On the Characterization of Bluetooth Scatternet Topologies

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Abstract—**Bluetooth [1] is an emerging short range, wireless networking standard. Using Bluetooth communication interfaces, a collection of devices can interconnect to form a *scatternet*. A set of n nodes can be organized into a scatternet in many different ways. This is in contrast to the classical narrowband packet radio networks or the wireless LAN systems where the topology of node interconnections is uniquely determined by the distances between the nodes.**

Understanding the topological structure of scatternets is an important first step towards the goal of building algorithms for self-organizing networks. Unfortunately, the space of all scatternet topologies is so large that it is computationally infeasible to search the space without proper understanding of its mathematical structure. Our contributions in this paper are two fold: 1) we identify the mathematical properties of the scatternets, and 2) we describe an efficient technique for enumerating all feasible Bluetooth scatternet topologies. Our results provide several key insights into the design of algorithms for Bluetooth networks, namely, quantification of the space of all possible scatternet topologies, characterization of several constrained subsets of topologies, and a lower bound on the message complexity of topology construction algorithms.

Keywords— **Bluetooth, wireless networking, graph enumeration, self-organizing networks.**

I. INTRODUCTION

LIKE PCs of the pre-Internet era, pervasive devices (cell phones, PDAs, pocket diaries, etc.) of today are used primarily as autonomous, non-networked entities. Some communication devies such as cell phones and pagers are connected to their respective service provider's network, but for the most part devices do not communicate with each other on a peer-to-peer basis. In some instances it is possible to connect devices together using wired cables (for example a cell phone to a PC), but this style of tethered communication is not only cumbersome, it cannot be used for interconnecting more than two devices together.

Bluetooth [1], an industry consortium is currently building specifications for a short range, low power, RF wireless link which can be integrated on devices such as laptops, cell phones, cameras, and PDAs. Using Bluetooth interfaces, a group of devices within close proximity will be able to interconnect and form an ad-hoc network known as *scatternet*. A collection of Bluetooth nodes can be organized into a scatternet in many different ways. This is because Bluetooth is spread spectrum based and nodes in proximity cannot communicate until they share the same spreading code. This is in contrast to the classical narrowband packet radio networks or the wireless LAN systems where the topology of node interconnections is uniquely determined by the distances between the nodes.

In less than a year from now, it will be feasible to build Bluetooth scatternets consisting of ten or more devices. Smart offices of the future, for example, will be filled with all sorts of intelligent devices: computers, PDAs, pagers, speakers, phones, white-boards, and sensors. A necessary pre-requisite for supporting any smart spaces application is to interconnect all devices into a network. Consequently, understanding the space of all feasible interconnection topologies and quantifying their number is an important first step towards building techniques for self-organizing networks. Unfortunately, the space of all scatternet topologies is so large that it is computationally infeasible to search the space without proper understanding of its mathematical structure. Our contributions in this paper are two fold: 1) we identify the mathematical properties of the scatternets, and 2) we describe an efficient technique for enumerating all feasible Bluetooth scatternet topologies. Our results provide several key insights into the design of algorithms for Bluetooth networks, namely, quantification of the space of all possible scatternet topologies, characterization of several constrained subsets of topologies, and a lower bound on the message complexity of topology construction algorithms.

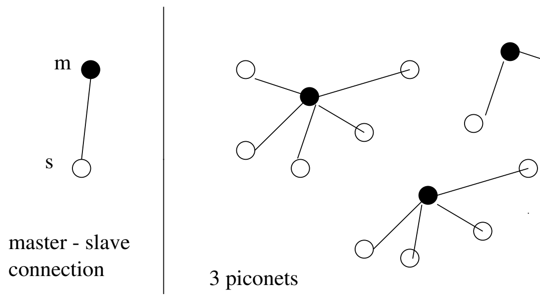

Fig. 1. Bluetooth piconets

II. BACKGROUND

Bluetooth is a frequency hopping spread spectrum communication system that is designed to operate in the 2.4GHz band [2]. Similar to any other frequency hopping system when two Bluetooth devices wish to communicate, they must first agree on a common frequency hopping sequence. In Bluetooth, this agreement is reached based the role (*master* or *slave*) that each device performs. Every Bluetooth device is capable of acting both as a master and a slave. When a pair of devices communicate, one of them acts as master and the other as slave. The master selects the hopping sequence and the slave follows that sequence for the entire duration of the connection. After the connection is established, the master and slave nodes share the channel using a slotted time division duplex protocol, The master node, again, is responsible for controlling the allocation of the time slots. Usually, alternate slots are reserved for the master to slave and the slave to master direction communication.

