. However, if it is not possible to provide each users an acceptable CIR, then the CMPA algorithm will converge to an infeasible solution and the new call would be blocked in order to continue to carry the existing calls.

In the event that user mobility and channel fading make it impossible to carry all calls, one or more calls must be dropped. Appropriate call dropping policies based on the CMPA algorithm are worthy of study. Furthermore, we believe that the CMPA algorithm will permit the consideration of CIR based call admission policies that are designed to reduce the dropping of calls in progress.

The asynchronous CMPA algorithm is a flexible totally asynchronous version of CMPA. Each user is permitted to adjust its transmitted power and base station assignment using possibly outdated estimates of the interference power at the base stations without waiting for all other users to perform updates. In addition, we note that a user update that includes a base station reassignment will require coordination between the user and the base stations and would take significantly longer than a simple power adjustment. Consequently, the asynchronous algorithm permits users to continue to adjust powers while other users perform more time consuming base station reassignments.

Chapter 5

Convergence of Iterative Power Control Algorithms

In wireless communication systems, mobile users adapt to a time varying radio channel by regulating transmitter power. It is important that a power control algorithm can quickly converge to a fixed point at which either all users have acceptable connections or an infeasibility can be detected. In this work, we show that a constrained iterative power control based on CIR measurements converges to a fixed point at a geometric rate. This conclusion is shown to hold when the assignment of users to base stations is fixed as well as when the users choose a base station in order to minimize transmitted power. The rate of convergence is evaluated by simulation of a one dimensional CDMA system.

5.1 Introduction

Previous chapters have proven convergence results but have not addressed the rate of convergence. This chapter is an effort to understand the *rate* of convergence for cellular radio systems employing iterative power control methods that are subject to maximum power constraints.

Convergence properties of unconstrained fixed base station assignment power control algorithms have been discussed in [8–10, 13]. Foschini and Miljanic [10] and Mitra [21] show that when the system can support all users at a common target CIR, an iterative power control will converge to a fixed point at a geometric rate.

A joint power control and base station assignment is considered by Hanly [24] and Yates and Huang [25] in the absence of transmitted power constraints. Under fixed base station assignment, Grandhi and Zander [27] examine the convergence problems when there are maximum power constraints. Yates and Huang [28] discuss the convergence problems for integrated power control and base station assignment in the presence of maximum power constraints. A framework for uplink power control yielding sufficient conditions for convergence are provided in [29].

In this work, we show that a constrained iterative power control based on CIR measurements converges to a fixed point at a geometric rate. This conclusion is shown to hold when the assignment of users to base stations is fixed as well as when the users choose a base station in order to minimize transmitted power. As described in [54], power control for CDMA systems is needed to compensate for the signal fluctuation caused by propagation loss, lognormal fading and slow Rayleigh fading. Error correction coding and interleaving can be used to mitigate the signal degradation caused by fast fading. In the proposed IS-95 CDMA system standard [2], power adjustments occur every 1.25 msec (800 times per second) for closed loop power control. In this context, we found that, according to simulation results, the CIR based power control algorithms are fairly practical in terms of convergence speed.

5.2 Interference Constraints

The power control algorithm we consider in this work permits the possibility that user i may choose a base station in order to minimize its transmitted power. Following [24], we will assume that each user i iteratively selects a base station k from a set K_i in order to minimize its transmitted power. In addition, a maximum power constraint q_i is imposed on each user i. The resulting iterative algorithm can be expressed as

$$\boldsymbol{p}(n+1) = \boldsymbol{T}(\boldsymbol{p}(n)) \tag{5.1}$$

where for user i,

$$p_i(n+1) = T_i(\boldsymbol{p}(n)) = \min\left\{\min_{k\in K_i} \boldsymbol{H}_i^{(k)}\boldsymbol{p}(n) + \delta_i^{(k)}, q_i\right\} \tag{5.2}$$

When user i is permanently assigned to base a_i, $K_i = \{a_i\}$ and Equation (5.2) is the fixed base station assignment constrained power control algorithm proposed in [27]. When K_i includes all possible base stations, we have the constrained minimum power assignment (MPA) algorithm analyzed in [28].

We number the actual base stations by $k = 1, 2, \ldots, M$. It will be useful to consider a user that must transmit at maximum power to be assigned to a *virtual* base station numbered $k = 0$. For this virtual base station, we define

$$\boldsymbol{H}_i^{(0)} = [0, \cdots, 0] \qquad \delta_i^{(0)} = q_i \tag{5.3}$$

In this case, the constrained iteration function $\boldsymbol{T}(\boldsymbol{p})$ of Equation (5.2) becomes

$$T_i(\boldsymbol{p}) = \min_{k \in K_i} \boldsymbol{H}_i^{(k)} \boldsymbol{p} + \delta_i^{(k)} \tag{5.4}$$

For a fixed base station assignment with maximum power constraints, $K_i = \{0, a_i\}$. When user i minimizes its transmitted power over all base stations, $K_i = \{0, \ldots, M\}$. In general, when K_i has a single element a_i for each i, we say the base station assignment is fixed. When each K_i has more than one element, each user attempts to minimize its transmitter power over the set of possible base station assignments. In this case, we call the power control iteration (5.4) a general minimum power assignment algorithm. In particular, the constrained power control iteration under fixed base station assignment can be viewed as a minimum power assignment algorithm in which each user i minimizes its transmitter power by choosing among its nominal assigned base a_i and the virtual base station 0.

A base station assignment is a vector $[a_1, \ldots, a_N]$ that specifies an assigned base a_i for each mobile i. We can number the base station assignments as $l = 1, 2, \ldots$ and define $a_i(l)$ such that under assignment l, user i is assigned to base $a_i(l)$. We can associate with each assignment l, a gain matrix $\boldsymbol{G}^{(l)}$, the normalized interference matrix under assignment l, as having i, jth element

$$G_{ij}^{(l)} = H_{ij}^{(a_i(l))} \tag{5.5}$$

Similarly, the normalized noise vector $\boldsymbol{\sigma}^{(l)}$ is defined to have ith component $\sigma_i^{(l)} = \delta_i^{(a_i(l))}$.

At every step n of the iteration (5.4), each user i chooses the base station $k_i \in K_i$ that requires minimum transmitted power in order to maintain a feasible connection. Thus, at step n, there is an assignment l_n such that $a_i(l_n) = k_i$ for all i and we say that assignment l_n was used at step n. When assignment l is used, the power control iteration function $\boldsymbol{T}(\boldsymbol{p})$ becomes

$$\boldsymbol{T}^{(l)}(\boldsymbol{p}) = \boldsymbol{G}^{(l)}\boldsymbol{p} + \boldsymbol{\sigma}^{(l)} \tag{5.6}$$

Thus, the iteration at step n can be expressed as

$$\boldsymbol{p}(n+1) = \boldsymbol{T}(\boldsymbol{p}(n)) = \boldsymbol{T}^{(l_n)}(\boldsymbol{p}(n)) = \boldsymbol{G}^{(l_n)}\boldsymbol{p}(n) + \boldsymbol{\sigma}^{(l_n)} \tag{5.7}$$

Since the base assignment is chosen to minimize the transmitted power, if assignment l_n is used at stage n, then for all assignments l,

$$\boldsymbol{T}^{(l_n)}(\boldsymbol{p}(n)) \leq \boldsymbol{T}^{(l)}(\boldsymbol{p}(n)) \tag{5.8}$$

From Equation 2.9, each assignment l specifies a cone of feasible power vectors

$$P^{(l)} = \left\{ \boldsymbol{p} \geq 0 | \boldsymbol{p} \geq \boldsymbol{G}^{(l)}\boldsymbol{p} + \boldsymbol{\sigma}^{(l)} \right\}$$

We say the assignment l is *feasible* if $P^{(l)}$ is not empty. By the Perron-Frobenius theorem [55], $P^{(l)}$ is empty iff the largest eigenvalue of $\boldsymbol{G}^{(l)}$, to be denoted $\lambda^{(l)}$, satisfies $\lambda^{(l)} > 1$ [10]. Under power vector $\boldsymbol{p} \in P^{(l)}$, each user i satisfying $a_i(l) > 0$ is meeting or exceeding its target CIR γ_i. When $a_i(l) = 0$, user i is transmitting at maximum power which only occurs if the user is below its target CIR.

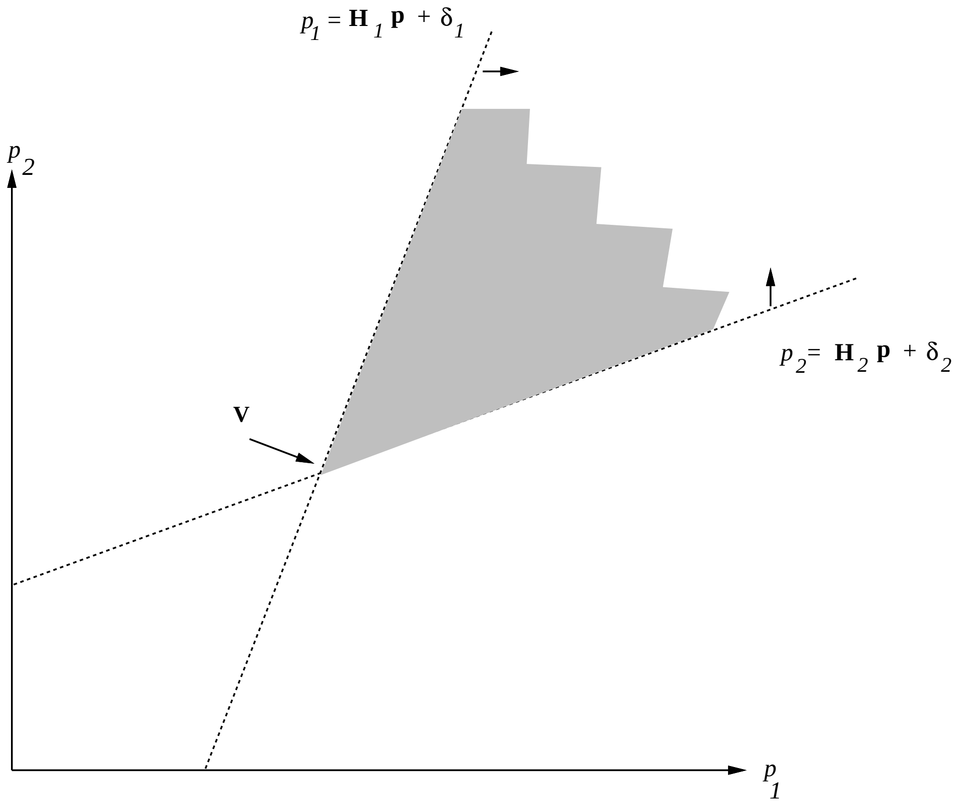


Figure 5.1: The cone $P^{(l)}$

In [28], it is verified that if $P^{(l)}$ is non-empty, then it has a unique vertex $\boldsymbol{v}^{(l)}$ satisfying $\boldsymbol{v}^{(l)} = \boldsymbol{G}^{(l)}\boldsymbol{v}^{(l)} + \boldsymbol{\sigma}^{(l)}$. For each cone $P^{(l)}$, there are three possibilities:

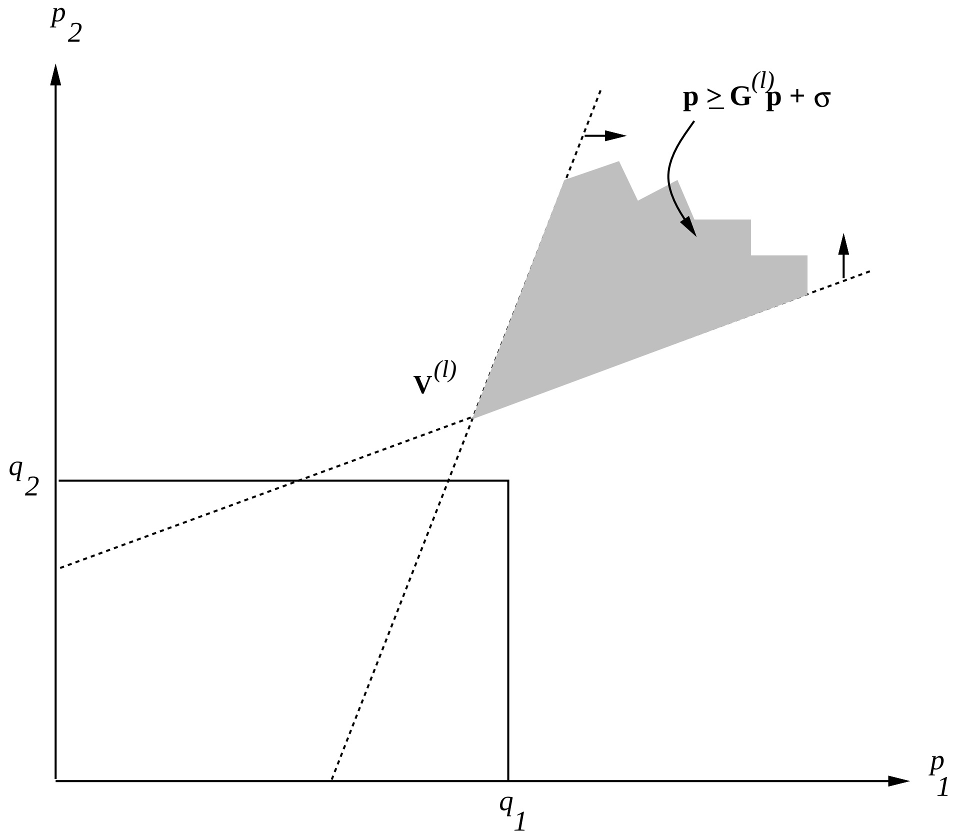
1. The cone $P^{(l)}$ is not empty and $\boldsymbol{v}^{(l)} \leq \boldsymbol{q}$; see Figure 5.1.

