Linear Multiuser Detector

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The large gaps in performance and complexity between the conventional single-user matched filter (MF) and the optimum multiuser detector motivate the search for other multiuser detectors that exhibit good performance/complexity tradeoffs [2]. This lecture examines various linear multiuser detectors. They can be implemented in a decentralized fashion where only users of interest need be demodulated, like the decorrelating detector which is optimal when the received amplitudes are completely unknown. The performance could be further improved if the information about the received signal-to-noise ratios is incorporable in the linear transformation. The lecture notes include the study of linear multiuser detectors both by decorrelating and non-decorrelating approaches, according to different criteria. The successive interference cancellation technique is introduced at the end.

I. Decorrelating Detector [3]

A. In the synchronous channel

Consider the output of the bank of K matched filters

$$\mathbf{y} = \mathbf{RAb} + \mathbf{n}, \tag{1}$$

where $\mathbf{n}$ is a Gaussian random vector with zero mean and covariance matrix $\sigma^2\mathbf{R}$. If we process the output vector as

$$\mathbf{R}^{-1}\mathbf{y} = \mathbf{Ab} + \mathbf{R}^{-1}\mathbf{n}, \tag{2}$$

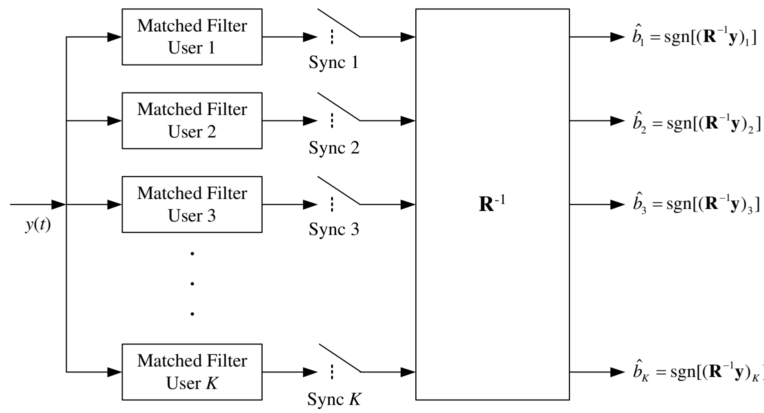

Fig. 1. Decorrelating detector for the synchronous channel.

clearly the kth component of vector $\mathbf{R}^{-1}\mathbf{y}$ is free from interference caused by any other users for any k (since $\mathbf{A}$ is diagonal). Note that the crosscorrelation matrix $\mathbf{R}$ is invertible if signature sequences are linear independent. If the background noise is vanishing, that is, $\sigma = 0$, then

$$\hat{b}_k = \text{sgn}((\mathbf{R}^{-1}\mathbf{y})_k) = \text{sgn}((\mathbf{A}\mathbf{b})_k) = b_k. \tag{3}$$

Hence, in absence of background noise, we get error free performance. In the presence of the background noise, decision is affected only by the background noise, that is,

$$\hat{b}_k = \text{sgn}((\mathbf{R}^{-1}\mathbf{y})_k) = \text{sgn}((\mathbf{A}\mathbf{b} + \mathbf{R}^{-1}\mathbf{n})_k). \tag{4}$$

This is why the detector is called the *decorrelating detector*.

From the implementation point of view, two desirable features of this multiuser detector are:

- It does not need for knowledge of the received amplitudes.
- It can be decentralized in the sense that the demodulation of each user is done separately.

To see the second property, denote $R^+_{kj} = (\mathbf{R}^{-1})_{kj}$. Note that the kth output of the linear transformation $\mathbf{R^{-1}}$ is

$$\begin{aligned}(\mathbf{R^{-1}y})_k &= \sum_{j=1}^{K} R_{kj}^{+} y_j \\ &= \sum_{j=1}^{K} R_{kj}^{+} \langle y, s_j \rangle \\ &= \langle y, \sum_{j=1}^{K} R_{kj}^{+} s_j \rangle \\ &= \langle y, \tilde{s}_k \rangle, \end{aligned} \tag{5}$$

where $\tilde{s}_k(t) = \sum_{j=1}^{K} R_{kj}^{+} s_j(t)$. Hence, the decorrelation for the kth user can be implemented as $\text{sgn}\{\int_0^T y(t)\tilde{s}_k(t)dt\}$, a modified match filter, where $y(t) = \sum_{j=1}^{K} A_k b_k S_k(t) + \sigma_n(t)$.