To facilitate communication between a group of devices, Bluetooth supports formation of what is known as a *piconet*. A piconet consist of exactly one master node and up to seven slave nodes, all of which are synchronized to the hopping sequence of the master. All master-slave pairs can communicate with each other, but slaves are not permitted to communicate directly. This is because all slaves only listen during the master to slave transmission slot. When a slave node transmits, only the master listens to the medium. Lack of communication capability at the physical layer, however, does not prevent two slave nodes from communicating. Using a higher layer forwarding protocol, a master node can always switch packets among all connected slaves nodes.

Since each piconet hops with a different hopping pattern, multiple piconets can co-exist in spatial proximity without interfering with each other. Figure 1, shows an example of three co-existing piconets. To facilitate communication between piconets, Bluetooth defines the concept of *scatternet*. A scatternet is formed by connecting multiple piconets using bridges. A node acting as a bridge is a member of two or more piconets and it participates in each piconets on a time sharing basis. After staying in a piconet for some duration, the node can turn to another piconet by switching its hopping sequence. By cycling through all member piconets, a node can send and receive packets in each piconet and also forward packets between piconets. Figure 2 shows an example scatternet which is formed by joining together the disjoint scatternets of the previous example. Notice that all nodes acting as bridges are slaves. Although it is possible to construct scatternet topologies where a node acts as master in one piconet and as slave in other piconets, such configurations result in poor bandwidth utilization and are therefore undesirable (see the discussion of connectivity constraint (b) in section III).

III. THE SPACE OF ALL SCATTERNET TOPOLOGIES:

It will be evident from the discussion in the previous section that only certain combinations of node arrangements are allowed by the Bluetooth system. A scatternet formed by a collection of n Bluetooth devices satisfies the following constraints:

1. **C1: Node Type constraint** Each node is either a master or a slave. A subset of slave nodes may act as bridges.
2. **C2: Degree Constraint**
   - A master can serve at most seven slaves, i.e., degree of each master is ≤ 7.
   - A bridge can participate in at most maxbridge piconets, i.e., degree of each bridge is ≥ 2 and $\leq max\_bridge\_degree$.
   - non-bridge slave nodes are connected to only one master, i.e, degree of each slave node $= 1$.
3. **C3: Connectivity Constraint**
   - (a) Two slaves cannot be connected directly
   - (b) Two masters should not be connected directly

Figure 4 shows the space of all scatternet topologies and its relation to the space of all possible graphs. The ellipse labeled as "Bluetooth scatternets" represents all graphs topologies satisfying all of the above constraints except (b). Notice that the connectivity constraint (b) is not posed by the Bluetooth physical layer. We introduce this constraint only to eliminate certain undesirable topologies. Due to the frequency hopping nature of the physical layer,

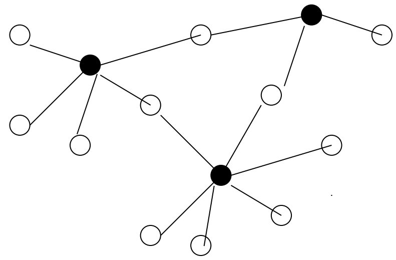

Fig. 2. Bluetooth scattenet example

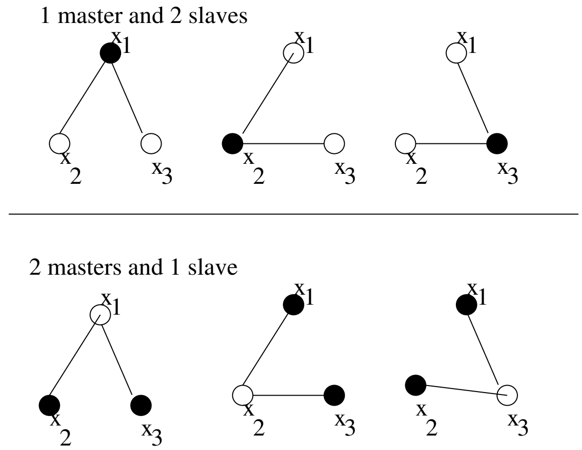

Fig. 3. Bluetooth scattenet example

two ends of any bluetooth link must form a master slave pair, i.e., if two master nodes are to be connected, one of the masters should also act as a slave of the other master. While switching between the master and slave roles may be feasible, it is wasteful because all communication in a piconet should be suspended when the master is away from its piconet. In the subsequent discussion without any loss of generality we'll use the term Bluetooth scatternet topologies to mean all graph topologies sastifying all of the above constraints. Inclusion of constraint (b) will constrain all such graphs to be bipartite.