2. The cone $P^{(l)}$ is nonempty but $\boldsymbol{v}^{(l)} \not\leq \boldsymbol{q}$; see Figure 5.2

3. The cone $P^{(l)}$ is empty; see Figure 5.3.

5.3 Review of Convergence Results

The convergence of the iteration defined by $\boldsymbol{T}(\boldsymbol{p})$ in Equation (5.4) is addressed in [28] and in [29]. In particular, in [29], an interference function $\boldsymbol{I}(\boldsymbol{p})$ is defined to be a *standard* interference function if it has the following properties:



The cone $P^{(l)}$ is nonempty and $\lambda^{(l)} < 1$, but for all $\boldsymbol{p} \in P^{(l)}$, $\boldsymbol{p} \geq q$.

Figure 5.2: The feasible region is outside maximum power constraints

- *Positivity* $\boldsymbol{I}(\boldsymbol{p}) > 0$
- *Monotonicity* If $\boldsymbol{p} \geq \boldsymbol{p}'$, then $\boldsymbol{I}(\boldsymbol{p}) \geq \boldsymbol{I}(\boldsymbol{p}')$.
- *Scalability* For all $\alpha > 1$, $\alpha \boldsymbol{I}(p) > \boldsymbol{I}(\alpha \boldsymbol{p})$.

It is easy to show that $\boldsymbol{T}(\boldsymbol{p})$ and $\boldsymbol{T}^{(l)}(\boldsymbol{p})$ are standard.

Throughout this work, we will use $\boldsymbol{z}(n)$, $\boldsymbol{p}(n)$ and $\boldsymbol{q}(n)$ to denote the result of n iterations of $\boldsymbol{T}(\boldsymbol{p})$ starting from the the all zero vector $\boldsymbol{z}$, an arbitrary power vector $\boldsymbol{p}$, and the maximum power vector $\boldsymbol{q}$ respectively. Similarly, we will use $\boldsymbol{z}^{(l)}(n)$, $\boldsymbol{p}^{(l)}(n)$ and $\boldsymbol{q}^{(l)}(n)$ to denote the corresponding quantities resulting from n iterations of the fixed assignment iteration function $\boldsymbol{T}^{(l)}(\boldsymbol{p})$.

Since $\boldsymbol{T}(\boldsymbol{p})$ and $\boldsymbol{T}^{(l)}(\boldsymbol{p})$ are standard, the following conclusions from [29] can be verified.

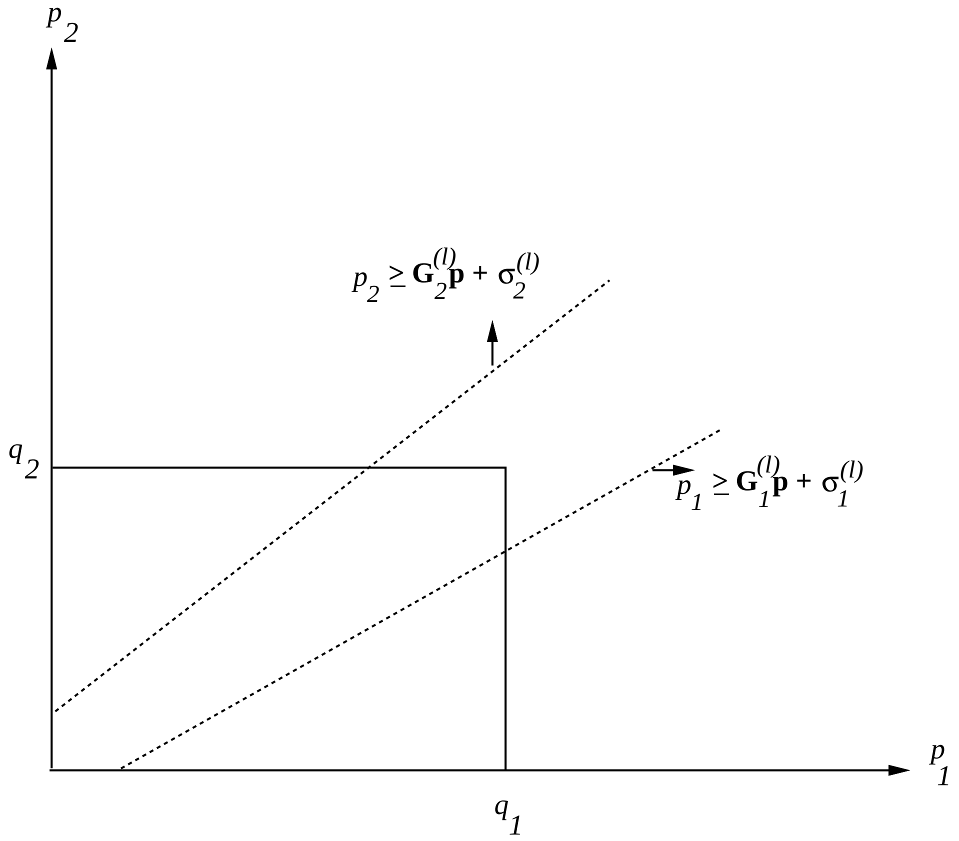


Figure 5.3: The cone $P^{(l)}$ is empty and $\lambda^{(l)} > 1$. No feasible solution exists for the assignment l.

Lemma 5.1 *$\boldsymbol{z}(n)$ and $\boldsymbol{z}^{(l)}(n)$ are monotone increasing sequences of power vectors.*

Lemma 5.2 *$\boldsymbol{q}(n)$ and $\boldsymbol{q}^{(l)}(n)$ are monotone decreasing sequences of power vectors*

Theorem 5.1 *If $\boldsymbol{T}(\boldsymbol{p})$ is a standard interference function such that there exists $\boldsymbol{p}'$ satisfying $\boldsymbol{p}' \geq \boldsymbol{T}(\boldsymbol{p}')$, then for any initial power vector $\boldsymbol{p} = \boldsymbol{p}(0)$, the sequence $\boldsymbol{p}(n)$ converges to a unique fixed point $\bar{\boldsymbol{p}}$.*

Note that Theorem 5.1 applies to all standard interference functions $\boldsymbol{T}(\boldsymbol{p})$. For $\boldsymbol{T}(\boldsymbol{p})$ defined by Equation (5.4), $\boldsymbol{q} \geq \boldsymbol{T}(\boldsymbol{q})$ and Theorem 5.1 always applies. For $\boldsymbol{T}^{(l)}(\boldsymbol{p})$, the feasibility condition $\boldsymbol{p}' \geq \boldsymbol{T}^{(l)}(\boldsymbol{p}')$ is met iff $P^{(l)}$ is nonempty.

Although the convergence of the iteration (5.2) was verified in [28, 29], nothing was proven about the rate of convergence, the subject of this paper.

5.4 Types of Convergence

It is worthwhile to define carefully several common forms of convergence for a synchronous iterative algorithm of the form $\boldsymbol{p}(n+1) = \boldsymbol{T}(\boldsymbol{p}(n))$ with respect to a norm $||\cdot||$. Although we state the definitions for an arbitrary norm, we assume throughout this work that $||\cdot||$ denotes the usual Euclidean norm.

Contraction Mapping There exists a nonnegative constant $\alpha < 1$ such that for all $\boldsymbol{p}, \boldsymbol{p}'$,

$$||\boldsymbol{T}(\boldsymbol{p}) - \boldsymbol{T}(\boldsymbol{p}')|| \leq \alpha||\boldsymbol{p} - \boldsymbol{p}'|| \tag{5.9}$$

Pseudo-contraction Mapping There exists a fixed point $\bar{\boldsymbol{p}}$ and a nonnegative constant $\alpha < 1$ such that for all $\boldsymbol{p}$,

$$||\boldsymbol{T}(\boldsymbol{p}) - \bar{\boldsymbol{p}}|| \leq \alpha||\boldsymbol{p} - \bar{\boldsymbol{p}}|| \tag{5.10}$$

Geometric Convergence A sequence $\{\boldsymbol{p}(n)\}$ is said to converge geometrically to $\bar{\boldsymbol{p}}$ if there exist nonnegative constants A and $\alpha < 1$ such that

$$||\boldsymbol{p}(n) - \bar{\boldsymbol{p}}|| \leq A\alpha^n$$

It is easy to show that contraction mapping $\boldsymbol{T}(\boldsymbol{p})$ is also a pseudo-contraction mapping and that a pseudo contraction mapping $\boldsymbol{T}(\boldsymbol{p})$ converges geometrically. In particular, if $\boldsymbol{T}(\boldsymbol{p})$ is either a contraction or pseudo-contraction mapping, then the sequence $\boldsymbol{p}(n) = \boldsymbol{T}^n(\boldsymbol{p})$ will converge to a fixed point $\bar{\boldsymbol{p}}$ at a geometric rate such that

$$||\boldsymbol{p}(n) - \bar{\boldsymbol{p}}|| \leq \alpha^n||\boldsymbol{p} - \bar{\boldsymbol{p}}|| \tag{5.11}$$

In these three types of convergence, the constant α denotes the rate of convergence.

5.5 Rate of Convergence

In this section, we examine the rate of convergence of the synchronous iteration (5.4) for both fixed assignment and the general minimum power assignment. We will see that in the presence of maximum power constraints, the sequence $\boldsymbol{p}(n)$ will converge to a fixed point at geometric rate whether there is a feasible solution or not.

5.5.1 Fixed Base Station Assignment

We discuss first the rate of convergence for a fixed assignment l in the absence of maximum power constraints. The results we derive for this case will aid our study of the general minimum power assignment iteration. Under fixed assignment, $K_i = \{a_i(l)\}$; that is, each user i has an assigned base $a_i(l) > 0$. The power control iteration is defined by

$$\boldsymbol{p}^{(l)}(n+1) = \boldsymbol{T}^{(l)}(\boldsymbol{p}^{(l)}(n)) = \boldsymbol{G}^{(l)}\boldsymbol{p}^{(l)}(n) + \boldsymbol{\sigma}^{(l)} \tag{5.12}$$

For this iteration, the following claim has been verified by Foschini and Miljanic [10] and Mitra [21].

Theorem 5.2 *When $\lambda^{(l)} < 1$, $\boldsymbol{T}^{(l)}(\boldsymbol{p})$ is a contraction mapping.*

Proof: For any two power vectors $\boldsymbol{p}$, $\boldsymbol{p}'$, we have

$$\begin{aligned}
||\boldsymbol{T}^{(l)}(\boldsymbol{p}) - \boldsymbol{T}^{(l)}(\boldsymbol{p}')|| &= ||\boldsymbol{G}^{(l)}\boldsymbol{p} + \boldsymbol{\sigma} - (\boldsymbol{G}^{(l)}\boldsymbol{p}' + \boldsymbol{\sigma})|| \\
&= ||\boldsymbol{G}^{(l)}(\boldsymbol{p} - \boldsymbol{p}')|| \\
&\leq ||\boldsymbol{G}^{(l)}||\,||\boldsymbol{p} - \boldsymbol{p}'|| \\
&= \lambda^{(l)}||\boldsymbol{p} - \boldsymbol{p}'||
\end{aligned}$$

□

When $\lambda^{(l)} \geq 1$, the transmitted power $p_i(n)$ of user i diverges if user i is assigned to a base station $a_i(l) > 0$. Only if the assigned base $a_i(l)$ is the virtual base station 0 does the transmitted power of user i not diverge. When $a_i(l) = 0$, $p_i(n)$ remains constant at q_i.

Lemma 5.3 *If $\lambda^{(l)} \geq 1$, then there exists $M_l < \infty$ such that for all i satisfying $a_i(l) > 0$, $p_i^{(l)}(M_l) > q_i$.*

Proof: Since $\boldsymbol{T}^{(l)}(\boldsymbol{p})$ is a monotone mapping, $\boldsymbol{p}^{(l)}(n) \geq \boldsymbol{z}^{(l)}(n)$. Thus, it is sufficient to show that there exists $M < \infty$ such that for i satisfying $a_i(l) > 0$, $z_i^{(l)}(M) > q_i$. Let $\hat{\boldsymbol{G}}$ and $\hat{\lambda}$ denote $\boldsymbol{G}^{(l)}$ and $\lambda^{(l)}$ respectively. We can write

$$\boldsymbol{z}^{(l)}(n) = \sum_{k=0}^{n-1} \hat{\boldsymbol{G}}^k \boldsymbol{\sigma}^{(l)} \tag{5.13}$$

Let $\boldsymbol{e}$ denote the eigenvector of $\hat{\boldsymbol{G}}$ corresponding to the largest eigenvalue $\hat{\lambda}$. By the Perron-Frobenius theorem [55], $\boldsymbol{e}$ is non-negative. If $a_i(l) > 0$, the ith row of $\boldsymbol{G}^{(l)}$ has positive entries and $e_i > 0$. Since $\hat{\boldsymbol{\sigma}} > 0$, we can choose $b > 0$ such that $b\boldsymbol{e} \leq \hat{\boldsymbol{\sigma}}$. Since $\hat{\boldsymbol{G}}^k$ is a nonnegative matrix, we observe that

$$\hat{\boldsymbol{G}}^k \hat{\boldsymbol{\sigma}} \geq \hat{\boldsymbol{G}}^k b\boldsymbol{e} = b\hat{\lambda}^k \boldsymbol{e}$$

Thus, from (5.13),

$$z_i^{(l)}(n) \geq b \sum_{k=0}^{n-1} \hat{\lambda}^k e_i$$

Since $\hat{\lambda} \geq 1$, there exists M such $z_i^{(l)}(M) > q_i$. $\square$

Lemma 5.3 implies that iterating with an infeasible assignment in the absence of maximum power constraints will result in diverging transmitter powers for all users assigned to real base stations. When we consider maximum power constraints, the power adjustment is not simply based on the assignment l but will be limited by the maximum power constraints. In this case, rate of convergence issues become more complicated.

5.5.2 General Minimum Power Assignment

For the general minimum power assignment algorithm with maximum power constraints defined by the iteration (5.4), our objective will be to show convergence to a fixed point will occur at a geometric rate. The difficulty in proving geometric convergence in this case is that it is possible that at many stages, an infeasible base station assignment is used in the iteration even if eventually the system converges to a feasible base station assignment. Thus, Theorem 5.2 has no direct generalization. Instead, we will show that maximum power constraints imply that an infeasible assignment can be used only a finite number of times. This fact will lead to a proof of geometric convergence.

