Wireless Communication Technologies

(ECE Course 16:332:559)

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Topics on Diversity Techniques

1. Introduction

In wireless communication, transmitted signals always suffer various kinds of fading before they reach the receiving terminal. Diversity is one very effective solution that exploits the principle of providing the receiver with multiple faded replicas of the same information that bears signal.

There are five approaches to achieve diversity: 1) frequency diversity, 2) time diversity, 3) polarization, 4) angle, 5) and space diversity. Frequency diversity is achieved by using multiple channels that are separated by at least the coherence bandwidth of the channel. Thus it is not bandwidth efficient. Time diversity is obtained by using multiple time slots that are separated by at least the coherence time of the channel. Polarization diversity is to exploit the property that scattering environment tends to depolarize a signal. Receiver antenna having different polarization can be used to obtain diversity. Angle diversity sometimes is also called directional diversity. Directional antennas are used to transmit the signal in the specified wave plane so that the receiver antennas obtain uncorrelated signals. Space diversity uses multiple receiver antennas. The frequency and time diversity require more bandwidth and the other methods usually involve more complex antenna system.

Generally speaking, The signal correlation, ρ, is a function of distance between two receiver antennas. Expressed mathematically,

$$\rho = J_0^2\left(\frac{2\pi d}{\lambda}\right) \qquad (1)$$

Where

$J_0(x)$ = Bessel function of zero order

d = distance between two antennas

λ = transmitted wave length

Figure 1 graphically presents the relationship between the correlation and antenna distance. If we want to achieve zero correlation, the distance between two antennas is approximately half-length of the transmitted waveform.

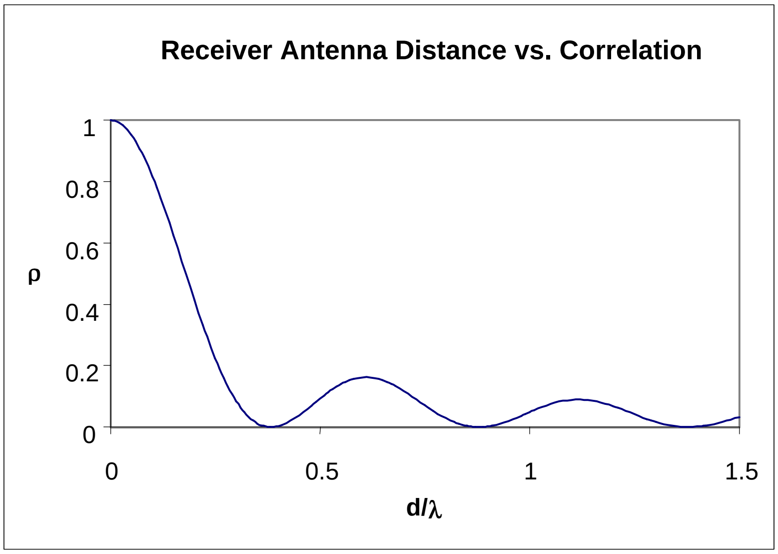

Figure 1. Correlation for different values of d/λ

For example, when f_c = 5MHz, the distance d is 30 meters. When f_c = 500MHz, the distance is reduced dramatically to 30 cm.

2. Combining Methods

Diversity combining is refereed to be a method by which the signals from the diversity branches are combined to detect the single. Diversity combining takes place before the detection is call the predetection combining, while diversity combining occurs after detection is called postdetection combining. If the detection is perfect, one has no obvious advantage over the other in coherent detection. For differential coherent detection, there is a slight difference between predetection and postdetection combinings. In the following section, we assume that coherent detection is ideal, thus we will not distinguish the difference between the predetection combining and postdetection combining.

There are many approaches to realize the detection combining. Typical methods include **Selection Diversity**, **Maximum Ratio Combining**, **Equal Gain Combining** and **Switched Combining**. They will be illustrated in detail in following paragraphs.

The fading gains of various diversity branches typically have some degree of correlation. The degree depends on the type of diversity being used and the propagation environment. For analytical purposes, the assumption of uncorrelated branches is employed because of the mathematical tractability. This usually yields a optimistic result. Nevertheless, we will evaluate the performance of various diversity techniques under the assumption of uncorrelated branches.

The fading distribution will also affect the diversity gain. The relative advantage of diversity is greater for Rayleigh fading than Rician in general. However, the performance of Rician fading is always better than Rayleight for a given average signal-to-noise ratio (SNR) in the same diversity. We will deal with performance analysis of Rayleigh fading.

3. Selection Diversity (SD)

3.1 CDF of SNR for Selection Diversity

We assume that there are *M* independent copies of signals available from *M* independent paths in a diversity system. Consider *M* diversity branches where SNR achieved on each branch is γ_i for $i = 1, 2, 3, \dots M$.

Let us assume that the received signal on each branch is independent and Rayleigh distributed with mean power of $2\sigma^2$. Then γ_i is exponentially distributed as,

$$p(\gamma_i) = \frac{1}{\gamma_0} \exp(-\frac{\gamma_i}{\gamma_0}) \qquad (2)$$

Where

$$\gamma_0 = 2\sigma^2 \frac{E_b}{N_0}$$, the mean of the received power on the branch

E_b/N_0 = SNR achieved without fading effect

E_b = bit energy

from equation 2, we can obtain the CDF for i^{th} branch as follows,

$$P\{\gamma_i \le \gamma\} = \int_0^{\gamma} p(x)dx = 1 - \exp(-\frac{\gamma}{\gamma_0}) \tag{3}$$