The set all scatternets with n nodes is a small subset of all possible graphs with n nodes. A simple method for enumerating all Bluetooth scatternets is to first generate all possible graphs with n nodes and then select the subset satisfying all of the above constraints. Figure 3 shows the application of this method for $n = 3$. Even with three nodes there are six different ways of connecting the nodes together. The first three configurations represent one master and two slaves nodes $(m = 1, s = 2)$ forming a single piconet. The next three configurations $(m = 2, s = 1)$ show two master nodes forming two separate piconets and a slave node acting as a bridge between the two piconets. As n grows large, the possible ways of forming scatternets increases exponentially $(2^{\frac{n(n-1)}{2}})$. For $n = 4$, it is possible to enumerate all configurations by hand, but for $n > 4$, the number is large enough that the process of enumeration cannot be managed without a computer. For $n \geq 7$, the space of all graph topologies is so large that it is not possible to find the feasible set of the topologies by brute force enumeration on a computer. In section IV we describe several efficient techniques for enumerating all scatternets.

Constrained subsets

For many applications, only a small subset of all possible scatternets may be of interest. For example, a spanning tree bridging algorithm [**?**] will only be interested in searching over the space of tree shaped scatternets. Similarly, an alternate path ad hoc routing algorithm may want to organize nodes such that all nodes have high degrees. Posing additional constraints on the structure of the scatternet will further constrain the search space, however this reduction may not always translate into complecxity reduction of the enumeration process. A simple albeit somewhat expensive solution would be to first generate all scatternet topologies and then select only the topologies of choice. This method, though feasible till some value of n, will fail for large n. In the next section, we'll show several techniques for pushing the computation further. Following are some example constraints for which our proposed technique will be applicable.

- **C4:** set of all trees shaped scatternets
- **C5:** set of all scatternets with $\leq K$ edges
- **C6:** set of all scatternets with given node degree constraint

IV. Counting the number of bluetooth configurations

In this section we describe how the Bluetooth scatternet topologies can be characterized in graph theoretic terms.

A. Graph Enumeration

There are numerous applications of graph theory in the study of Computer networks. (cite Data Networks, Bertsekas, Gallagher ?) The hosts are represented by nodes of the graph while the edges represent the links. We will be interested in *labeled* graphs where the nodes are distinguished from each other. Denote the set of nodes by $\mathcal{X} = \{X_1, \cdots, X_N\}$ and the sequence of their degrees (number of edges incident on each node) by $(d_1, \cdots, d_N)$.

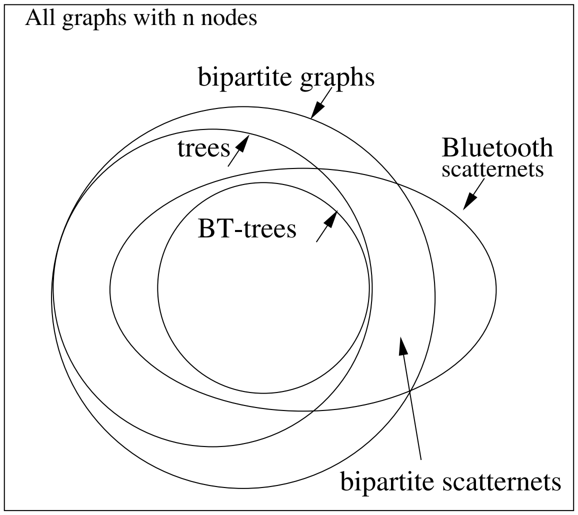

Fig. 4. The space of all Bluetooth topologies

It is easy to see that $\sum_{i=1}^{N} d_i = 2k$, where k is the number of total edges. In number theory, the integer sequence $(d_1, \cdots, d_N)$ is referred to as a *partition* of the integer $2k$. The parts of a partition are usually given in a non-increasing order. For example, 4 has five partitions: (4), (3,1), (2,2), (2,1,1), and (1,1,1,1). A partition $(p_1, p_2, \cdots,)$ of an integer l satisfies $\sum_i p_i = l$. An alternative way to represent a partition is by collecting repeated parts together; e.g. (2,1,1) can be written as $(1^2 2^1)$ and (3,1) as $(1^1 2^0 3^1)$. Note that a partition $(1^{v_1} 2^{v_2} \cdots)$ of l satisfies $\sum_i i v_i = l$.