Let $m_l(n)$ denote the number of times assignment l is used in first n iterations starting from $\boldsymbol{z}$. We will need the following lemma.

Lemma 5.4 $\boldsymbol{z}(n) \geq \boldsymbol{z}^{(l)}(m_l(n))$

Before giving the proof, we observe that $\boldsymbol{z}(n)$ denotes the power vector obtained by n iterations of $\boldsymbol{T}(\boldsymbol{p})$ starting from $\boldsymbol{z}$, the all zero vector. Also starting from $\boldsymbol{z}$, $\boldsymbol{z}^{(l)}(m_l(n))$ denotes the power vector obtained by $m_l(n)$ iterations under the fixed assignment iteration function $\boldsymbol{T}^{(l)}(\boldsymbol{p})$. In generating the sequence $\boldsymbol{z}(0), \ldots, \boldsymbol{z}(n)$, the iteration function $\boldsymbol{T}^{(l)}(\boldsymbol{p})$ was used $m_l(n)$ times. Thus, one way to obtain $\boldsymbol{z}^{(l)}(n)$ is to perform only the $m_l(n)$ iterations of $\boldsymbol{T}(\boldsymbol{p})$ that used assignment l. Since $\boldsymbol{z}(n)$ is a monotone increasing sequence, omitting those iterations that used assignments other than l should result in a smaller power vector. The proof formalizes this argument.

Proof: We prove the claim by induction. For $n = 0$, we trivially have $\boldsymbol{z} = \boldsymbol{z}(0) = \boldsymbol{z}^{(l)}(0)$. Assuming the claim holds for $n-1$ yields $\boldsymbol{z}(n-1) \geq \boldsymbol{z}^{(l)}(m_l(n-1))$. By Lemma 5.1, $\boldsymbol{z}(n) \geq \boldsymbol{z}(n-1)$. If at iteration n, assignment l is not used, then $m_l(n) = m_l(n-1)$ and

$$\boldsymbol{z}(n) \geq \boldsymbol{z}(n-1) \geq \boldsymbol{z}^{(l)}(m_l(n-1)) = \boldsymbol{z}^{(l)}(m_l(n))$$

If at iteration n, assignment l is used, then $m_l(n) = m_l(n-1) + 1$ and the monotone property of $\boldsymbol{T}^{(l)}(\boldsymbol{p})$ implies

$$\boldsymbol{z}(n) = \boldsymbol{T}^{(l)}(\boldsymbol{z}(n-1)) \geq \boldsymbol{T}^{(l)}(\boldsymbol{z}^{(l)}(m_l(n-1)) = \boldsymbol{z}^{(l)}(m_l(n-1)+1) = \boldsymbol{z}^{(l)}(m_l(n))$$

□

We will now use the result of Lemma 5.4 to show that the number of times an infeasible assignment l can be used is upper bounded by M_l from Lemma 5.3.

Lemma 5.5 *If* $\lambda^{(l)} \geq 1$, *then* $\lim_{n\to\infty} m_l(n) \leq M_l$.

Proof: Suppose there exists n such that $m_l(n) \geq M_l$. By Lemma 5.4, $\boldsymbol{z}(n) \geq \boldsymbol{z}^{(l)}(m_l(n))$. Since $\boldsymbol{z}^{(l)}(n)$ is monotone increasing in n, $\boldsymbol{z}^{(l)}(m_l(n)) \geq \boldsymbol{z}^{(l)}(M_l)$ Since $\lambda^{(l)} \geq 1$, $\boldsymbol{G}^{(l)}$ has at least one nonzero row i for which $a_i(l) > 0$ since otherwise $\boldsymbol{G}^{(l)}$ would be an all zero matrix. Thus, $z_i^{(l)}(M_l) > q_i$. By Lemma 5.4,

$$z_i(n) \geq z_i^{(l)}(m_l(n)) \geq z_i^{(l)}(M_l) > q_i$$

which is a contradiction since $\boldsymbol{T}(\boldsymbol{p}) \leq \boldsymbol{q}$ for all $\boldsymbol{p}$. □

Lemma 5.6 *The sequence* $\boldsymbol{z}(n)$ *converges geometrically to the fixed point* $\bar{\boldsymbol{p}}$.

Proof: By Lemma 5.1 and Theorem 5.1, $\boldsymbol{z}(n)$ is monotone increasing sequence that converges to $\bar{\boldsymbol{p}}$. Thus,

$$\boldsymbol{z}(n) \leq \boldsymbol{z}(n+1) \leq \bar{\boldsymbol{p}} \tag{5.14}$$

Since the fixed point $\bar{\boldsymbol{p}}$ is unique, if $\boldsymbol{z}(n) \neq \bar{\boldsymbol{p}}$, then $\boldsymbol{z}(n) \neq \boldsymbol{z}(n+1)$. Thus, there exists $\alpha_n \in [0, 1)$ such that

$$||\boldsymbol{z}(n) - \bar{\boldsymbol{p}}|| \leq \alpha_n ||\boldsymbol{z}(n+1) - \bar{\boldsymbol{p}}|| \tag{5.15}$$

Let L denote the set of feasible assignments l satisfying $\lambda^{(l)} < 1$. Let $\overline{L}$ equal the complement of L. Both L and $\overline{L}$ are finite sets. Let $l(n)$ denote the assignment used at iteration n. We can define

$$I = \{n | l(n) \in L\} \tag{5.16}$$

$$\bar{I} = \{n | l(n) \in \bar{L}\} \tag{5.17}$$

Since an infeasible assignment $l \in \overline{L}$ is never used more than M_l times, the cardinality of $\overline{I}$ satisfies $|\overline{I}| \leq \sum_{l \in \overline{L}} M_l < \infty$. Since $\alpha_n < 1$ for all n and since $\overline{I}$ is a finite set,

$$\max_{n \in \overline{I}} \alpha_n < 1 \tag{5.18}$$

If at step n, a feasible assignment $l(n) \in L$, is used, Theorem 5.2 implies that $\alpha_n \leq \lambda^{(l_n)} < 1$. Since in each iteration $n \in I$ an assignment $l(n) \in L$ is used,

$$\max_{n \in I} \alpha_n \leq \max_{l \in L} \lambda^{(l)} < 1$$

Thus, for all n,

$$\alpha_n \leq \alpha = \max \left\{ \max_{n \in \overline{I}} \alpha_n, \max_{n \in I} \alpha_n \right\} < 1$$

In other words, there exists $\alpha \in [0, 1)$ such that for all n,

$$||\boldsymbol{z}(n+1)) - \bar{\boldsymbol{p}}|| \leq \alpha ||\boldsymbol{z}(n) - \bar{\boldsymbol{p}}|| \tag{5.19}$$

Hence $||\boldsymbol{z}(n) - \bar{\boldsymbol{p}}|| \leq \alpha^n ||\bar{\boldsymbol{p}}||$ and the sequence $\boldsymbol{z}(n)$ converges geometrically to $\bar{\boldsymbol{p}}$. $\square$

Lemma 5.7 *The sequence $\boldsymbol{q}(n)$ converges to the fixed point $\bar{\boldsymbol{p}}$ at a geometric rate.*

Proof: Associated with the fixed point $\bar{\boldsymbol{p}}$ is a corresponding assignment $\bar{l}$ such that $\bar{\boldsymbol{p}} = \boldsymbol{G}^{(\bar{l})}\bar{\boldsymbol{p}} + \boldsymbol{\sigma}^{(\bar{l})}$. Let l_n denote the assignment used at step n. From Equation (5.8), we observe that for all l,

$$\boldsymbol{q}(n+1) = \boldsymbol{G}^{(l_n)}\boldsymbol{q}(n) + \boldsymbol{\sigma}^{(l_n)} \leq \boldsymbol{G}^{(l)}\boldsymbol{q}(n) + \boldsymbol{\sigma}^{(l)} \tag{5.20}$$

Since (5.20) holds for $l = \bar{l}$,

$$\begin{aligned} ||\boldsymbol{T}(\boldsymbol{q}(n)) - \bar{\boldsymbol{p}}|| &= ||\boldsymbol{G}^{(l_n)}\boldsymbol{q}(n) + \boldsymbol{\sigma}^{(l_n)} - (\boldsymbol{G}^{(\bar{l})}\bar{\boldsymbol{p}} + \boldsymbol{\sigma}^{(\bar{l})})|| \\ &\leq ||\boldsymbol{G}^{(\bar{l})}(\boldsymbol{q}(n) - \bar{\boldsymbol{p}})|| \\ &\leq ||\boldsymbol{G}^{(\bar{l})}||||\boldsymbol{q}(n) - \bar{\boldsymbol{p}}|| \\ &= \lambda^{(\bar{l})}||\boldsymbol{q}(n) - \bar{\boldsymbol{p}}|| \end{aligned}$$

where $\lambda^{(\bar{l})} \in [0, 1)$. Thus, the sequence $\boldsymbol{q}(n)$ converges geometrically to the fixed point $\bar{\boldsymbol{p}}$. □

Theorem 5.3 *Starting from any initial power vector $\boldsymbol{p}$, the sequence $\boldsymbol{p}(n) = \boldsymbol{T}^n(\boldsymbol{p})$ converges geometrically to the fixed point $\bar{\boldsymbol{p}}$.*

Proof: The monotonicity of $\boldsymbol{T}(\boldsymbol{p})$ implies that for any initial power vector $\boldsymbol{p} = \boldsymbol{p}(0)$,

$$\boldsymbol{z}(n) \leq \boldsymbol{p}(n) \leq \boldsymbol{q}(n) \tag{5.21}$$

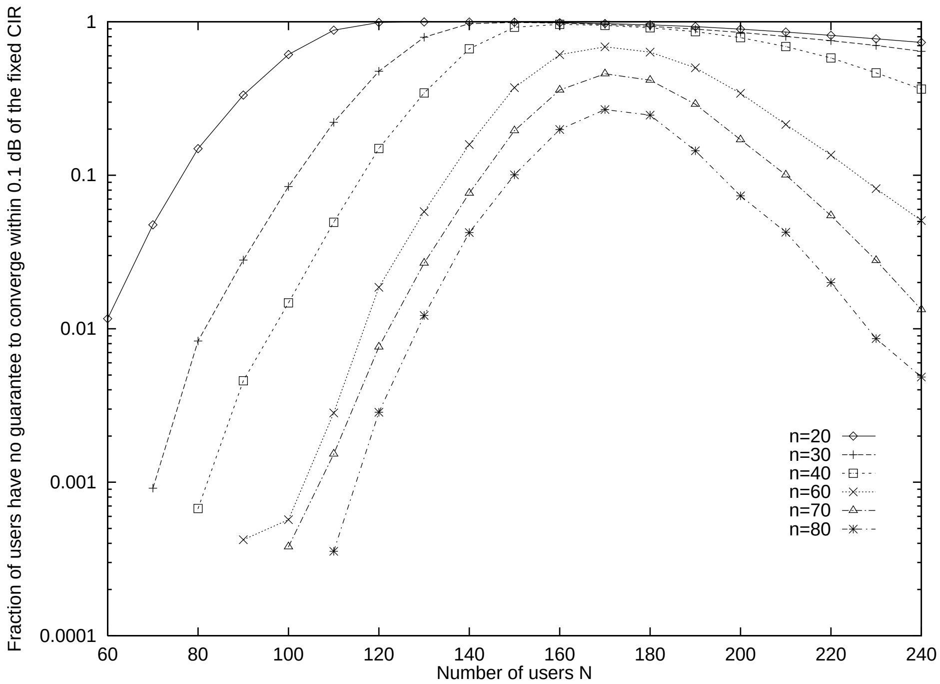
Since $\boldsymbol{z}(n) \leq \bar{\boldsymbol{p}} \leq \boldsymbol{q}(n)$, we must have

$$|p_i(n) - \bar{p}_i| \leq |q_i(n) - z_i(n)|$$

Thus

$$\begin{aligned} ||\boldsymbol{p}(n) - \bar{\boldsymbol{p}}|| &\leq ||\boldsymbol{q}(n) - \boldsymbol{z}(n)|| \\ &= ||\boldsymbol{q}(n) - \bar{\boldsymbol{p}} - (\boldsymbol{z}(n) - \bar{\boldsymbol{p}})|| \\ &\leq ||\boldsymbol{q}(n) - \bar{\boldsymbol{p}}|| + ||\boldsymbol{z}(n) - \bar{\boldsymbol{p}}|| \end{aligned}$$

Since there $\boldsymbol{z}(n)$ and $\boldsymbol{q}(n)$ converge geometrically to $\bar{\boldsymbol{p}}$, there exists $A > 0$ and $\alpha \in [0, 1)$ such that $||\boldsymbol{q}(n) - \bar{\boldsymbol{p}}|| \leq (A/2)\alpha^n$ and $||\boldsymbol{z}(n) - \bar{\boldsymbol{p}}|| \leq (A/2)\alpha^n$. Thus, $||\boldsymbol{p}(n) - \bar{\boldsymbol{p}}|| \leq A\alpha^n$. □



As a function of the number of iterations n and the number of mobiles N, this plot records the fraction of stationary mobiles i that fail to meet the sufficient condition (5.22) to guarantee a CIR within 0.1 dB of the fixed point CIR $\bar{\gamma}_i$.