Selection diversity picks up the best signal among M branches in terms of SNR. Expressed mathematically, the selection criterion is defined as,

$$\begin{aligned} P\{\gamma\} &= P_r\{\max\{\gamma_i\} \le \gamma\} \\ &= P_r\{\gamma_1 \le \gamma,\ \gamma_2 \le \gamma, \cdots,\ \gamma_M \le \gamma\} \\ &= \prod_{i=1}^{M} P_i\{\gamma_i \le \gamma\} \end{aligned} \tag{4}$$

After we substitute equation 3 into equation 4, we obtained the commutative distribution function as follows.

$$P\{g\} = \left[1 - \exp(-\frac{g}{l_0})\right]^M \tag{5}$$

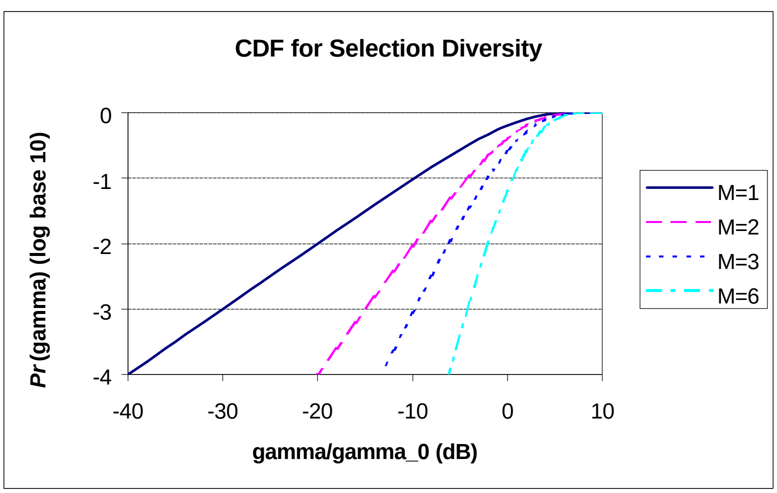

Figure 2. CDF for Selection Diversity with M = 1, 2, 3, 6

Where,

gamma/gamma_0 = γ/γ_0

Pr(gamma) (log base 10) = $\log_{10}(Pr(\gamma))$

Note that the probability of very low SNR decrease rapidly with *M* increases. When SNR is fare below γ_0, we can approximate equation 5 by

$$P\{g\} = \left[1 - \exp(-\frac{g}{l_0})\right]^M = \left(\frac{g}{g_0}\right)^M \qquad (6)$$

Where we use the following approximation for small *x*

$$\exp(-x)=1-x \qquad (7)$$

Taking the logarithm of equation 6 yields,

$$\log P\{g\} = M \log\left(\frac{g}{g_0}\right) \qquad (8)$$

Equation 8 indicates that $\log P\{\gamma\}$ increases linearly with the number of branches *M*. This factor is also shown in Figure 2.

3.2 Average SNR in Selection Diversity

Sometimes, we might be interested in the average SNR for selection diversity with *M* branches. The average SNR is defined as follows,

$$E[\max\{\gamma_i\}] = \int_0^\infty \gamma\, p(\gamma) d\gamma \qquad (9)$$

Since we have obtained the CDF of γ, the pdf of γ can be determined by taking the derivative with respect to γ in equation 5 as follows,

$$p(\gamma)=\frac{dP(\gamma)}{d\gamma}=\frac{M}{\gamma_0}\exp(-\frac{\gamma}{\gamma_0})\left[1-\exp(-\frac{\gamma}{\gamma_0})\right]^{M-1} \tag{10}$$

Substitute equation 10 into 9, and make some algebraic manipulation, we have

$$\begin{aligned}
E[\max\{\gamma_i\}] &= \int_0^\infty \frac{M\gamma}{\gamma_0}\exp(-\frac{\gamma}{\gamma_0})\left[1-\exp(-\frac{\gamma}{\gamma_0})\right]^{M-1} d\gamma \\
&= \int_0^\infty \frac{M\gamma}{\gamma_0}\exp(-\frac{\gamma}{\gamma_0})\sum_{k=0}^{M-1}\left\{\binom{M-1}{k}(-1)^{M-1-k}\exp(-\frac{(M-1-k)\gamma}{\gamma_0})\right\}d\gamma \\
&= \frac{M}{\gamma_0}\sum_{k=0}^{M-1}\binom{M-1}{k}(-1)^{M-1-k}\int_0^\infty \gamma\exp(-\frac{(M-k)\gamma}{\gamma_0})d\gamma \\
&= \gamma_0\sum_{k=0}^{M-1}\binom{M-1}{k}(-1)^{M-1-k}\frac{M}{(M-k)^2} \\
&= \gamma_0\sum_{k=1}^{M}\frac{1}{k}
\end{aligned} \tag{11}$$

Rather than using direct integration technique, we can also use induction method to obtain equation 11. Now let us make some comparisons as M increases from 1 to 3 and see how many gains can be achieved. For example, if we increase M = 1 to M = 2, the average gain

$$10\log\left(\frac{g_0(1+1/2)}{g_0}\right)=1.8dB$$

If we increase M=2 to M=3, the average gain is

$$10\log\left(\frac{g_0(1+1/2+1/3)}{g_0(1+1/2)}\right)=0.9dB$$

For most practical applications, M = 2 is always chosen as the optimal value. The disadvantage associated with selection diversity is that signals must be monitored at a rate that is faster than the fading process.

4. Maximum Ratio Combining (MRC)

4.1 CDF of SNR for MRC

MRC uses the information of all signals from all branches simultaneously. Each branch is weighted with a gain factor that is proportional to its own SNR. MRC results in a ML receiver channel, therefore it gives the best possible performance among the diversity combining techniques. Figure 3 illustrates the basic idea of the MRC method.