B. Performance analysis for the synchronous case

In presence of the background noise, the decorrelation enhances the background noise (while suppressing the interference). Referring to the output of the modified matched filter (2), we see that the output matched to $\tilde{s}_k$ only has two components: $A_k b_k$ and the background noise which is Gaussian with zero mean and variance equal to the kk component of the covariance matrix

$$\begin{aligned} E[(\mathbf{R^{-1}n})(\mathbf{R^{-1}n})^T] &= \mathbf{R^{-1}} E[\mathbf{nn^T}]\mathbf{R^{-1}} \\ &= \mathbf{R^{-1}} \sigma^2 \mathbf{R} \mathbf{R^{-1}} \\ &= \sigma^2 \mathbf{R^{-1}}. \end{aligned} \tag{6}$$

That is, $(\sigma^2 \mathbf{R^{-1}})_{kk} = \sigma^2 R_{kk}^{+}$.

Consequently, the kth user's bit-error-rate (BER) is

$$P_k^d(\sigma) = Q\left(\frac{A_k}{\sigma\sqrt{R_{kk}^{+}}}\right). \tag{7}$$

In general, $R_{kk}^{+} > 1$, so the noise is enhanced. $R_{kk}^{+} = 1$ if and only if the kth user is orthogonal to the other users. Then, the decorrelator coincides with the single-user matched filter.

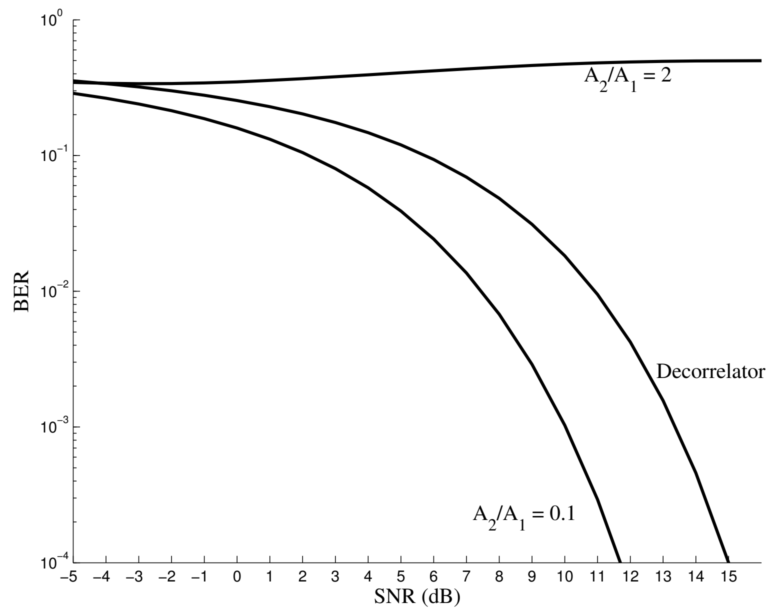

Fig. 2. BER comparison of decorrelator and single-user matched filter with two users and $\rho = 0.75$.