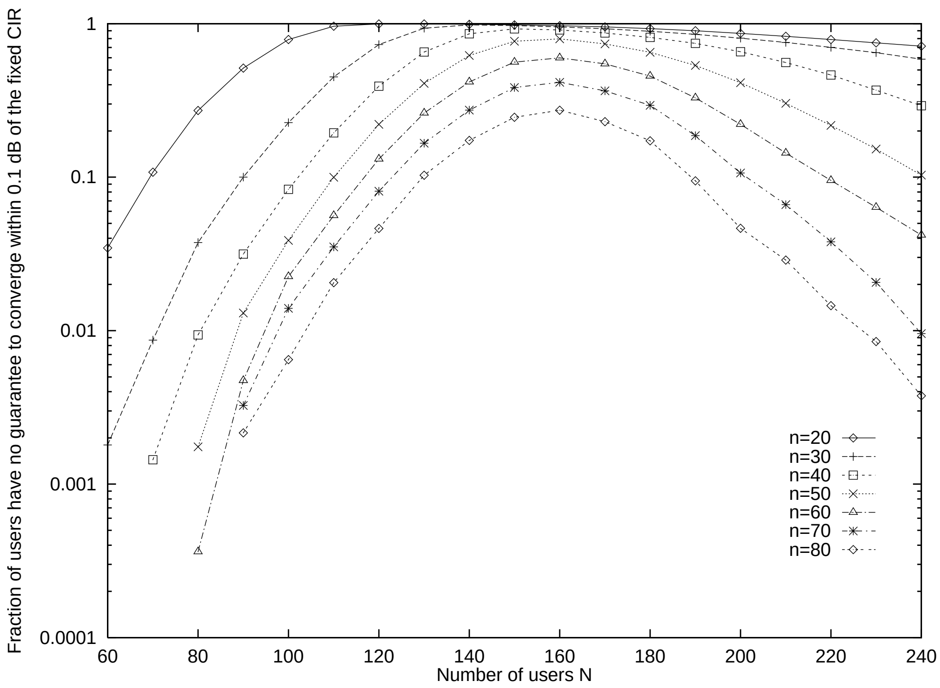
Figure 5.4: Convergence of the MPA algorithm.

5.6 Simulation Results

Although we have verified that the general minimum power assignment algorithm converges geometrically, the rate constant α depends on the number of users in the system, the uplink gains h_{ki} and the maximum power $\boldsymbol{q}$. Thus, in this section we examine by simulation the rate of convergence for the minimum power assignment (MPA) scheme in which each user tries to minimize its transmitter power over the set of all base stations and also for a fixed base station assignment scheme.

We repeatedly place stationary mobiles on the ring and examine the rate of convergence of the MPA algorithm and the fixed assignment by recording as a function of the number of iterations n, the fraction of mobiles that are guaranteed to be within 0.1 dB of the eventual CIR at the fixed point.

We would like power control algorithms to converge quickly since this would suggest that



As a function of the number of iterations n and the number of mobiles N, this plot records the fraction of stationary mobiles i that fail to meet the sufficient condition (5.22) to guarantee a CIR within 0.1 dB of the fixed point CIR $\bar{\gamma}_i$.

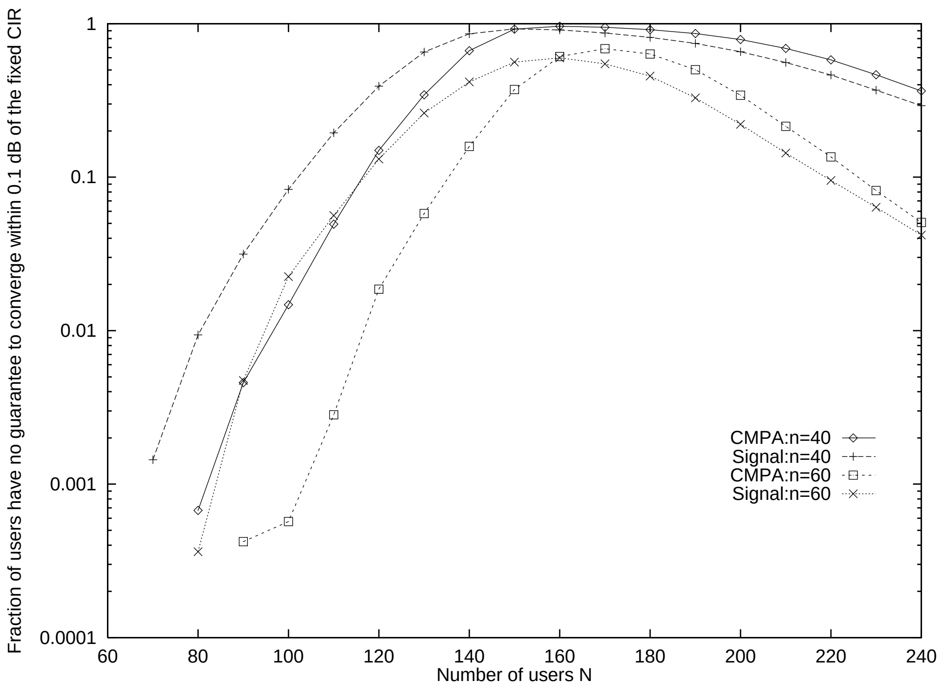
Figure 5.5: Convergence of the Signal based assignment algorithm.

in the event that a set of CIR targets are feasible, the power control algorithm can track the variations in the radio propagation. Alternatively, fast convergence to the fixed point will permit the system to detect quickly an infeasible situation. To provide some indication of the rate of convergence, we repeatedly placed N mobiles randomly on the ring and we recorded the fraction of mobiles i that were guaranteed to be within 0.1 dB of the CIR $\bar{\gamma}_i$ obtained at the fixed point. We considered a number of mobiles N ranging from 60 to 240.

Under power vector $\boldsymbol{p}$, let the CIR of user i at the base station where its CIR is maximized be denoted by $\boldsymbol{\Gamma}_i(\boldsymbol{p})$.

Lemma 5.8 *If*

$$q_i(n) \leq (1+\epsilon) z_i(n) \tag{5.22}$$



As a function of the number of iterations n and the number of mobiles N, this plot records the fraction of stationary mobiles i that fail to meet the sufficient condition (5.22) to guarantee a CIR within 0.1 dB of the fixed point CIR $\bar{\gamma}_i$.

Figure 5.6: Convergence of the MPA algorithm and Signal based assignment algorithm.

then the CIR of user i after n iterations satisfies

$$\frac{1-\epsilon}{1+\epsilon}\bar{\gamma}_i \leq \boldsymbol{\Gamma}_i(\boldsymbol{p}(n)) \leq \frac{1+\epsilon}{1-\epsilon}\bar{\gamma}_i \tag{5.23}$$

Proof: By the monotonicity property of $\boldsymbol{T}(\boldsymbol{p})$, $z_i(n) \leq \bar{p}_i \leq q_i(n)$. Thus, $q_i(n) \leq (1+\epsilon)z_i(n)$ implies

$$q_i(n) \leq (1+\epsilon)\bar{p}_i \tag{5.24}$$

and that

$$(1-\epsilon)\bar{p}_i < \frac{\bar{p}_i}{1+\epsilon} \leq \frac{q_i(n)}{1+\epsilon} \leq z_i(n) \tag{5.25}$$

Combining the (5.24) and (5.25) yields

$$(1-\epsilon)\bar{p}_i < z_i(n) \leq p_i(n) \leq q_i(n) \leq (1+\epsilon)\bar{p}_i \tag{5.26}$$

From Equations (2.5), (2.7) and (5.5), the CIR of user i under power vector $\boldsymbol{p}$ and assignment l can be written as

$$\boldsymbol{\Gamma}_i^{(l)}(\boldsymbol{p}) = \frac{p_i}{\boldsymbol{G}_i^{(l)}\boldsymbol{p} + \sigma_i^{(l)}} \tag{5.27}$$

Let l_n denote the base station assignment after n iterations and let $\bar{l}$ denote the assignment corresponding to the fixed point $\bar{\boldsymbol{p}}$. Thus, $\boldsymbol{\Gamma}_i^{(l_n)}(\boldsymbol{p}(n)) = \boldsymbol{\Gamma}_i(\boldsymbol{p}(n))$ and $\boldsymbol{\Gamma}_i^{(\bar{l})}(\bar{\boldsymbol{p}}) = \boldsymbol{\Gamma}_i(\bar{\boldsymbol{p}})$. Since at every iteration, the MPA algorithm maximizes the CIR over all base station assignments l, we observe that $\boldsymbol{\Gamma}_i^{(l_n)}(\boldsymbol{p}(n)) \geq \boldsymbol{\Gamma}_i^{(l)}(\boldsymbol{p}(n))$ for all l. In this case, (5.26) and (5.27) imply that

$$\begin{aligned} \boldsymbol{\Gamma}^{(l_n)}(\boldsymbol{p}(n)) &\leq \frac{(1+\boldsymbol{\epsilon})\bar{p}_i}{\boldsymbol{G}_i^{(l_n)}(1-\boldsymbol{\epsilon})\bar{\boldsymbol{p}}(n) + (1-\boldsymbol{\epsilon})\sigma_i^{(l_n)}} && (5.28)\\ &= \frac{1+\boldsymbol{\epsilon}}{1-\boldsymbol{\epsilon}}\boldsymbol{\Gamma}_i^{(l_n)}(\bar{\boldsymbol{p}}) && (5.29)\\ &\leq \frac{1+\boldsymbol{\epsilon}}{1-\boldsymbol{\epsilon}}\boldsymbol{\Gamma}_i^{(\bar{l})}(\bar{\boldsymbol{p}}) && (5.30)\\ &= \frac{1+\boldsymbol{\epsilon}}{1-\boldsymbol{\epsilon}}\bar{\gamma}_i && (5.31) \end{aligned}$$

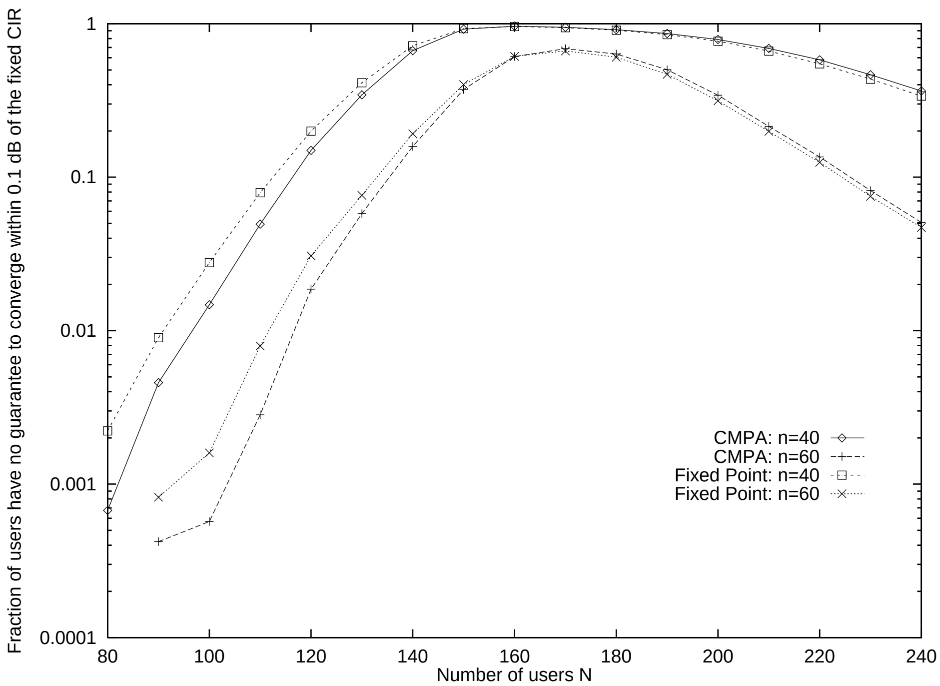
Next, we observe that

$$\boldsymbol{\Gamma}_i^{(l_n)}(\boldsymbol{p}(n)) \geq \boldsymbol{\Gamma}_i^{(\bar{l})}(\boldsymbol{p}(n)) \geq \frac{\bar{p}_i(1-\boldsymbol{\epsilon})}{\boldsymbol{G}_i^{(\bar{l})}\bar{\boldsymbol{p}}(1+\boldsymbol{\epsilon}) + \sigma_i^{(\bar{l})}(1+\boldsymbol{\epsilon})} = \frac{1-\boldsymbol{\epsilon}}{1+\boldsymbol{\epsilon}}\bar{\gamma}_i \tag{5.32}$$

to complete the proof. □

Thus (5.22) is a sufficient condition that guarantees that the CIR of user i is within $10\log_{10}[(1+\boldsymbol{\epsilon})/(1-\boldsymbol{\epsilon})]$ dB of the fixed point CIR $\bar{\gamma}_i$. In particular, for $\epsilon = 0.0115$, (5.22) implies that user i is within 0.1 dB of the fixed point SIR.

For each random placement of the mobile users, we recorded the number of users i for which the sufficient condition (5.22) (for $\epsilon = 0.0115$) was not satisfied as a function of the number of synchronous MPA iterations n. In Figure 5.4, we graph the fraction of mobiles i that failed to satisfy the sufficient condition (5.22) over a large number of trials for $n = 20$ to 80. We see that for a small number of users, $N = 100$ or less, 40 iterations is sufficient for almost all users to be guaranteed to be within 0.1 dB. As the number of users increases, the required number of



As a function of the number of iterations n and the number of mobiles N, this plot records the fraction of stationary mobiles i that fail to meet the sufficient condition (5.22) to guarantee a CIR within 0.1 dB of the fixed point CIR $\bar{\gamma}_i$.