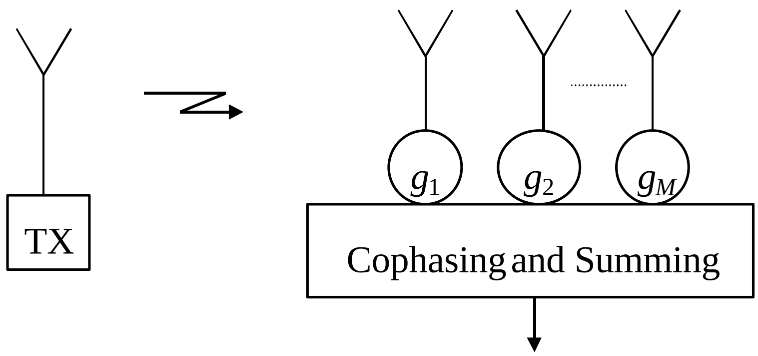

Figure 3. MRC Implementation Diagram

Where the gain of i^{th} branch is defined to be proportional to its SNR, i.e. $g_i \propto (S/N)_i$ for all i = 1, 2, …, M. Cophasing is also important because the received signals may be out of phase due to the reflection and fading, etc. The realization of this optimal combining technique is based on the assumption that channel attenuation and phase shifts are known.

Assuming that a_i is the envelope of i^{th} branch. Then the combined signal envelope for M branches is given by,

$$a = \sum_{i=1}^{M} a_i g_i \qquad (12)$$

We further assume that noise components are *i.i.d.* for each branch. Then the total noise power for M branches can be estimated as follows,

$$N_t = N_0 \sum_{i=1}^{M} g_i^2 \tag{13}$$

Finally, the resulting SNR in the receiver is given by,

$$\gamma = \frac{a^2 E_b}{N_t} = \frac{E_b \left(\sum_{i=1}^{M} a_i g_i \right)^2}{N_0 \sum_{i=1}^{M} g_i^2} \tag{14}$$

Using the Schwarz inequality, we can obtain the up-bound for equation 14 as follows,

$$\begin{aligned} \gamma = \frac{a^2 E_b}{N_t} &= \frac{E_b \left(\sum_{i=1}^{M} a_i^2 g_i^2 \right)}{N_0 \sum_{i=1}^{M} g_i^2} \\ &\leq \frac{E_b \sum_{i=1}^{M} a_i^2 \sum_{i=1}^{M} g_i^2}{N_0 \sum_{i=1}^{M} g_i^2} \\ &= \frac{E_b}{N_0} \sum_{i=1}^{M} a_i^2 \end{aligned} \tag{15}$$

where

$g_i = ka_i$ with k is a constant

Thus the maximum SNR is achieved when $g_i = a_i$ as follows,

$$g = \frac{E_b}{N_0} \sum_{i=1}^{M} a_i^2 = \sum_{i=1}^{M} \frac{E_b}{N_0} a_i^2 = \sum_{i=1}^{M} g_i \tag{16}$$

This equation also indicates that the resulting SNR is the sum of individual SNR of each branch.

To obtain the pdf of combined signal, we observe that SNR in i^{th} branch is given by

$$g_i = \frac{E_b}{N_0} a_i^2 = \frac{E_b}{N_0}(x_i^2 + y_i^2) \qquad (17)$$

Where x_i and y_i are two independent Gussian random variables with zero mean and common variance of σ^2. The sum of two squared Gussian random variables is Chi-square distributed with degree of two. Since we have M branches and each branch has two degrees, thus the total degree of freedom for γ is $2M$. The pdf of γ is given as,

$$p(g) = \frac{1}{(M-1)!} \frac{g^{M-1}}{g_0^M} \exp(-\frac{g}{g_0}), \qquad \gamma \geq 0 \qquad (18)$$

where

$$g_0 = \frac{2s^2 E_b}{N_0} \text{ which is mean SNR in each branch}$$

Finally, the CDF is computed by,

$$P(\gamma) = \int_0^{\gamma} \frac{1}{(M-1)!} \frac{x^{M-1}}{\gamma_0^M} \exp(-\frac{x}{\gamma_0}) dx = 1 - \exp(-\frac{\gamma}{\gamma_0}) \sum_{i=1}^{M} \frac{1}{(i-1)!} \left(\frac{\gamma}{\gamma_0} \right)^{i-1} \qquad (19)$$

Equation 19 can be obtained through the integration by parts. Due to limited space, the detailed procedure is omitted in this note.

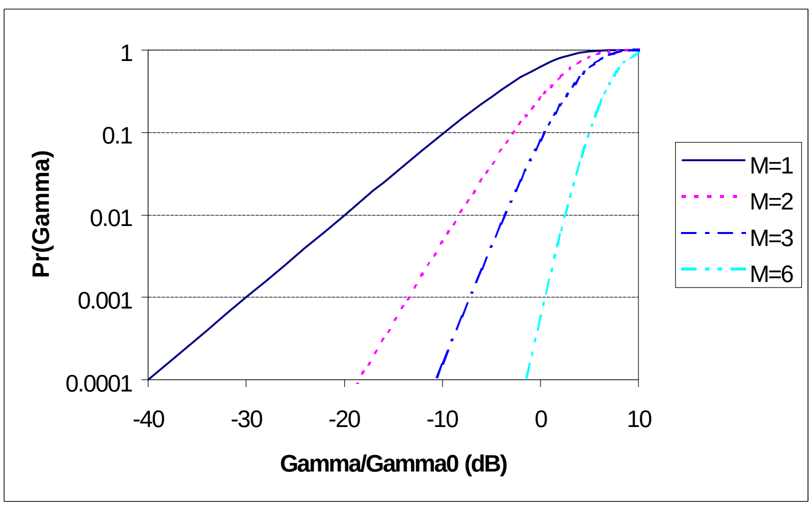

Figure 4. CDF of MRC

Note: Gamma/Gamma0 = γ/γ_0

4.2 Average SNR for MRC

The average SNR for MRC is relatively easy to compute by summing the average SNR of individual branch.

$$E[\sum_{i=1}^{M} g_i] = \sum_{i=1}^{M} E[g_i] = Mg_0 \qquad (20)$$

Here the mean SNR for MRC increases considerably with M as compared to the mean of SNR for selection diversity in equation 11.