For a 2-user system, we have $R^{+}_{kk} = (1-\rho^2)^{-1}$ and

$$P_k^d(\sigma) = Q\left(\frac{A_k\sqrt{1-\rho^2}}{\sigma}\right), \quad k = 1, 2. \tag{8}$$

Given the error probability of the single-user matched filter detector

$$P_1^c(\sigma) = \frac{1}{2}Q\left(\frac{A_1 - A_2\rho}{\sigma}\right) + \frac{1}{2}Q\left(\frac{A_1 + A_2\rho}{\sigma}\right), \tag{9}$$

the performance of two detectors is compared in Fig. 2 for the two-user case. The figure shows that the error probability of the decorrelating detector is independent of A_2, whereas (9) takes values in the range

$$Q\left(\frac{A_1}{\sigma}\right) \leq P_1^c(\sigma) \leq \frac{1}{2}. \tag{10}$$

Therefore, if the interfering amplitude is small enough, the conventional matched filter detector is preferable to the decorrelator, because the effective noise has variance σ^2 for the former but variance $\sigma^2/(1-\rho^2)$ for the latter. Thus, the price paid for the complete elimination of multiaccess interference is noise enhancement.

How is the decorrelator optimum (if in any sense)? Since (7) is the BER of a single user channel with SNR equal to

$$\frac{A_k^2}{\sigma^2 R_{kk}^+}, \tag{11}$$

the multiuser efficiency is equal to $\eta_k^d = 1/R_{kk}^+$, which does not depend on either the noise level or the interfering amplitudes, and thus it is equal to the asymptotic multiuser efficiency and to the near-far resistance

$$\bar{\eta}_k^d = \frac{1}{R_{kk}^+}. \tag{12}$$

We see that the decorrelating detector achieves the maximum near-far resistance.

The decorrelating detector can be obtained as the solution to various optimization problems. Consider the optimum detector when the detector knows nothing about the amplitude. Then the problem is a joint maximum likelihood (ML) estimation of amplitude and transmitted bits. Since the noise is AWGN, the most likely bits and amplitude are those that best explain the received waveform in a mean-square sense, that is, the arguments that achieve

$$\min_{\mathbf{b}\in\{-1,1\}^K} \min_{\substack{A_k\geq 0\\ k=1,\ldots,K}} \int_0^T \left[y(t) - \sum_{k=1}^{K} A_k b_k s_k(t) \right]^2 dt. \tag{13}$$

Let $c_k = A_k b_k$, then the above is equivalent to

$$\max_{\mathbf{c}\in R^K} 2\mathbf{c}^T\mathbf{y} - \mathbf{c}^T\mathbf{R}\mathbf{c}. \tag{14}$$

Taking the gradient with respect to $\mathbf{c}$ and set to zero, we have

$$\mathbf{y} = \mathbf{R}\mathbf{c}, \tag{15}$$

so the maximum of (14) is achieved by $\mathbf{c}^* = \mathbf{R}^{-1}\mathbf{y}$. Now, the arguments that minimize (13) are

$$\mathbf{b}^* = \mathrm{sgn}(\mathbf{c}^*),$$

$$\hat{b}_k = \mathrm{sgn}(c_k^*) = \mathrm{sgn}((\mathbf{R}^{-1}\mathbf{y})_k),$$

and

$$\hat{A}_k = |c_k^*|.$$

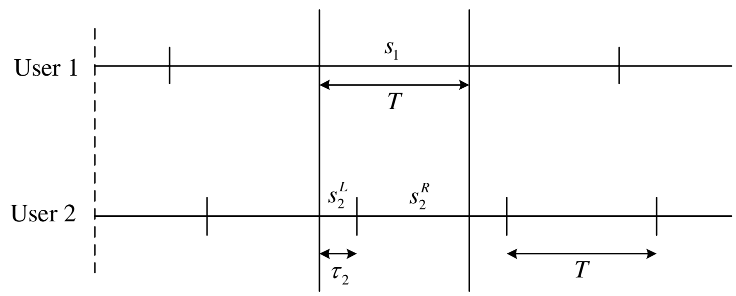

Fig. 3. Asynchronous two-user channel.

Therefore, the decorrelating detector is a ML detector when received amplitudes are unknown.