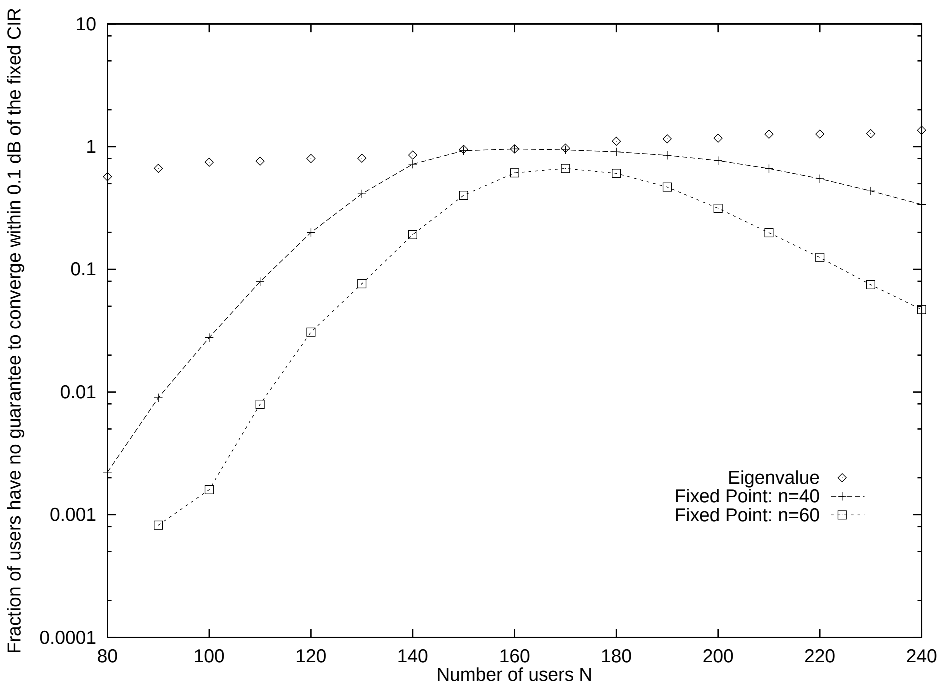
Figure 5.7: Convergence of the MPA algorithm and Fixed Point assignment

MPA iterations also increases.

Note that with an CIR target of $\gamma = 1/20$, a single base station in isolation would be able to support 20 mobiles. Hence, in the simulated system of ten base stations with an CIR target of $\gamma = 1/20$, it is impossible to carry more than 200 calls, even in the absence of maximum power constraints. Thus, as the system was overloaded with more than 200 users, we observed that the required number of MPA iterations to ensure that a particular fraction of users will be within 0.1 dB of the fixed point CIR began to decrease. This occurred because every possible assignment l was infeasible, and as a result, starting from zero initial power, the powers rapidly increased, and thus converged quickly, to the fixed point $\bar{\boldsymbol{p}}$.

With the same convergence criteria, we also simulate the rate of convergence for signal based assignment algorithm where a user will choose a base station with strongest pilot signal strength. When comparing the convergence of the MPA algorithm to a constrained iteration using a fixed assignment l, Equation (5.8) has two implications that result from the fact that the MPA algorithm tries to minimize the transmitter powers. First, $\boldsymbol{z}(n) \leq \boldsymbol{z}^{(l)}(n)$. That is, starting at zero power MPA converges more slowly because powers increase as slowly as possible. Starting at maximum power $\boldsymbol{q}$ however, $\boldsymbol{q}(n) \leq \boldsymbol{q}^{(l)}(n)$. That is, starting at maximum power, the effort to reduce powers results in faster convergence. As we can see in Figure 5.5 and Figure 5.6, the results for signal strength based assignment algorithm are similar but the convergence speed is slower than that of the MPA algorithm at $n = 160$ or lower. This is because starting from maximum power, the MPA algorithm converges faster than signal based assignment scheme. Under a reasonable traffic load, the MPA is not likely to be operate at a cone which will slow down the rate of convergence. On the other hand, when the offered traffic is high ($n = 180$ or higher), the MPA algorithm will converge slower than signal based assignment algorithm. In higher traffic load, the final fixed point will move close to maximum power vector. In this case, the convergence speed will be limited by the convergence speed of starting from zero power vector $\boldsymbol{z}$. As discussed, the MPA is trying always to use minimum power. This explains why at high traffic load the MPA algorithm converges slower than than signal based assignment algorithm.

In Figure 5.7, we compare the convergence speed of the CMPA algorithm to that of the Fixed Point assignment where we iterate using the ultimate assignment at the fixed point. The results are the same as in Figure 5.6. We find that that the convergence speeds are almost identical when the system has high traffic ($N > 160$). This can be explained that the ultimate assignment for Fixed Point assignment is similar to the assignment during the CMPA convergence process. The relationship between the maximum eigenvalue, λ, of the ultimate assignment $\boldsymbol{G}^{(\bar{l})}$ and the convergence speed of the Fixed Point assignment is shown in Figure 5.8. The results show that when a system has feasible solutions (i.e. $\lambda < 1$) the system can quickly converge to a fixed point. But, when the λ gradually increases and approaches to 1, the rate of convergence will become slow. But, this is not true when $\lambda > 1$ where it is not possible to provide all users feasible connections. The required transmitter power will also quickly converge to a fixed point



As a function of the number of iterations n and the number of mobiles N, this plot records the fraction of stationary mobiles i that fail to meet the sufficient condition (5.22) to guarantee a CIR within 0.1 dB of the fixed point CIR $\bar{\gamma}_i$.

Figure 5.8: Convergence of the MPA algorithm and Fixed Point assignment

as the value of λ further increases. We can see that λ is convergence constant which can be used to indicate the rate of convergence for different power control algorithms.

5.7 Discussion

In wireless communication systems, mobile users adapt to a time varying radio channel by regulating transmitter power. This power control is intended to provide each user an acceptable connection, as measured by a carrier to interference ratio (CIR), by eliminating unnecessary interference. It is important that a power control algorithm can effectively converge to a desirable solution when a fixed radio channel permits such a solution. We have shown that no matter whether the system can support all users the target CIR or not, iterative power control can guarantee the sequence $\{\boldsymbol{p}(n)\}$ converges to a fixed point at a geometric rate for both fixed base station assignment and MPA assignment.

Simulation results are based on a CDMA system. As described in [54], power control for CDMA systems is needed to compensate for the signal fluctuation caused by propagation loss, lognormal fading and slow Rayleigh fading. Error correction coding and interleaving can be used to mitigate the signal degradation caused by fast fading. In the proposed IS-95 CDMA system standard [2], power adjustments occur every 1.25 msec (800 times per second) for closed loop power control. In this context, we found that, according to simulation results, the iterative power control algorithms are fairly practical in term of convergence speed.

Chapter 6

Call Admission Control

In this chapter, we study call admission control in power controlled CDMA systems. In CDMA systems, the number of users that each cell can support is limited by the total interference received at each base station and will vary with time. When a system is congested, admitting a new call can only make the link quality of ongoing calls worse and may result in call dropping. Thus, a system needs a call admission policy for new call requests to maintain acceptable connections for ongoing users. In this work, we use a weighted combination of call blocking and call dropping probabilities as a system performance measure. Based on different concepts for protecting ongoing users, we study transmitter power based and received power based call admission control schemes to improve system performance.

6.1 Introduction

In wireless communication systems, resources are limited. When number of admitted users exceed system capacity, there are not enough resources to support the feasible connections for all users and may result in call dropping. Due to user mobility, new access from a base station may also be requested from ongoing users. When a handoff user can not get access from the new base station, the call may be dropped. We believe that dropping a call is more serious than blocking a call. Thus, a system needs a call admission policy for new call requests to maintain acceptable connections for ongoing users.

In this work, we use a weighted combination of call blocking and call dropping probabilities as a system performance measure. Call dropping may occur when the system is congested. A call admission policy that can effectively reduce call dropping without causing too many blocked calls can improve system performance. Based on different concepts for protecting ongoing users, we study transmitter power based and received power based call admission control schemes to improve system performance.

As described in Chapter 4 and also in [27,28], in a system with maximum power constraints, infeasibility can be detected at the convergence point of the power control algorithm. When the system can not support all users at the target CIR, it means that some users have not achieved the target CIR while transmitting at maximum power. As a result, we consider a Transmitted Power based Call Admission Control (TPCAC) scheme in which we block a new user when there are users transmitting at maximum power. Essentially, the TPCAC admits a call if it is possible to maintain acceptable CIR for all ongoing calls.

Considering user mobility and the fact that capacity at each base station is shared by all active users, we show that total received power at base station can reflect the cell congestion level. As a result, we consider Received Power based Call Admission Control (RPCAC) scheme in which a base station blocks new calls when the total received power measured at the base station exceeds a threshold. As the number of users near a particular base station increases, the total received power at that base station increases. During these congested periods, high total received power at a base station makes it difficult to carry both ongoing calls assigned to the same base station and handoffs from neighboring cells. Thus, high received power levels increase the likelihood that some ongoing calls are dropped. Essentially, the RPCAC attempts to block new call attempts during periods of congestion and can further reserve capacity for handoff calls by setting different threshold values of allowable total received power at base stations.

6.2 Power Control and Base Station Assignment

In cellular wireless communication systems, transmitted power is regulated to provide each user an acceptable connection by limiting the interference caused by other users. Several models have been considered including: (1) Fixed base station assignment [8–10, 13, 15, 22] where the assignment of users to base station is fixed, (2) Minimum power assignment [24, 25, 28] where a user is iteratively assigned to the base station at which its CIR is highest, and (3) Soft handoff assignment , based on IS-95 standard [2] , where a base station will become a candidate when the pilot signal to total received signal ratio exceeds a predefined threshold value. During soft handoff[1](more than 2 base stations assigned to one user) period, the base station with the highest CIR will be used to specify user's signal quality.

Fixed Assignment (FA): We will denote by a_i the assigned base of user i, which we assume to be fixed and specified by pilot signal strength. The CIR requirement of user i at its assigned base a_i can be written

$$\begin{aligned} \boldsymbol{p}(n+1) = \boldsymbol{T}^{FA}(\boldsymbol{p}(n)) &= \min\{\boldsymbol{G}\boldsymbol{p}(n) + \boldsymbol{\sigma}, \boldsymbol{q}\} && (6.1) \\ p_i(n+1) &= \min\{\frac{\gamma}{\gamma_i(n)} p_i(n), q_i\} && (6.2) \end{aligned}$$

In FA, the base station is chosen by pilot signal strength and is independent of CIR measures but the power control is based on CIR measures.

Constrained Minimum Power Assignment (CMPA): At each step of the CMPA algorithm [28], user i is assigned to the base station at which its CIR is maximized. The power adjustment of users can be written

$$\begin{aligned} \boldsymbol{p}(n+1) = \boldsymbol{T}^{CMPA}(\boldsymbol{p}(n)) &= \min\left\{\min_l\{\boldsymbol{G}^{(l)}\boldsymbol{p}(n) + \boldsymbol{\sigma}^{(l)}\}, \boldsymbol{q}\right\} && (6.3) \\ p_i(n+1) &= \min\{\frac{\gamma}{\max_l \gamma_i^{(l)}(n)} p_i(n), q_i\} && (6.4) \end{aligned}$$

Equation (6.4) can be derived from Equation (2.5).

In fact, the CMPA is a choice over all possible base station assignments for which the required transmitted power is minimized to achieve target CIR.

[1] Soft handoff [2] is a special feature in CDMA systems where several base stations can be simultaneously assigned to one user. Among those base stations, the mobile switching center (MSC) will chose one with the best signal quality as assigned base station for the user. The assignment will be changed frame by frame during the soft handoff.

Soft Handoff Assignment (SHA): According to IS-95 [2], power control and base station assignment is based on the best received signal quality among the candidates which has higher pilot signal to total received power ratio than a predefined threshold value. In IS-95, this base station assignment method is called soft handoff. In this work, we examine a soft handoff assignment (SHA) algorithm that is based on the method of IS-95. In particular, we define

$$B_i^{\mathrm{SHA}} = \{l | T_l \geq = T_{\mathrm{ADD}}\} \tag{6.5}$$

where T_l is the value of received pilot signal from base station l to total received power at user i and T_{ADD} is the threshold value such that when $T_l \geq T_{\mathrm{ADD}}$ the base station l will become an active base station for user i. As a result, the B_i^{SHA} is a set of base stations assigned to user i during soft handoff. The power adjustment for user i can be written

$$\begin{aligned} p_i(n+1) = \tilde{\boldsymbol{T}}_i(\boldsymbol{p}(n)) &= \min\left\{ \min_{l \in B_i^{\mathrm{SHA}}} \{\boldsymbol{G}_i^{(l)} \boldsymbol{p}(n) + \boldsymbol{\sigma}^{(l)}\}, q_i \right\} & (6.6) \\ &= \min\{ \frac{\gamma}{\max_{l \in B_i^{\mathrm{SHA}}} \gamma_i^{(l)}(n)} p_i(n), q_i \} & (6.7) \end{aligned}$$

When users are aiming for a common CIR such that $\gamma_i = \gamma$, the CIR based power control schemes are balanced power control schemes in which users assigned to the same base station have the same received power at that base station.

6.3 CDMA Cell Load Measurement

In this section, we discuss cell load measurement in CDMA systems. Since the capacity at each base station is shared by all users, a meaningful cell load measure should also include the interference caused by users assigned to adjacent cells. We show that cell load can be measured by total received power at the base station.

In this study, we assume that each user i has an identical CIR requirement $\gamma_i = \gamma$. All users assigned to base station k will have equal received powers R_k. We now develop a characterization of the cell capacity. When each user assigned to base station k has the same received power R_k, we can write the CIR requirement in terms of the received powers as

$$\frac{R_k}{(N_k - 1)R_k + I_k + \eta_k} \geq \gamma \tag{6.8}$$

where at base k, N_k is the number of users assigned, I_k is the external interference, and η_k is the receiver noise.

We know that the external interference I_k will depend not only on the cell load but also on the spatial distribution of mobile users. The required received power for a particular configuration of users in the system can be expressed as

$$R_k \geq \frac{\gamma(\eta_k + I_k)}{1 - (N_k - 1)\gamma} \tag{6.9}$$

and will depend on the internal users N_k and the external interference I_k.

In CDMA systems, the maximum capacity for each cell is achieved when there is no other interference except the users assigned to the same base station. That is $I_k = \eta_k = 0$. From Equation (6.8), the number of users assigned to base station k must satisfy the following condition.

$$N_k \leq 1 + \frac{1}{\gamma} \tag{6.10}$$

We have the maximum capacity for base station k is

$$N_{\max} = 1 + 1/\gamma \tag{6.11}$$

6.3.1 Single Cell System

For single cell systems, we have $I_k = 0$. The CIR requirement of a user can be written

$$\frac{R}{(N-1)R+\eta} \geq \gamma \tag{6.12}$$

The minimum required received power to support N users in a single cell system is

$$R = \frac{\eta}{1/\gamma + 1 - N} \tag{6.13}$$

With a fixed background noise, the received power R is only a function of the number of users, N, actually assigned to the system and the quality requirement γ. The N can be calculated by user's received power at base station.

$$N = \frac{(1/\gamma + 1)R - \eta}{R} \tag{6.14}$$

Hence, in order to have a feasible received power requirement R, we have $N < 1 + 1/\gamma = N_{\max}$ for nonzero background noise.