4.3 Comparison Between Selection Diversity and MRC

Figure 5 graphically draws the two cases for selection diversity and MRC with M = 1 and 2 respectively. As we can see that the MRC is always better than diversity selection at the same M.

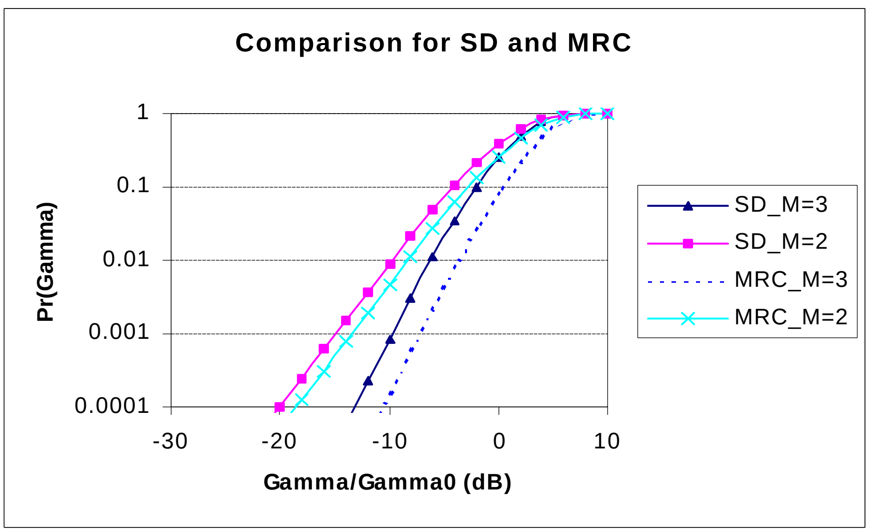

Figure 5. Comparison Between Selection Diversity and MRC.

Notes:

SD_M=3 for selection diversity with M=3
SD_M=2 for selection diversity with M=2
MRC_M=3 for maximum ratio combining with M=3
MRC_M=2 for maximum ratio combining with M=2

5. Equal Gain Combining (EGC)

5.1 CDF of SNR for EGC

Equal gain combining is a special case of MRC by setting all $g_i = 1$ for $i = 1, 2, 3, \ldots M$. This yields the following,

$$a = \sum_{i-1}^{M} a_i \tag{21}$$

$$\gamma = \frac{a^2 E_b}{MN_0} = \frac{E_b}{MN_0}\left(\sum_{i=1}^{M} a_i\right)^2 \tag{22}$$

In other words, SNR is the sum of sequence Rayleigh distribution. Although no close form is available for the CDF of γ when $M > 2$, it turns out that the performance of equal gain combining is better than the selection diversity, and worse than MRC.

5.2 Average SNR for Equal Gain Combining

$$E[g] = \frac{E_b}{MN_0} E\left[\left(\sum_{i=1}^{M} A_i\right)^2\right] = \frac{E_b}{MN_0} E\left[\sum_{i=1}^{M}\sum_{j=1}^{M} A_i A_j\right] \tag{23}$$

If branches are uncorrelated, and for the Rayleigh distribution with $E[A_i] = \sqrt{\frac{p}{2} s^2}$ and $E[A_i^2] = 2s^2$, the mean of the SNR for equal gain combining is obtained as

$$\begin{aligned} E[\gamma] &= \frac{E_b}{MN_0} E\left[\sum_{i=1}^{M}\sum_{j=1}^{M} A_i A_j\right] = \frac{E_b}{MN_0}\left[2M\sigma^2 + M(M-1)\frac{\pi\sigma^2}{2}\right] \\ &= \gamma_0\left[1 + (M-1)\frac{\pi}{4}\right] \end{aligned} \tag{24}$$

If M is large the mean of SNR can be further simplified as follows

$$E[g] = \frac{Mg_0 p}{4} \tag{25}$$

5.3 Comparison for Selection Diversity, MRC and Equal Gain Combining

Figure 6 presents the average SNR at different *M* for selection diversity, MRC and equal gain combining. As we can see MRC is the best and equal gain combining is always better than selection diversity, but worse than MRC.

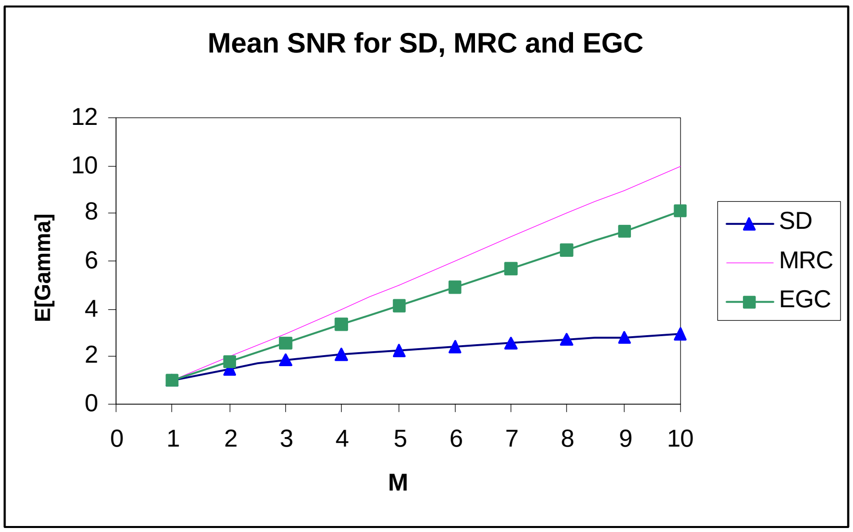

Figure 6. Comparison for Selection Diversity, MRC and Equal Gain Combining

Notes:

SD = selection diversity

MRC = maximum ratio combining

EQC = equal gain combining

6. Switched Diversity

6.1 CDF of SNR for Switched Diversity

One branch at a time is used and the strategy is to remain using the current branch until the signal envelope drops below the pre-determined threshold "*x*". The advantage of this strategy is to attack the "ping-pong" effect that occurs in selection diversity approach. There are two types of switched diversity.

1) Switch and stay---select other branch only above the threshold and stay till it drops to the threshold. (see Figure 7)

2) Switch and exam---select the branch only above the threshold. If both branches are below the threshold, but the current branch is higher that the other, this current branch will continue to be hold. (see Figure 7)

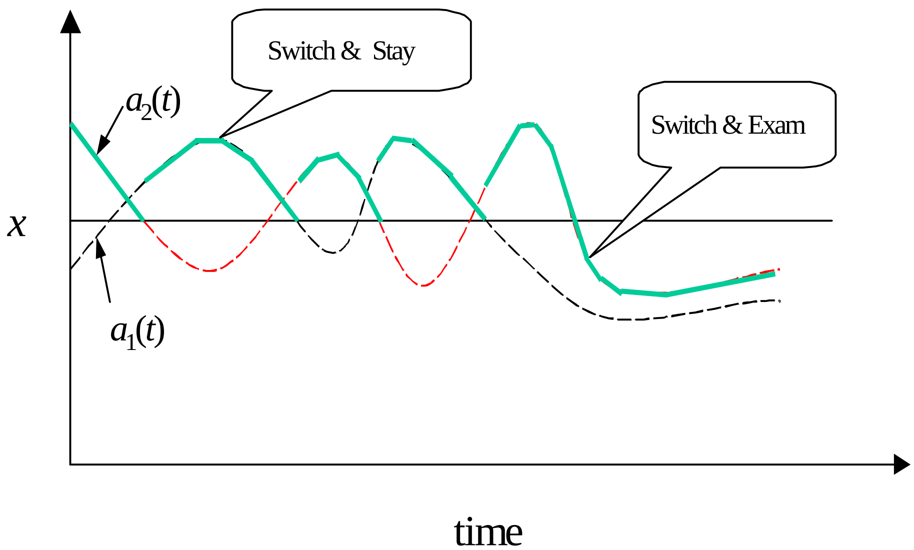

Figure 7. Switched Diversity Technique

Note that $a_1(t)$ and $a_2(t)$ are independent Rayleigh processes. We also assume that they have the identical mean and the same variance. Then the combined process $r(t)$ is corresponded to the discontinuous wide-dark line in Figure 7.

Let γ_x be SNR corresponding to level x. Then the probability that SNR of any one branch is below than γ_x is given by

$$q_x = P_r\{\gamma_{(a_1\,or\,a_2)} \leq \gamma_x\} = 1 - \exp(-\frac{\gamma_x}{\gamma_0}) \qquad (26)$$

Where

γ_0 = mean SNR

The SNR of combined signal has pdf

$$p_c(\gamma) = \begin{cases} (1-q_x)P_r(x), & \gamma \geq \gamma_x \\ q_x P_r(x), & \gamma < \gamma_x \end{cases} \qquad (27)$$

Where $P_r(\gamma)$ is exponentially distributed is each branch. Based on equation 27, the CDF can be obtained as follows,

$$P(\gamma) = \begin{cases} (1+q_x)\left[1-\exp\left(-\frac{\gamma}{\gamma_0}\right)\right] - q_x, & \gamma \geq \gamma_x \\ q_x\left[1-\exp\left(-\frac{\gamma}{\gamma_0}\right)\right] & \gamma < \gamma_x \end{cases} \qquad (28)$$

6.2 Comparison between Switched Diversity and Selection Diversity

Note that switched diversity is much worse than selection diversity. The tractability is that switched diversity reduces the complexity of system compared to selection diversity.

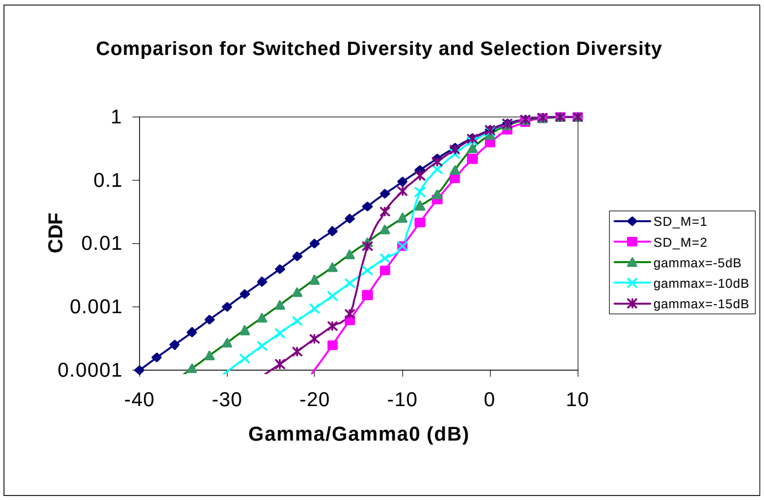

Figure 8. Comparison for Switched diversity and Selection Diversity

Notes:

SD_M=1 for selection Diversity with M=1

SD_M=2 for selection Diversity with M=2

Gamma/Gamma0 for γ/γ_0

gammax=-5dB for γ_x/γ_0 = -5dB

gammax=-10dB for γ_x/γ_0 = -10dB

gammax=-15dB for γ_x/γ_0 = -15dB

Advantages: Simplicity in implementation as only one receiver (frontend) is required in contrast with selection diversity where all channels must be monitored simultaneously.