C. In the asynchronous channel

Consider a two-user case as depicted in Fig. 3, the demodulation of user 1 in asynchronous case is being interfered with by effectively 2 interferers, which have unit-energy signature waveforms:

$$s_2^L(t) = \begin{cases} \frac{1}{\sqrt{\theta_2}} s_2(t+T-\tau_2), & \text{if } 0 \leq t \leq \tau_2; \\ 0, & \text{if } \tau_2 < t \leq T, \end{cases} \tag{16}$$

$$s_2^R(t) = \begin{cases} 0, & \text{if } 0 \leq t \leq \tau_2; \\ \frac{1}{\sqrt{1-\theta_2}} s_2(t-\tau_2), & \text{if } \tau_2 < t \leq T, \end{cases} \tag{17}$$

where θ_2 is the partial energy of the interfering signal over the left overlapping interval. Hence, we have a "three-user synchronous" system, where user 2 modulates s_2^L and user 3 modulates s_2^R. The crosscorrelation matrix of this "truncated window" detector is

$$\mathbf{R} = \begin{bmatrix} 1 & \rho_{21}/\sqrt{\theta_2} & \rho_{12}/\sqrt{1-\theta_2} \\ \rho_{21}/\sqrt{\theta_2} & 1 & 0 \\ \rho_{12}/\sqrt{1-\theta_2} & 0 & 1 \end{bmatrix}. \tag{18}$$

The strategy adopted in obtaining the above decorrelating detector can be generalized (in order to improve performance) so as to extend the observation interval beyond $[0, T]$ while keeping a sliding observation window that spans several symbol length. Complexity increases with the number of fictitious interferers included in the window, and performance also improves up to a point where it is distinguishable from that of the ideal asynchronous decorrelator that observes the whole transmitted sequence.

Since the BER of the decorrelating detector is independent of the amplitude of the interferers, the power-tradeoff region (permissible region of SNR so that the BER of all users does not exceed P) is always a quadrant as shown in Fig. 4 and 5. It is observed that the two-user optimum detector offers marginal gains with respect to the decorrelating detector when both amplitudes are equal. [htb]

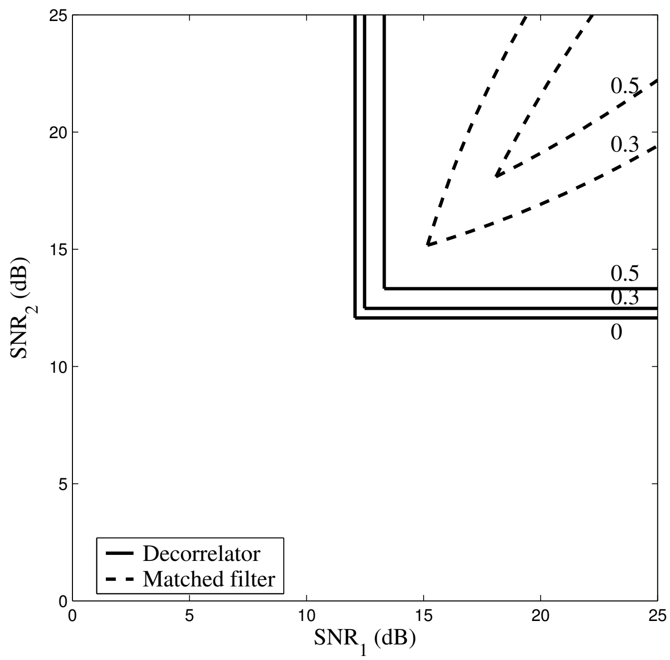

Fig. 4. SNR necessary to achieve BER $\leq 3\times 10^{-5}$ for both user. Shown for $|\rho| = 0, 0.3, 0.5$, and compared with the single-user matched filter detector regions.