Measuring the Cell Load

In general, cell load is defined as the ratio of the number of users assigned to a cell to the maximum capacity of that cell. For single cell systems, the normalized cell load, X, can be expressed as

$$X = \frac{N}{N_{\max}} = \frac{N}{1/\gamma + 1} \tag{6.15}$$

Proposition 6.1 *The cell/system load X is the ratio of the sum of the active users' received powers to the total received power*

Proof: Proposition 6.1 Suppose F is the ratio of the sum of the active users' received powers to the total received power. By employing the balanced power control, we can write F as

$$F = \frac{NR}{NR + \eta} = \frac{N}{N + \eta/R} \tag{6.16}$$

From Equation (6.13), F can be rewritten as

$$F = \frac{N}{N + 1/\gamma - N + 1} = \frac{N}{1/\gamma + 1} \tag{6.17}$$

□

Before discussing the multicell system, we make two observations. First, for a single cell system, the number of active users N and the cell load X , can be measured by user's received power and total received power respectively at base station. Second, the single cell system provides an upper bound on the number of users that each cell can support in the multicell system.

6.3.2 Multicell System Modeled as Single Cell System

We now characterize the capacity at cell k in terms of the received power requirement R_k. Let $L_k = I_k/R_k$ denote an effective number of cell k users caused by external interference. That is, we model I_k, the interference at base k caused by users not assigned to base k, by an equivalent number of base k users. In this case, we can restate Equation (6.8)

$$\frac{R_k}{(N_k + L_k - 1)R_k + \eta_k} \geq \gamma \tag{6.18}$$

The current minimum required received power is

$$R_k = \frac{\eta_k}{1/\gamma + 1 - N_k - L_k} \tag{6.19}$$

Now, let $T_k = N_k + L_k$ denote the effective users caused by all active users. That is, we view base station k as a single cell system with T_k effective users. The minimum received power requirement for having T_k effective users is

$$R_k = \frac{\eta_k}{1/\gamma + 1 - T_k} \tag{6.20}$$

The effective number of users caused by all active users is

$$T_k = N_{\max} - \frac{\eta_k}{R_k} \tag{6.21}$$

which can be calculated by the received power R_k. In addition, T_k is always upperbounded by $N_{\max} = 1 + 1/\gamma$. With that assumption, we view $N_{\max} - T_k = \eta_k/R_k$ as the *residual* capacity for base station k.

Measuring Cell Load

As mentioned, the cell load is defined as the ratio of the number of users assigned to a cell to the maximum capacity of that cell. However, in CDMA systems, we know the number of users that each base station can support is not fixed. It depends strongly on the load of the adjacent cells. This is because all active users share the same frequency spectrum. In order to calculate a meaningful cell load, we need to take the effect of external interference into account. So, we have the cell load of base station k, X_k, is

$$\begin{aligned} X_k &= \frac{N_k + L_k}{N_{\max}} \\ &= \frac{N_k + L_k}{1/\gamma + 1} \end{aligned} \tag{6.22}$$

Theorem 6.1 *The cell load of base station k, X_k, is the ratio of the sum of all active users' received powers to the total received power.*

Proof: Theorem 6.1 Suppose F_k is the ratio of the sum of the active users' received powers to the total received power at base station k. By employing the balanced power control, we can write F as

$$F_k = \frac{N_k R_k + I_k}{N_k R_k + I_k + \eta_k} = \frac{N_k + L_k}{N_k + (I_k + \eta_k)/R_k} \tag{6.23}$$

From Equation (6.20), we can rewrite F_k as

$$F_k = \frac{N_k + L_k}{1/\gamma + 1} = X_k \tag{6.24}$$

□

The relationship between the effective number of users and the cell load can be expressed as

$$T_k = N_{\max} X_k \tag{6.25}$$

Let $Z_k = N_k R_k + I_k + \eta_k$ denote the total received power at base station k. From Theorem 6.1, we can express X_k^*, a desired cell load, in terms of the total received power at base station k

$$X_k^* = \frac{Z_k^* - \eta}{Z_k^*} \tag{6.26}$$

where the corresponding total received power Z_k^* is

$$Z_k^* = \frac{\eta_k}{1 - X_k^*} \tag{6.27}$$

According to Equation (6.27), total received power Z_k can be used to measure cell load at base station k. From Equation (6.25), the effective number of users can be also measured by the total received power.

$$T_k = N_{\max} \frac{Z_k^* - \eta}{Z_k^*} \tag{6.28}$$

The following propositions can be seen as examples for Theorem 6.1.

Proposition 6.2 *For power vector* $\boldsymbol{p}$ *and* $\boldsymbol{p}'$*, if* $\boldsymbol{p} \leq \boldsymbol{p}'$*, we have* $X_k(\boldsymbol{p}) \leq X_k(\boldsymbol{p}')$ *for all* $k \in \{1, ..., M\}$.

Proof: Proposition 6.2 The current cell load at base station X_k can be expressed as

$$X_k(\boldsymbol{p}) = \frac{\sum_i p_i h_{ki}}{\sum_i p_i h_{ki} + \eta_k}$$

When the system adds an additional user, the new cell load at base station k can be written as

$$\begin{aligned} X_k(\boldsymbol{p}') &= \frac{\sum_i p_i' h_{ki}}{\sum_i p_i' h_{ki} + \eta_k} \\ &= \frac{\sum_i p_i h_{ki}(1+\epsilon)}{\sum_i p_i h_{ki}(1+\epsilon) + \eta_k} \end{aligned}$$

where ϵ is the effect caused by the new users. Due to monotone property proved in [25, 28], we have $\epsilon > 0$

$\square$

Proposition 6.3 *When there is no feasible solution, in the unconstrained MPA iteration, the cell load* X_k *for all* k *will converge to 1.*

Proof: Proposition 6.3 As defined in Equation (2.8), $\boldsymbol{G}$ is the normalized interference matrix. From Equation (2.9), we have

$$\boldsymbol{p} = \boldsymbol{G}\boldsymbol{p} + \boldsymbol{\sigma}$$

According to **Perron - Frobenius** theorem, there exists a unique eigenvalue of $\boldsymbol{G}$ such that the corresponding eigenvector $\boldsymbol{p}$ is a positive power vector. The feasible solution exists when $||G|| < 1$ [17, 18]. From [56], we know that when there is no feasible solution, we have $||G|| \geq 1$,

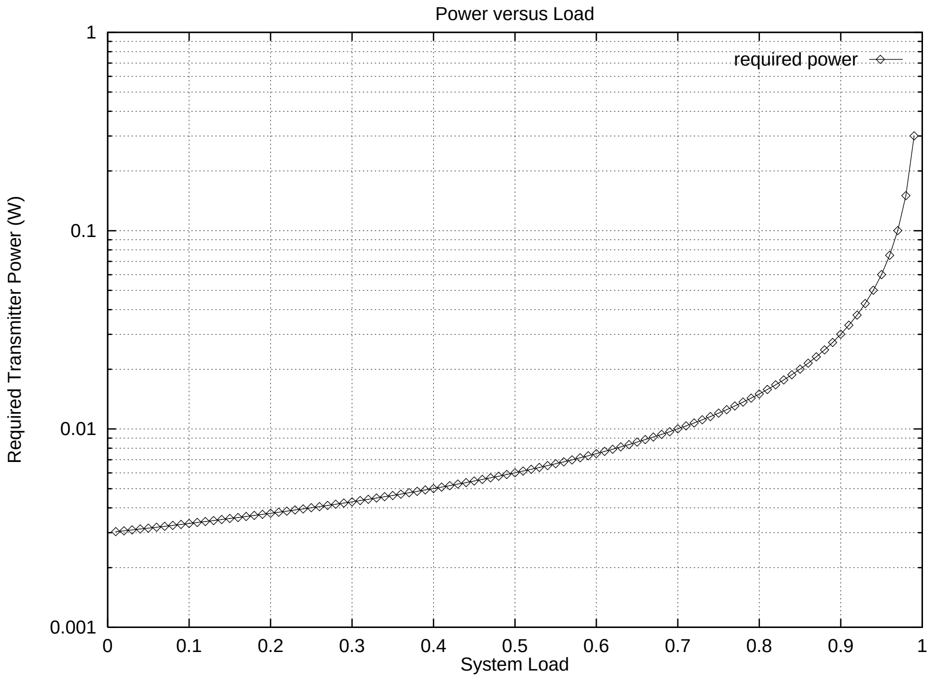


Figure 6.1: Required transmitted power versus the cell load

then the transmitted power vector $\boldsymbol{p}$ grows without bound and $X_k \cong 1$.

$\square$

We just show that the cell load can be measured by total received power at the receiver front end of each base station. The system can reserve capacity by setting maximum allowable total received power for maximum allowable cell load X_k^*. Whenever the total received power Z_k is greater than Z_k^*, we know $X_k > X_k^*$ has to be true.

6.4 The Role of Maximum Power

The maximum transmitter power should be chosen such that users can have sufficient transmitter power to achieve their quality requirements when the system reaches the allowable system load. The cell capacity is determined by transmitter power budget, channel environments and coverage area.

From Theorem 6.1, we have

$$1 - X_k = \frac{\eta_k}{\sum_j p_j h_{kj} + \eta_k} \tag{6.29}$$

We know that the CIR of user i at the base station k is

$$\frac{p_i h_{ki}}{\sum_j p_j h_{kj} - p_i h_{ki} + \eta_k} = \gamma \tag{6.30}$$

From Equation (6.29) and Equation (6.30), we have the required transmitted power of user i at base station k

$$p_i = \frac{\gamma}{1+\gamma} \frac{\eta_k}{h_{ki}(1-X)} \tag{6.31}$$

With the fixed γ and η_k, it is clear that the required transmitted power depends on the cell load and the gain between the user and the base station. Figure 6.1 shows that the required maximum transmitter power versus the allowable cell load. The required transmitted power grows rapidly when the cell load goes above 90%. The allowable cell load region should be in the range of 70% to 90% where the required transmitted power increases smoothly.

6.5 Call Admission Control

In CDMA systems, capacity is shared by all active users. A user with higher transmitted power will create more interference at the adjacent cells. In other words, users with higher transmitted power will borrow more capacity from the adjacent cells. This power related capacity sharing makes us believe that in a CDMA system the call admission control should be based on either required transmitted power or total received power.

As mentioned, call dropping occurs when the connected cell has insufficient resources to maintain the signal quality. In CDMA systems, since the resources (capacity) are fully shared by all active users, two questions should be asked before designing the call admission control. The first question is how can we accurately measure the load at each base station. The second question is how does the call dropping actually happen.

The first question has been discussed in Section 6.3. Basically, we can model a multicell system as an isolated single cell system and count the external interference as the interference caused by effective intra cell users. We show that total received power at the desired base station can be used to measure the congestion level of that cell.

For the second question, it is clear that when the system admits too many users at a base station some users may have their link quality fall below γ_d and will result in dropping. In this case, if we can accurately measure the cell load, the system blocks calls as the system becomes congested. Another possible reason for call dropping is user mobility. The user mobility will change the interference seen at all base stations. At this case, call dropping can also occur when a user moves close to a fully loaded cell. Capacity reservation is needed when call dropping caused by handoff traffic is significant.

6.5.1 Transmitter Power Based Call Admission Control (TPCAC) Scheme

The TPCAC is based on the consequences of the occurrence of infeasibility in which some users transmit maximum power but still can not achieve target CIR. Admitting a new call can only make ongoing link quality worse. As discussed in [28], if a call can be carried, a CIR based power control scheme will converge to a feasible power vector and a corresponding base station assignment. However, if it is not possible to provide each user an acceptable CIR, the power control scheme will still converge to a fixed point. This fixed point will lie on the boundary of the constrained power set where there are users transmitting at maximum power level without achieving the target CIR. As a result, we consider Transmitter Power based Call Admission Control (TPCAC).

As mentioned, $a_i(\boldsymbol{p}(n))$ is base station assigned to user i after n power adjustments. Let W_k denote a subset of base stations that are close to base station k. To simplify the description of the TPCAC, we define

$$U_k(n) = \{i|\ a_i(\boldsymbol{p}(n)) = k\} \tag{6.32}$$

$$D_k(n) = \cup_{j\in W_k} U_j(n) \tag{6.33}$$

That is, U_k(n) is the current set of users assigned to base station k after n CMPA iteration adjustments. The $D_k(n)$ is the set of mobiles assigned to base stations nearby to base station k.

In the TPCAC, there is a call set up phase (N power adjustments) in which the new call will join the applied power control algorithm. The decision of call admission will be based on the power control iterations of the users in the decision set $D_k(n)$. Let the power vector $\bar{\boldsymbol{p}}$ denote the initial power vector when new call arrives. We have $\bar{\boldsymbol{p}} = (\boldsymbol{p}, 0)$. After N power adjustments, if no users (including new user) in the decision set D_k transmit at maximum power, the new call will be accepted.

Algorithm 6.1 TPCAC. *During the nth power adjustments, the following rule is checked for call admission:*

- Block the new call if $\boldsymbol{T}^n(\bar{\boldsymbol{p}}) \geq q_i$ for any $i \in D_k(N)$ and $n \leq N$.
- Admit call iff $\boldsymbol{T}^N(\bar{\boldsymbol{p}}) < \boldsymbol{q}$ for all $i \in D_k(N)$.

As we can see, the TPCAC admits the call if it is possible to support all users with feasible connections. Since the new call will be blocked as soon as there is user transmitting maximum power, the TPCAC can be seen as an active link protection call admission control algorithm. Andersin, Rosberg and Zander [45] consider it as interactive admission control and implement this admission control algorithm in orthogonal channel systems. There is a tradeoff in the choice of N. With large N, the system can have better protection for ongoing users but the set up phase will last longer which will increase implementation complexity.