Disadvantages: Discontinuous signals are obtained at switching instant, this implies that amplitude and phase may have upright transition.

7. Correlated Branches

Previous lecture explored the situation where branches are mutually independent. The envelopes of signals are Rayleigh distributed. However, the real situation does not exactly match with these assumptions. It is quite often that received signals on branches are correlated.

All the diversity approaches require the assumption that branches are independent. What if they are dependent with each other. This question will be addressed in following paragraphs.

Correlated Signals: For practical systems, it is difficult to achieve independent signals and branches. The limited frequency separation between frequency diversity and closely mounted antennas are major reasons for the correlated signals.

For $M > 2$, it is very difficult to analyze the correlation explicitly. Thus we only examine the case with $M = 2$.

7.1. For Maximum Ratio Combining

The CDF for SNR with correlation at the received terminal is given by

$$P(g) = 1 - \frac{1}{2r}\left[(1+r)\exp(-\frac{g}{g_0(1+r)}) - (1-r)\exp(-\frac{g}{g_0(1+r)})\right] \qquad (29)$$

Where ρ is a parameter that captures the normalized covariance of the two signal envelopes. ρ^2 = 1 means full correlated branches, i.e. no diversity. ρ^2 = 0 means no correlation. Equation 29 is graphically presented in Figure 9 together with selection diversity for M = 1 and M = 2. Note that, for M =1, it corresponds to ρ^2 = 1, and for M = 2, it is equivalent to ρ^2 = 0.

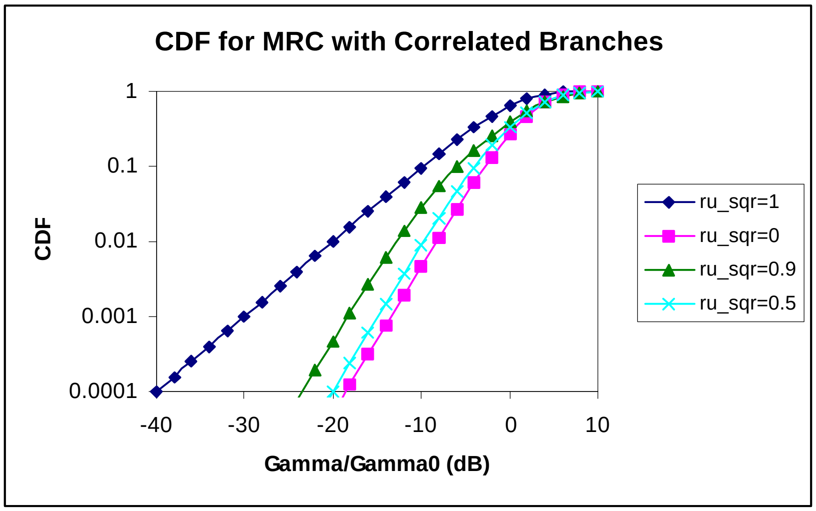

Figure 9. CDF of MRC with Correlated Branches for M = 2

Notes:

ru_sqr=1 for $\rho^2 = 1$

ru_sqr=0 for $\rho^2 = 0$

ru_sqr=0.9 for $\rho^2 = 0.9$

ru_sqr=0.5 for $\rho^2 = 0.5$

From Figure 9, even ρ^2 = 0.9, there are still substantial diversity gains obtained. This shows that diversity technique really improves the communication performance.

7.2. For Selection Diversity

$$P(\gamma) = 1 - \exp(-\frac{\gamma}{\gamma_0})[1 - Q(a,b) + Q(b,a)] \quad (30)$$

where

$$Q(a,b)=\int_b^{\infty} x\exp\left(-\frac{1}{2}(a^2+x^2)\right)I_0(ax)dx$$

$$b=\sqrt{\frac{2g}{g_0(1-r^2)}}$$

$$a=br$$

Even with high correlation, considerable gains are possible compare to no diversity.

7.3. Effect of Correlated Noise

The noise received in one branch may not be uncorrelated with noise in another branch. Selection and switched diversity do not have this effect because at each time instance, there is only one branch is used. However, maximum ratio combining and equal gain combining do suffer the correlated noise because they use multiple branches at any time instance.

Let us denote the noise signal in i^{th} branch as n_i. Without loss of generality, let us assume that $E[n_i]=0$ and $Var(n_i)=1$. Let $\rho_{ij} = E[n_i n_j] = \rho$ be the correlation between n_i and n_j for all i and j ($i \neq j$) The resulting noise process in the system with equal weight is

$$\begin{aligned} E[n^2] &= E\left[\left(\sum_{i=1}^{M} n_i\right)^2\right] \\ &= \sum_{i=1}^{M} E[n_i^2] + \sum_{i=1}^{M}\sum_{j=1, j\neq i}^{M} E[n_i n_j] \\ &= M[1+(M-1)\rho] \end{aligned} \quad (31)$$

For uncorrealted noise on M branches, we have

$$E[n^2]=M \quad (32)$$

The resulting noise power in a correlated system is increased by a factor of $1+(M-1)\rho$. This indicates that increasing the number of branches does not guarantee the improvement of received signal SNR.