Not every partition $(p_1, p_2, \cdots,)$ of an even integer is *graphical*, meaning that there is a graph with degrees p_i. For example, it can be verified readily that only the last two partitions of 4 given above ((2,1,1), and (1,1,1,1)) are graphical. Erdos and Gallai theorem gives necessary and sufficient conditions for a partition to be graphical. A partition $(d_1, \cdots, d_N)$ of $2k$ with $d_1 \geq d_2 \geq \cdots d_N$ is graphical if and only if for each integer r, $1 \leq r \leq N-1$,

$$\sum_{i=1}^{r} d_i \leq r(r-1) + \sum_{i=r+1}^{N} min\{r, d_i\}.$$

Given a graphical partition, there is a simple algorithm by Havel and Hakimi to construct a graph with that partition.

Next we consider the problem of counting labeled graphs with a given degree sequence. There has been extensive research in the past in the area of graph enumeration with applications in several areas. See for example [3]. Associate with each node X_i a variable x_i and consider the expansion of the product

$$\Pi_{i=1}^{j} \Pi_{j=1}^{N} (1 + x_i x_j) \; = \; \sum n(\bar{d}) {x_1}^{d_1} \cdots {x_N}^{d_N} \qquad (1)$$

Each term in the expansion is obtained by selecting $x_i x_j$ from some factors and 1 from the rest of the factors. Thus each term corresponds to a labeled graph which contains the edge $X_i X_j$ if the term $x_i x_j$ has been chosen. The coefficient $n(\bar{d})$ of the monomial ${x_1}^{d_1} \cdots {x_N}^{d_N}$ in the expansion gives the number of graphs with the degree sequence $\bar{d} = (d_1, \cdots, d_N)$. The brute force approach to enumeration by expansion (1) becomes unmanageable as the number of nodes N increases. For example, for $N = 10$, the product has 2^{45} terms.

The *adjacency matrix* $A = [a_{ij}]$ of a labeled graph with N nodes is the $N \times N$ matrix with $a_{ij} = 1$ if node X_i is adjacent to node X_j (i.e. there is an edge between X_i and X_j) and $a_{ij} = 0$ otherwise. Thus there is a one-to-one correspondence between labeled graphs with N nodes and $N \times N$ symmetric binary matrices with zero diagonal [4]. The graph enumeration problem described above can be cast in terms of enumeration of binary matrices.

As we noted above, bluetooth scatternets have additional structure which limits the number of their topologies. Since the possible links are only between a master and a slave and not between two masters or two slaves, we can model these topologies using *bipartite* graphs. Bipartite graphs have two disjoint sets $\mathcal{X} = \{X_1, \cdots, X_m\}$ and $\mathcal{Y} = \{Y_1, \cdots, Y_n\}$ of nodes and each edge joins a node in $\mathcal{X}$ with one in $\mathcal{Y}$. $\mathcal{X}$ here represents the set of masters and $\mathcal{Y}$ represents the set of slaves. Associating variables x_i and y_j with nodes X_i and Y_j respectively, the expansion

$$\Pi_{i=1}^{m} \Pi_{j=1}^{n} (1 + x_i y_j) \; = \; \sum n(\bar{p}, \bar{q}) {x_1}^{p_1} \cdots {x_m}^{p_m} {y_1}^{q_1} \cdots {y_n}^{q_n} \qquad (2)$$

now enumerates all labeled bipartite graphs. The coefficient $n(\bar{p}, \bar{q})$ of the monomial ${x_1}^{p_1} \cdots {x_m}^{p_m} {y_1}^{q_1} \cdots {y_n}^{q_n}$ on the right side gives the number of bipartite graphs where $\mathcal{X}$ has the degree sequence $\bar{p} = \{p_1, \cdots, p_m\}$ and $\mathcal{Y}$ has degrees $\bar{q} = \{q_1, \cdots, q_n\}$. For $m = 3$ and $n = 7$, there are now only 2^{21} terms in the expansion which is still quite large.