6.5.2 Received Power based Call Admission Control (RPCAC) Scheme

The RPCAC is based on the current total received power at the desired base station to decide whether to admit a call or not. Due to the user mobility, handoff traffic caused by user moving from one base station to another base station should have higher priority than a new user for using resources. Capacity reservation is required for handoff traffic if the call dropping caused by handoff traffic is significant and can be done by setting threshold values for total received power.

In the RPCAC, a new user will be accepted as long as the total received power at base station k is less than a predefined threshold value Z_K^*. We can consider the RPCAC as a non-iterative admission control.

Algorithm 6.2 RPCAC. *Based on the current load of the desired cell , the following rule is checked for call admission:*

- Admit at base station k iff $Z_k(\boldsymbol{p}) \leq Z_k^*$.

The approach of RPCAC is to predict cell congestion and to reserve capacity at each base station by assuming the call dropping will be caused by handoff traffic. The concept of reservation becomes important if call dropping caused by users mobility is significant.

6.6 Simulation Results

We examine the performance of different power control algorithms in a simulation of mobile users arriving and departing on the ring of base stations. In this case, call blocking and call dropping that results from user mobility define the system performance.

We assume that new calls (users) arrive as an independent Poisson process. Each call is randomly started on the ring. Call holding times are independent and exponentially distributed with a mean time of 120 seconds. A mobile's speed follows a truncated Gaussian distribution with mean speed of 90 km/hr, minimum speed of 60 km/hr and maximum speed of 120 km/hr. The "clockwise" and "counterclockwise" directions are equally likely. The velocity of a terminal

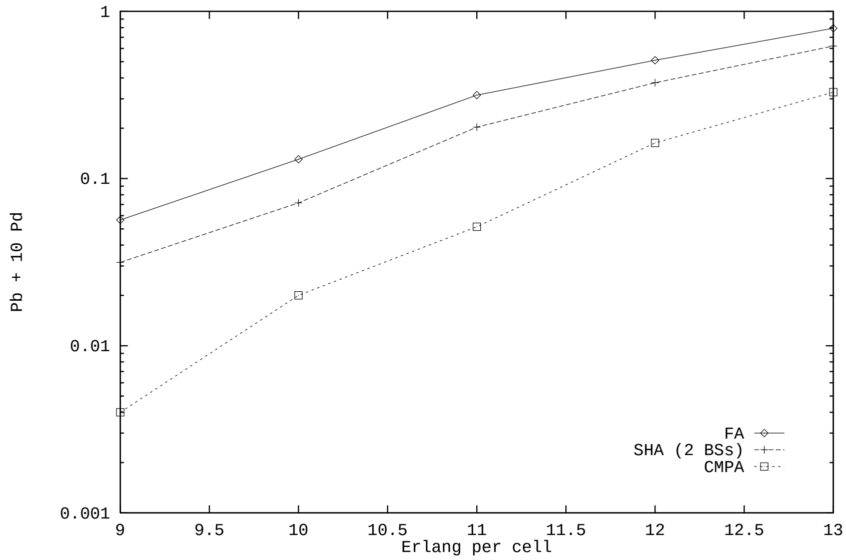


Figure 6.2: Cost Function: FA, SHA with 2 BSs, CMPA

remains fixed throughout duration of a call. By varying the call arrival rate, we consider offered traffic loads ranging from 9 to 13 Erlangs per base station.

In particular, we wish to identify whether the proposed call admission control can effectively improve the system performance. Let P_b and P_d denote the fraction of mobile calls that are blocked and dropped respectively during a simulation. We define a cost function, $10P_d + P_b$ in which we assume that dropping a call is 10 times worse than blocking a new call. Blocking of new call attempts occurs when the required transmitted power of the new call exceeds the maximum power and call dropping occurs if at any time a user has CIR 3 dB lower than the target CIR.

In this study, the allowable cell load is 0.9. When a base station's cell load exceeds 0.9, users assigned to the base station can not maintain the target CIR at boundary. The system performance is different for different power control algorithms. As shown in Figure 6.2, the CMPA has the best results when no admission control is introduced.

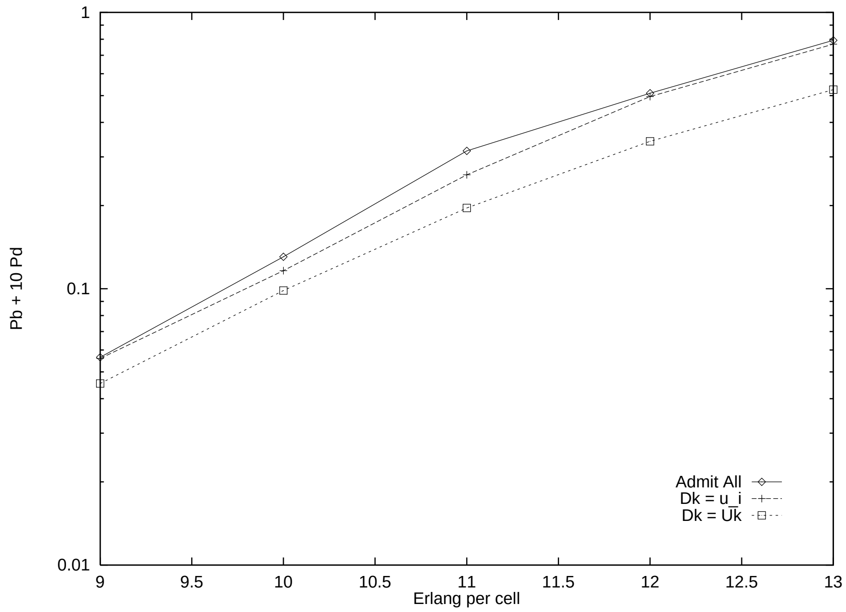


Figure 6.3: TPCAC-Cost Function: Fixed Assignment

6.6.1 TPCAC Performance

In this section, we will evaluate the proposed TPCAC. For TPCAC, after N power adjustments, the decision will be made based on power vector in decision set, defined in (6.33). If there exists user transmitting at maximum power, the new call u_i will be blocked. We consider two decision sets $D_k = u_i$ and $D_k = U_k$ (users in the same base station) for this study.

For fixed assignment, as shown in Figure 6.3 the TPCAC can not effectively improve system performance for $D_k = u_i$. Especially, at low traffic load, there is no improvement by simply checking the required transmitted power of the new user i. This is because the required transmitted power not only depends on system congestion level but also depends on the gain between user and the connected base station. Simply based on the transmitted power of just the new user does not reflect the system congestion or the connection problems of other on going users. The system may already be congested but the required transmitted may be still less than maximum power. More protection for ongoing users by checking all users assigned to the same base station can further improve system performance but the tradeoff is that more information is required to make the decision.

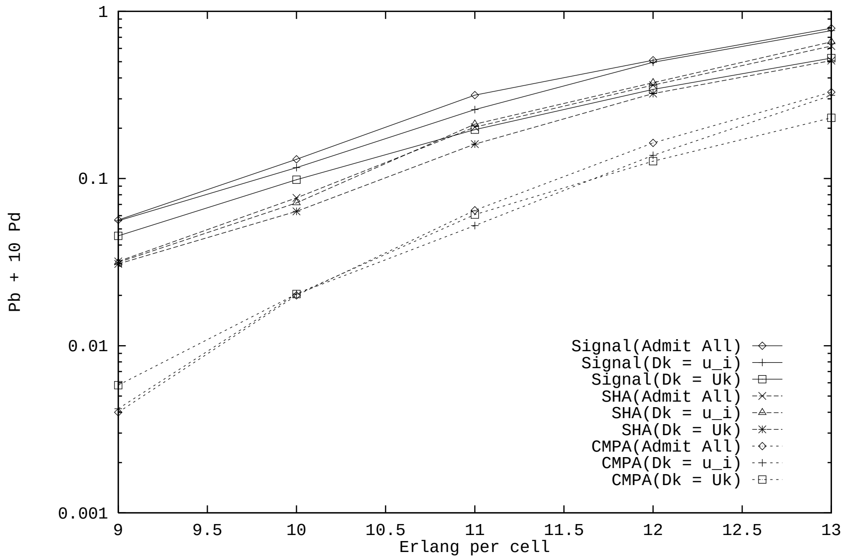


Figure 6.4: TPCAC-Cost Function: FA, SHA with 2 BSs and CMPA

The improvement in performance will become less effective when the system implements better power control schemes. As shown in Figure 6.4, the SHA based and the CMPA based power control shames can support higher traffic load but have less or no improvement in system performance at lower traffic load. When the cell is congested call admission can always improve system performance. This can be explained in Figure 6.10 by considering the CMPA case. We can see that system performance will be improved when the dropping probability exceeds 0.002 for either $D_k = u_i$ or $D_k = U_k$ cases. According to simulation results, user's transmitter power can still be used to reflect the cell congestion level even though the improvement is not significant .

6.6.2 RPCAC Performance

In this section, we will identify the proposed RPCAC whether capacity reservation is required for handoff traffic. Based on Equation (6.27), we reserve capacity by setting the corresponding threshold values for operating call admission control.

The effect on system performance for different reservation thresholds is shown in Figures 6.6

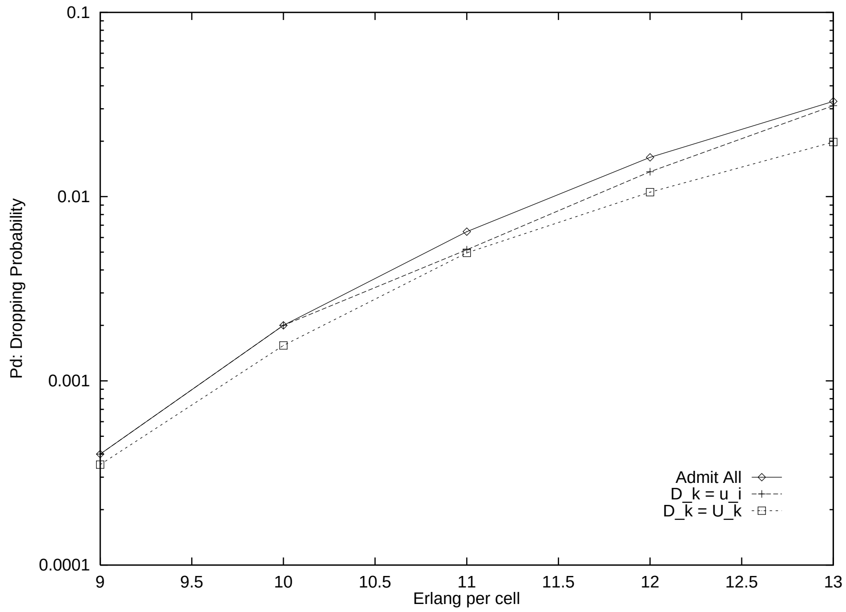


Figure 6.5: TPCAC-Dropping Probability: MPA Algorithm

through 6.9. It is clear that for fixed assignment, the RPCAC can effectively improve system performance through reservation. When the traffic load decreases, the extent of the improvement is reduced. This appears to occur because when system is less congested, the reservation causes more unnecessary call blocking which will degrade system performance. The improvement in performance is continuous (even at low traffic load) until the capacity is reserved more than 10%. With the cost of increasing blocking probability, higher (15% or above) reservation is less desirable. With the same traffic load, the SHA based power control scheme can still improve system performance by limiting new calls when the desired cell is congested. As shown in Figure 6.8, admission control is required to improvement system performance. Reservation for handoff traffic is only needed when the offered traffic is high. The same results occur when system employs the CMPA power control scheme. As mentioned before, due to the robustness of the CMPA, call admission is less desired when offered traffic is very low. According to Figure 6.9 and Figure 6.10, the total received power can be used to indicate whether the desired cell is congested. The system performance will be improved when the dropping probability exceeds 0.002.

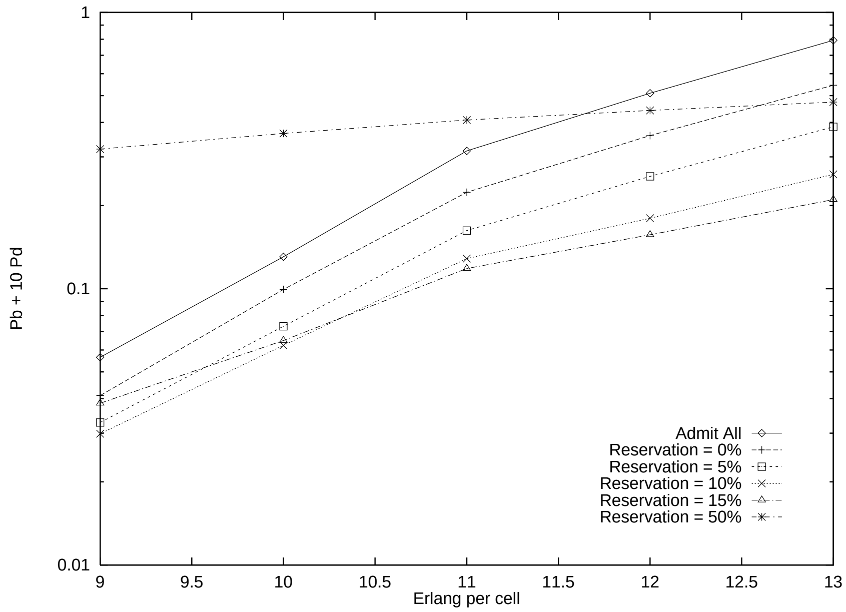


Figure 6.6: RPCAC-Cost Function: Fixed Assignment

6.7 Discussion

In wireless communication systems, resources are limited. When number of admitted users exceed system capacity, there are not enough resources to support the feasible connections for all users and may result in call dropping. Due to user mobility, new access from a base station may also be requested from ongoing users. When a handoff user can not get access from the new base station, The call may be dropped We believe that dropping a call is more serious than blocking a call. Thus, a system needs a call admission policy for new call requests to maintain acceptable connections for ongoing users. Two different concepts of call admission control, protection control and reservation control, are considered in this paper.