8. Bit Error Performance of Diversity Scheme

8.1. MRC with *M* Branches

Bit error probability (BER) is obtained by evaluating probability of error conditioned on SNR. We will evaluate the BER for different modulation schemes such as BPSK, FSK, DPSK and non-coherent FSK.

8.1.1 BPSK

We assume detector makes decisions on the output signal from MRC. The resulting SNR is given by (assuming $g_i = a_i$ for all i = 1, 2, 3, …, M)

$$\gamma = \sum_{i=1}^{M} \gamma_i = \frac{E_b}{N_0} \sum_{i=1}^{M} a_i^2 \qquad (33)$$

Conditioned on $\{a_i\}$ or γ, the BER is given as,

$$P_e(\gamma) = Q(\sqrt{2\gamma}) \qquad (34)$$

Then

$$P_d = \int_0^{\infty} p_e(\gamma) p_{MRC}(\gamma) d\gamma \qquad (35)$$

Notice that

$$P_{MRC} = \frac{1}{(M-1)!} \frac{\gamma^{M-1}}{\gamma_0^M} \exp(-\frac{\gamma}{\gamma_0}) \qquad (36)$$

Substitute into equation

$$P_d = \int_0^{\infty} Q(\sqrt{2\gamma}) \frac{1}{(M-1)!} \frac{\gamma^{M-1}}{\gamma_0^M} \exp(-\frac{\gamma}{\gamma_0}) d\gamma$$

$$= \left(\frac{1-\mu}{2}\right)^M \sum_{i=0}^{M-1} \binom{M-1+i}{i} \left(\frac{1+\mu}{2}\right)^i \qquad (37)$$

Where

$$\mu = \sqrt{\frac{\gamma_0}{1+\gamma_0}}$$

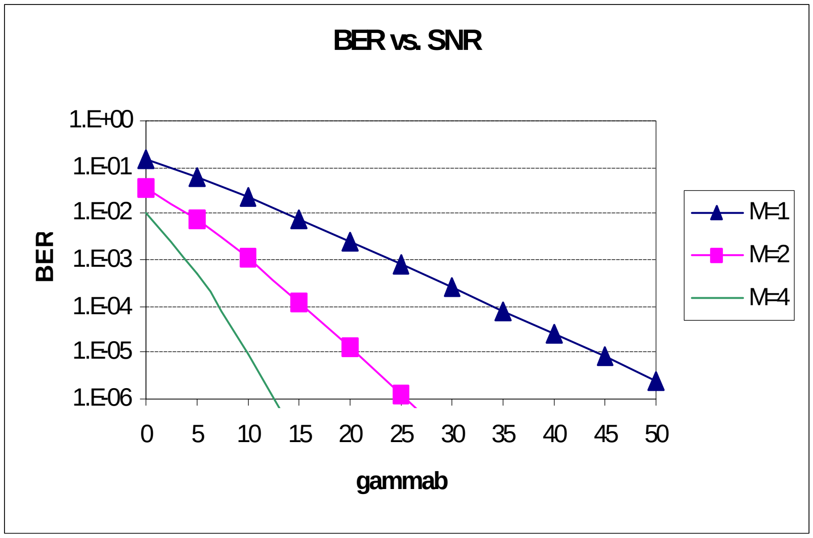

Figure 10. Bit Error Rate for BPSK with MRC Diversity

Note: Gammab = γ_b

For time and frequency diversity, $\gamma_b = M\gamma_0$. For space, angle and polarization diversity, $\gamma_b = \gamma_0$. This conclusion can be summarized in following table.

Table 1. Gain in dB when $P_d = 10^{-5}$

	Time and Frequency Diversity	Space, Polarization and Angle Diversity
M=2 over *M*=1	20 dB	29dB
M=4 over *M*=1	23dB	35dB

When the mean SNR, γ_0, is much larger than one, we have

$$\frac{1+\mu}{2} = 1 \tag{38.a}$$

and

$$\frac{1-\mu}{2} = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma}{1+\gamma_0}}\right)$$

$$= \frac{1}{2}\left(1 - \sqrt{1 - \frac{1}{\gamma_0 + 1}}\right)$$

$$\approx \frac{1}{4\gamma_0} \tag{38.b}$$

and

$$\sum_{i=0}^{M} \binom{M-1+i}{i} = \binom{2M-1}{M} \tag{39}$$

Thus for large SNR, equation 37 is approximated as

$$P_d = \binom{2M-1}{M}^M \left(\frac{1}{4\gamma_0}\right)^M \tag{40}$$

8.1.2 Binary FSK Modulation

The error probability FSK is given by

$$P_e(\gamma) = Q(\sqrt{\gamma}) \tag{41}$$

If we substitute equation 36 and 41 into equation 35, we have

$$P_d = \binom{2M-1}{M}^M \left(\frac{1}{2\gamma_0}\right)^M \tag{42}$$

8.1.3.Binary DPSK Modulation

Similarly we can obtain the error probability for DPSK as follows,

$$P_d = \binom{2M-1}{M}^M \left(\frac{1}{2\gamma_0}\right)^M \tag{43}$$

8.1.4. Binary for non-coherent Orthogonal FSK

We can also obtain the error probability for non-coherent FSK as follows,

$$P_d = \binom{2M-1}{M}^M \left(\frac{1}{\gamma_0}\right)^M \tag{44}$$

Therefor diversity really helps to improve the performance of fading channels. From the graph, we find that when M=2, the non-coherent FSK has no obvious disadvantage over coherent FSK.