In the case of bipartite graphs also, there is a $m \times n$ binary matrix $B = [b_{ij}]$ corresponding to the bipartite graph such that $b_{ij} = 1$ if there is an edge between nodes X_i and Y_j and $b_{ij} = 0$ otherwise. Thus the enumeration of biparitie graphs with degree sequences $\bar{p} = \{p_1, \cdots, p_m\}$ and $\bar{q} = \{q_1, \cdots, q_n\}$ is identical to enumeration of $m \times n$ binary matrices with row sum vector $\bar{p}$ and column sum vector $\bar{q}$.

The *conjugate* $\bar{p}' = \{p_1', \cdots, p_{m'}'\}$ of a partition $\bar{p} = \{p_1, \cdots, p_m\}$ is obtained by choosing p_i' as the number of parts of $\bar{p}$ that are $\geq i$. Gale, Ryser theorem gives the necessary and sufficient condition for the existence of a binary matrix with given row sum vector $\bar{p}$ and $\bar{q}$: such a matrix exists if and only if $\bar{q}$ is majorized by the conjugate of $\bar{p}$ i.e.

$$\sum_i^k q_i \leq \sum_i^k p_i' forall 1 \leq k \leq n.$$

For example $\bar{p} = (4, 2, 1)$ and $\bar{q} = (2, 2, 2, 1)$. The conjugate of $\bar{p}$ is $(3, 2, 1, 1)$ which clearly majorizes $(2, 2, 2, 1)$.

B. Enumeration of Bluetooth topologies

In addition to the bipartiteness, we now consider other constraints on the Bluetooth topologies. Suppose that the set of slaves has a given degree sequence. We will describe how the enumeration of the topologies simplifies and illustrate with a few examples. The case of known degrees for the masters can be handled similarly.

Suppose the slave nodes $Y_1, \cdots, Y_n$ have degrees $q_1, \cdots, q_n$ respectively. From the expansion (2), the term factoring ${y_1}^{q_1} \cdots {y_n}^{q_n}$ is the polynomial

$$\sum_{\bar{p}} n(\bar{p}, \bar{q}) {x_1}^{p_1} \cdots {x_m}^{p_m}$$

where the summation is over all master degree sequences $\bar{p}$ such that $(\bar{p}, \bar{q})$ are graphical. This polynomial enumerates the constrained Bluetooth topologies in this case. Considering the product of factors in equation (2) columnwise, i.e. as $(1 + x_1 y_1) \cdots (1 + x_m y_1) \cdots (1 + x_1 y_n) \cdots (1 + x_m y_n)$, clearly the factor ${y_1}^{q_1} \cdots {y_n}^{q_n}$ is obtained by choosing $x_i y_1$ from q_1 of the factors in the first column and 1 from the rest and so on. Using the identity

$$\begin{aligned} \Pi_{i=1}^n (1 + z_i) & \\ &= 1 + (z_1 + \cdots + z_n) + (z_1 z_2 + \cdots + z_{n-1} z_n) + \cdots \\ &= \sum_{k=0}^n a_k(z_1, \cdots, z_n), \end{aligned}$$

where the functions $a_k(z)$ are known as elementary symmetric functions, the desired factor of $y_1^{q_1}$ from the first column is seen to be $a_{q_1}(x_1, \cdots, x_m)$ etc. Thus we have

$$\begin{aligned} a_{q_1}(x_1, \cdots, x_m) \cdots a_{q_n}(x_1, \cdots, x_m) \\ = \sum_{\bar{p}} n(\bar{p}, \bar{q}) {x_1}^{p_1} \cdots {x_m}^{p_m}. \end{aligned}$$

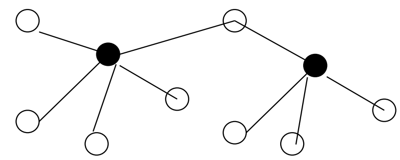

Fig. 5. An example scatternet corresponding to the parition (2,1,1,1,1,1,1,1)

Let us consider some examples. Suppose we have a scatternet with 2 masters and 8 slaves arranged in a tree topology. One of the 8 slaves acts a bridge between the two piconets thus resulting in a slave degree sequence of the type $(2, 1, 1, 1, 1, 1, 1, 1)$. Here we have $k = 9$ edges. Figure 5 illustrates this example. The enumerating polynomial is given by

$$a_2(x_1, x_2) a_1^7(x_1, x_2) = (x_1 + x_2)^7 x_1 x_2.$$

We can now obtain the number of all topologies with the given constraint by substituting $x_1 = x_2 = 1$ as 8.2^7 the factor 8 representing the 8 possible choices of bridge nodes among the 8 slaves. Similarly the number of topologies with a specific master degree sequence can obtained easily. Note that this polynomial could have been written down simply by inspection since each slave node with degree 1 can either connect to master node X_1 or X_2 leading to a factor $(x_1 + x_2)$ each and the bridge node connects to both, giving rise to $x_1 x_2$.