The TPCAC, based on transmitted power measures, is to protect ongoing users by limiting new calls when ongoing users can not maintain the target CIR. More protection for ongoing calls can be provided by considering a larger decision set but at the cost of more information (users' transmitted powers). Basically, under the same traffic load, the FA power control algorithm can provide a better improvement in system performance than SHA based and CMPA power control algorithms. When the system is congested, no matter which power control algorithm is chosen, the TPCAC based admission policy can improve system performance.

Based on total received power, the RPCAC algorithm can reserve capacity by simply setting the corresponding threshold values for call admission control operation. Due to the differences in weights between call blocking and call dropping, the system performance under FA is effectively improved through reservation. For the CMPA, reservation is not necessary but new call still should be blocked when the system is congested. According to the simulation results, total received power is a better measure of cell load than user's transmitted power. Especially, the decision of the RPCAC can be made by using only local information. In fact, the total received power can be easily measured at the receiver front end of each base station.

We believe that the proposed call admission control scheme can be further extended to a system which considers to provide multimedia services. Due to the heterogeneous requirements, call admission control can be more effective in improving system performance [57].

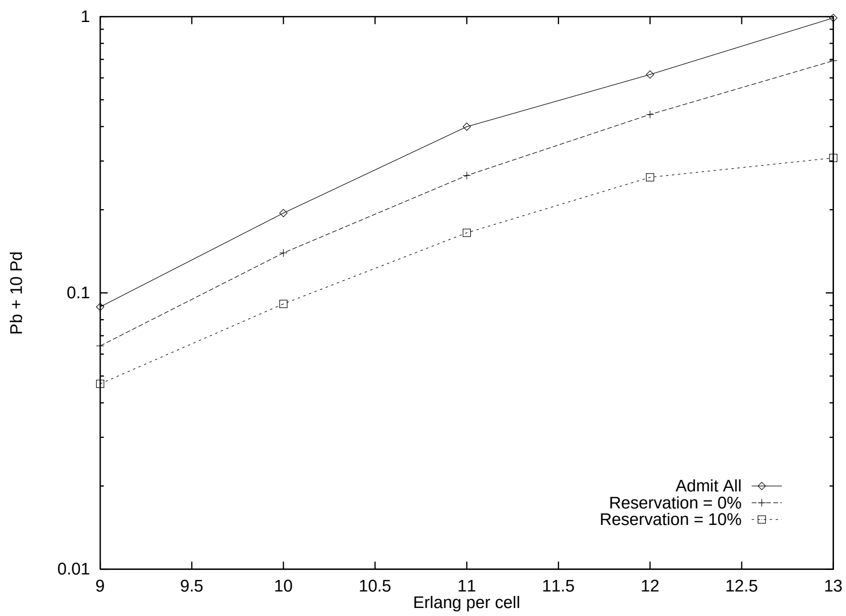


Figure 6.7: RPCAC-Cost Function: SHA with 1 BS (only consider one base station for reassignment)

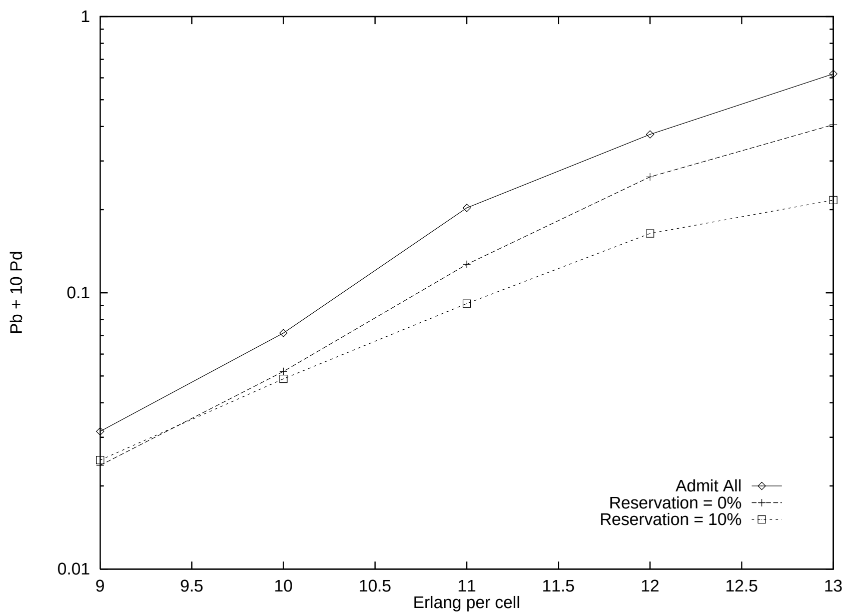


Figure 6.8: RPCAC-Cost Function: SHA with 2 BSs (always consider two base station for reassignment)

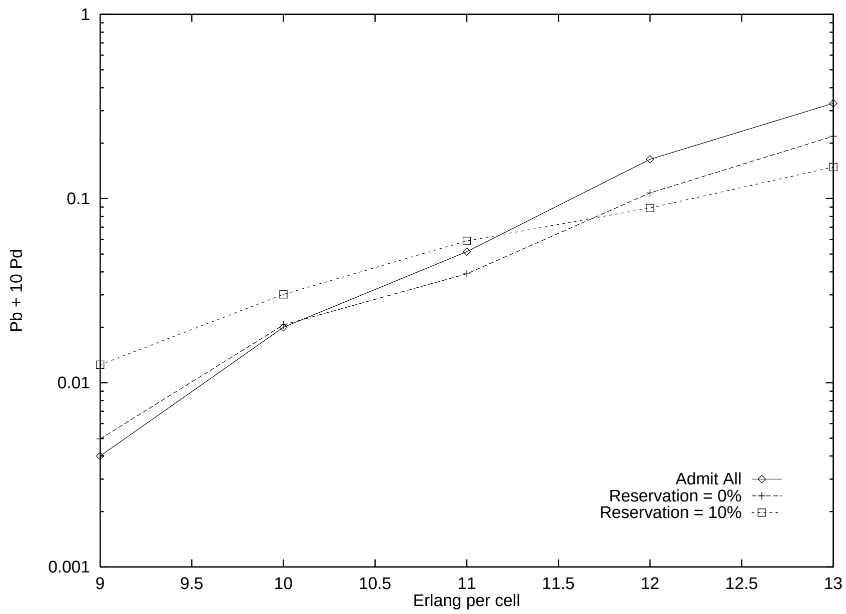


Figure 6.9: RPCAC-Cost Function: CMPA Algorithm

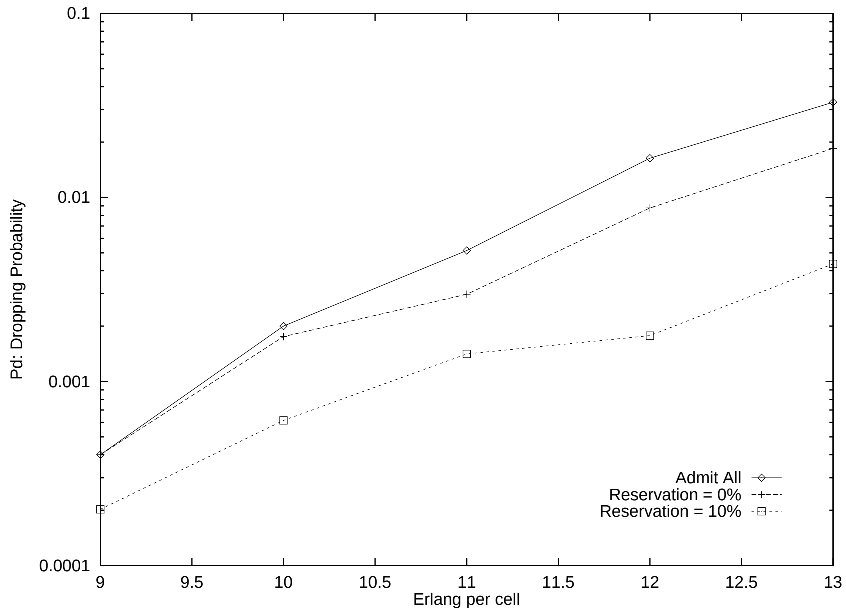


Figure 6.10: RPCAC-Dropping Probability: CMPA Algorithm

Chapter 7

Discussion

7.1 Conclusions

In this thesis, we study resource management problems in power controlled CDMA systems. With increasing demand for wireless communication services, an improved radio resource management is essential to minimize the use of resources and to improve system performance. We propose an integrated approach to power control, base station assignment and call admission to fulfill the above goals.

Several results for resource management have been reported in literature [7,8,18,23,58–60] to maintain feasible connections for all users. These works decomposed the resource management problem into separate power control, base station assignment and channel assignment problems. In this thesis, an integrated power control and base station assignment were considered in the call maintenance stage. Due to the fact that channel (code) assignment is not based on the quality measures, we did not consider code assignment in resource management.

During the call maintenance stage, no new call will try to get access to the system. At this stage, minimizing the use of resources is the major concern. As a result, we formulate the minimum transmitted power (MTP) problem. We find synchronous and asynchronous distributed minimum power assignment (MPA) algorithms that converge to a fixed point. When there exists a feasible solution, the fixed point is optimal in the sense of minimizing the total transmitted power subject to maintaining an individual target CIR for each mobile. In the presence of maximum power constraints, when there is no feasible solution, the infeasibility can be detected at the fixed point which lies on the boundary of the maximum power set. In other words, when system is congested some users will transmit at maximum power without achieving target CIR.

The rate of convergence for MPA algorithm was also studied. It was shown that iterative power control will converge to a fixed point at a geometric rate. This is important in the maintenance of good service under rapid variations in channel quality and user mobility. Simulation results also indicate the rate of convergence for MPA algorithm is fairly practical compared to current CDMA standard.

During the call set up stage, due to limited radio resources, call admission control is required to improve system performance. In this work, a weighted call blocking and dropping probability is considered as system performance. We believe that dropping a call in progress is more serious than blocking a new call. Thus a system needs a call admission policy for new requests to maintain acceptable connections for existing users. We consider two different concepts of call admission control, protection control and reservation control. The TPCAC, based on transmitted power measures, is to protect existing users by limiting new admission when existing users can not maintain the target CIR. Greater protection and improvement of system performance can be provided by considering larger set of users. The tradeoff between the choices of decision set is that larger decision set requires more information (users' transmitted powers). With the concept of reservation control, the RPCAC algorithm, based on total received power, can measure cell congestion levels and reserve capacity by setting the threshold values of allowable total received power for call admission control operation. As discussed, when the system is congested, call admission is required to improve system performance. The total received power at a base station is a good measure for the cell congestion level. The concept of reservation was found to be less effective when we considered the MPA algorithm.

7.2 Future Studies

7.2.1 Multiple Channel Systems

Many mobile communications systems currently operate with orthogonal channels. As such, there is significant capital investment in such radio equipment. This motivates us to consider the extension of our work to systems where user/system must choose not only base station and power assignment but also from a set of orthogonal channels. Notice that each channel might have different propagation characteristics and therefore different h_{kj}.

This multiple channel problem is fundamentally more difficult than the single channel problem. Specifically, for single channels, changing a user's base station assignment results in no degradation of CIR for other users. Changing a user's channel, however, can strongly degrade the CIR of another user. Nonetheless, we believe that the minimum power principle of the single channel MPA algorithm may serve to motivate good solutions for the multiple channel problem.

7.2.2 Call Admission for Multimedia Services

In the future, multimedia services will be considered in wireless communications systems. In fact, short message and data services have been included in the second generation cellular systems. This motivates us to consider call admission control for multimedia wireless systems. Notice that different service types require different quality of services (QoS). The improvement in system performance is of significant proportions by employing call admission control for these heterogeneous QoS requirements [57].

The call admission problem is more difficult for multimedia services. Two problems are needed to be resolved in order to design a good call admission policy. The first problem is about the measurement of the system congestion. In a single class service system, transmitted power is regulated such that user assigned to the same base station will have the same received power at that base station. In other words, they will generate the same interference to other users assigned to the same base station. For multimedia services systems, however, the interference seen from users assigned to the same base station is different. How to measure system congestion will be the first problem needed to be resolved. In this case, we believe that total received power at a base station can be used for cell load measurement. The second problem is to unify system performance measures. For single class service, we can use the same system performance measures but for multimedia services the performance measures are different and is difficult to consider jointly. For example, for voice calls, we can consider weighted call blocking and dropping probability as performance measures but for data calls, delay will be used as a performance measure. We believe that a good unified performance measure is required for call admission design for multimedia services wireless communication systems.

7.2.3 Utility based Power Control

A more general framework for power control is to define a user's preference defined as a utility function of transmitted power and connection quality (CIR). Higher quality with less transmitted power is desirable to individual user. But due the interference caused by other users, how to optimize individual utility and social utility will be the motivation for this work.

In this thesis, transmitted power and base station assignment is regulated to provide each user the target CIR. Among feasible solutions, the power vector that minimizes the total transmitted power would benefit mobile users with limited battery life. The fixed target CIR based power control problem allows us to prove that there exists an optimal solution to minimizing total transmitted power. In the utility based power control problem, this is not generally true. A better understanding of this general power control problem is needed.

7.2.4 Pricing for Wireless Networks

Currently, in cellular radio systems, each call is charged according to its call holding time by either peak rate or off-peak rate. This simple pricing algorithm is based on human behavior and is not based on the use of resources. This motivates us to design a good pricing scheme for wireless communication systems.

With limited radio resources, mobile users should be limited to guarantee all on going users can have feasible connection at all time. When the system is less congested, there are unused resources. In order to encourage more users to take advantage of these resources, lower charge could be applied when the system is less congested. On the contrary, when system is congested, users should be discouraged by sharing the resources.

We believe that the pricing problems for cellular radio networks can be studied based on the understanding of resource management. We know that total received power at a base station can be used to measures the congestion level at that base station. This received power based pricing scheme should be further investigated in the future studies.

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