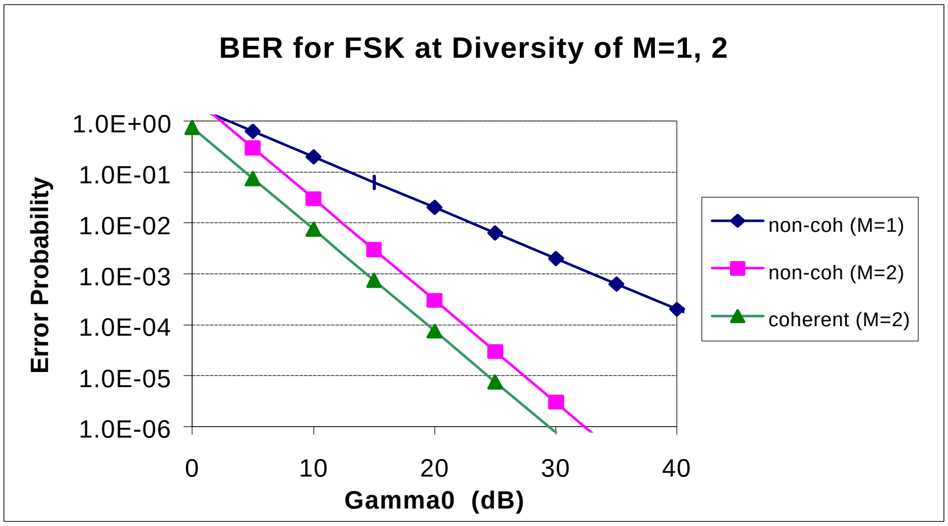

Figure 11. Bit Error Rate for BFSK with MRC Diversity

Note: Gamma0 = γ_0

8.2. Selection Diversity

The probability error for the selection diversity can be computed in the similar manner as the MRC.

$$P_d = \int_0^\infty P_e(\gamma) p_{SD}(\gamma) d\gamma \tag{45}$$

For binary non-coherent FSK, the error probability is

$$P_e = \frac{1}{2}\exp(-\frac{\gamma}{2}) \tag{46}$$

and

$$p_{SD}(\gamma) = \frac{M}{\gamma_0}\exp(-\frac{\gamma}{\gamma_0})\left(1-\exp(-\frac{\gamma}{\gamma_0})\right)^{M-1} \tag{47}$$

By substituting equations 45 and 46 into equation 47, we can obtain the bit error probability expression as follows

$$P_d = M\sum_{i=0}^{M-1}(-1)^i \binom{M-1}{i} \frac{1}{\gamma_0 + 2 + 2i} \qquad (48)$$

9. Macro Diversity (Base Station Diversity)

Diversity that is used to combat the effects of envelope fading is called microscopic diversity. Another type of diversity that can be used in mobile systems is macroscopic diversity. This is used to mitigate the shadowing fading. In macroscopic diversity, the mobile terminal receives multiple signals from two or more base stations that are geographically separated. A diversity advantage is achieved when signals experience some degrees of uncorrelated shadowing. It is quite safe to assume that signals on the different macroscopic diversity branches will suffer independent envelope fading.

Let S_k be received signal from k^{th} base station. All received signals are averaged and thus the Rayleigh fading effect is gone. The shadow fading is the major concern in the macro diversity.

Note that S_k is log-normally distributed with mean $E[S_k]=m_s$ (dB) and $Var(S_k)=\sigma^2$ (dB^2). Then the pdf is given as

$$p(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\ -\frac{1}{2\sigma^2}(\log_{10} x - m_s)^2 \qquad (49)$$

For macro diversity, the probability of average signal less the threshold is defined as

$$P_r\{S \le \gamma\} = \sum_{k=1}^{M} P_r\{S_k \le \gamma\} \qquad (50)$$

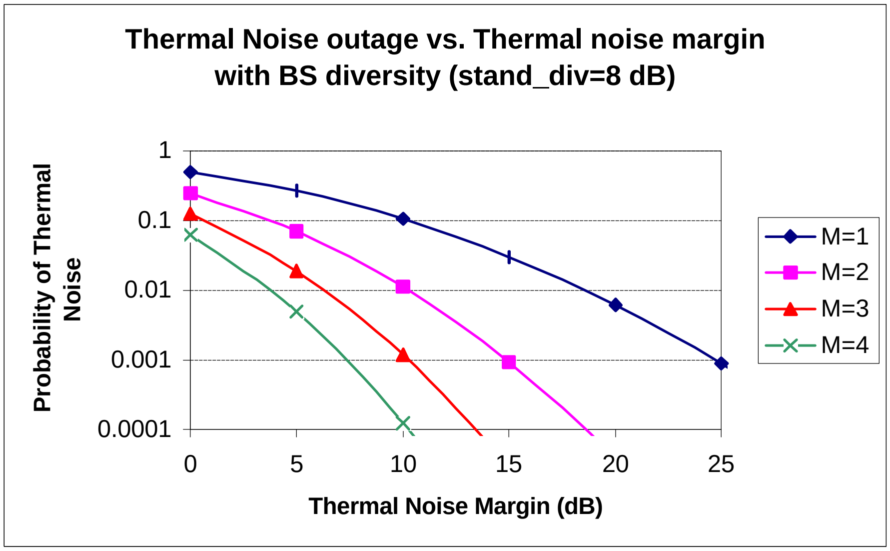

Figure 12. Macro Diversity

10. Summary

Microscopic diversity and macroscopic diversity are two basic diversities that are used in wireless communication to combat the fading. Microscopic diversity is related to the mobile terminal, while macroscopic diversity is implemented on base stations. Both diversity techniques will enhance the SNR in a considerable degree.

References

1. Narayan B. Mandayam, Wireless Communication Technologies, (Class Notes), Spring Semester, 2000. (Notes are obtained mainly based on this).

2. Gordon L Stuber, Principles of Mobile Communications, Kluwer Academic Publishers, 1996

3. John G. Proakis, Digital Communications, 3rd Edition, McGraw-Hill Inc., 1996