As a further example, suppose we wish to enumerate all topologies with 3 masters and 7 slaves with the number of edges being 12 or less. Since we are interested primarily in connected topologies, we will take the minimum number of edges to be 9 with 2 slaves acting as bridges thus resulting in the tree topology. Since k can be 9, 10, 11 or 12, we have the enumerating polynomial

$$a_1^5(x_1, x_2, x_3) a_2^2(x_1, x_2, x_3) + a_1^4(x_1, x_2, x_3) a_2^3(x_1, x_2, x_3) + a_1^3(x_1, x_2, x_3) a_2^4(x_1, x_2, x_3) + a_1^2(x_1, x_2, x_3) a_2^5(x_1, x_2, x_3)$$

and the resulting total number of topologies is $(2.\frac{7!}{2!5!} + 3.\frac{7!}{3!4!})3^7$.

The number of scatternet topologies for the two examples considered here and for other cases are listed in Table below.

Masters	Slaves	Edges	No. of Topologies
2	8	9	1024
2	8	11	1792
2	8	≤ 12	6848
3	7	9	45927
3	7	11	76545
3	7	≤ 12	244944
4	6	9	276480
4	6	11	186624
4	6	≤ 12	820800

As the number of nodes and the number of edges in a scatternet increases, the approach described above for deriving the enumerating polynomial as a product of elementary symmetric functions becomes difficult to handle both manually and on the computer. In such a case there is an approach that efficiently reduces the number of terms to be handled, thereby allowing the enumeration of topologies for larger scatternets than otherwise would be possible. We will outline the approach below. For details, refer to [5]. In the final version of the paper, we will illustrate this approach with an example.

It is known that the elementary symmetric functions $a_k(\cdot)$ can be written as polynomials in the power sums $s_l = \sum_i x_i^l$. For example, $a_2(x_1, x_2) = \frac{1}{2}(s_1^2 - s_2)$. Suppose we are interested in enumerating topologies with a given slave degree sequence and with master degrees less than or equal to d. Then the polynomial in power sums can be simplified by setting s_{d+1}, s_{d+2} etc. to zero since any term involving one of these will result in a graph with maximum degree greater than d. The reduced polynomial in the power sums can now be expanded back into a polynomial in x_i's.

V. Discussion & Conclusion

Although our discussion was focussed on Bluetooth scatternet topologies, the techniques presented in this paper are generally applicable to other ad hoc networks as well. In general when the network topology can be modeled as a bipartite graph, it is possible to exploit several structural properties of bipartite graphs for building efficient enumeration techniques. Otherwise, a less compact, NxN adjacency matrix representation can always be used. It must be evident from the examples that without proper understanding the mathematical structure of the underlying topologies even small sized networks (between 5-7 seven nodes) cannot be enumerated.

In graph theory literature, a number of results about the properties of 0-1 matrices already exist. Our aim in this paper was not to invent new graph theory results, but to investigate how some of the Bluetooth topology enumeration problems can be reduced to well known results in graph theory. The examples in this paper illustrate how some of the classical results can be interpreted ls in the context of Bluetooth. Some of our key observations are that:

- a piconet is a degree constrained star shaped network, where several slave nodes of degree one are connected to one master node.
- Bluetooth scatternets are bipartite graphs
- enumeration of all scatternets with given degree constraints is same as counting all rectangular matrices with given row and column sums
- enumerating all tree shaped scatternets is a special case of the above with a special structure on the slave degree sequence
- enumeration of all scatternets with $\leq K$ edges can be found by applying the observation 3 over all possible graphical partions of K edges
- the enumerating polynomial of a given degree constrained scatternet can be expressed as a product of elementric symmetric functions
- the expansion of the above product can be simplified by substituting elementric symmetric functions by respective power sum expressions

The result presented in this paper is only a step towards understanding the structure of Bluetooth scatternets. A number of problems remain open, namely: finding a topology construction algorithm, method of addressing and routing packets in Bluetooth networks, interconnecting a Bluetooth scatternet to the IP backbone. We hope that some of the insights presented !lain this paper will encourage other researchers to investigate other open research issues in this emerging, exciting new area in mobile computing.

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