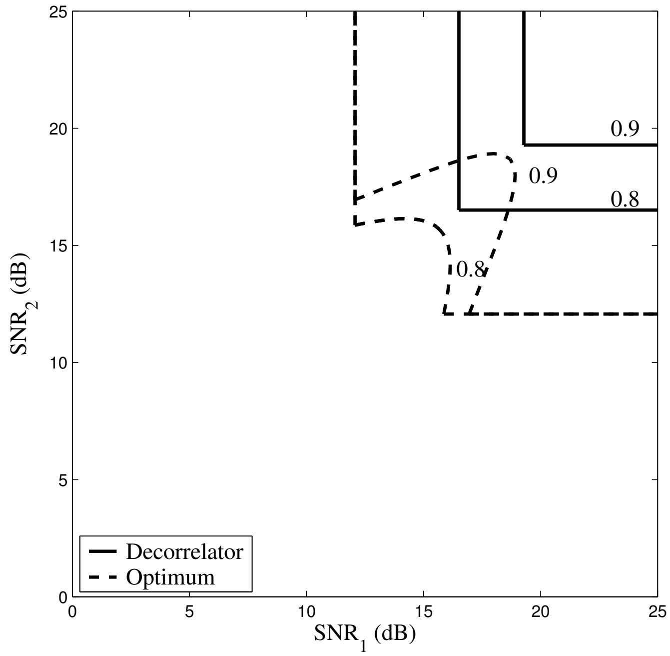

Fig. 5. SNR necessary to achieve BER $\leq 3 \times 10^{-5}$ for both user. Shown for $|\rho| = 0.8, 0.9$, and compared with the optimal regions.

II. Approximate Decorrelator [4]

If the normalized crosscorrelation matrix $\mathbf{R}$ is such that crosscorrelation among all signature waveforms is very small, then $\mathbf{R}$ is strongly diagonal. Hence, the inverse matrix

$$\mathbf{R}^{-1} = (\mathbf{I} + \delta\mathbf{M})^{-1} = \mathbf{I} - \delta\mathbf{M} + o(\delta). \tag{19}$$

The result is that, for the kth user, the approximation results in a modified matched filter

$$\tilde{s}_k(t) \approx s_k(t) - \sum_{j \neq k} \rho_{kj} s_j(t) \tag{20}$$

in the synchronous case and

$$\tilde{s}_k(t) \approx s_k(t) - \sum_{j \neq k} \rho_{jk} s_j(t - \tau_j) - \sum_{j \neq k} \rho_{kj} s_j(t - \tau_j + T) \tag{21}$$

in the asynchronous case.

Whenever the crosscorrelation are not known in advance and the detector coefficients have to be computed on-line, the approximation in (20) has the advantage that it does not need any processing of the crosscorrelations supplied by the crosscorrelations of the replicas of the signature waveforms. The reduced complexity of the approximate decorrelator and performance gains over the conventional matched filter makes it a viable alternative for implementation in practical CDMA systems, in particular in those where the signature waveforms span many symbol intervals. The near-far resistance of the approximate decorrelator is zero, but its BER is much superior to that of the conventional matched filter. In fact, as long as the load factor $K/N < 1/3$ in a K-user CDMA system, the BER performance of the approximate decorrelator is better than that of the conventional matched filter. A typical load factor in cellular systems is $1/5$.

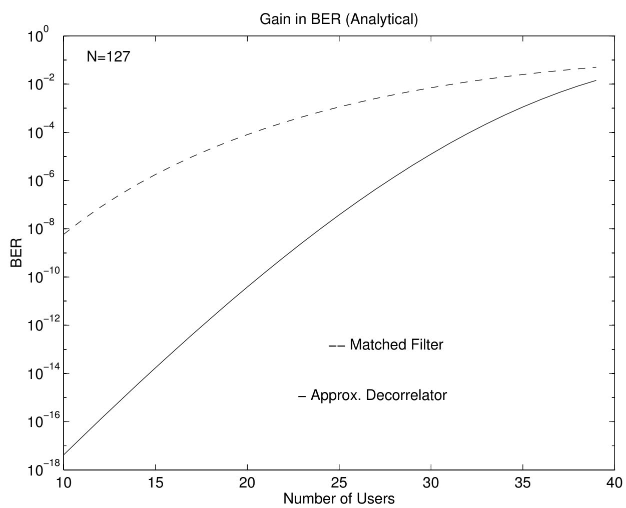

Fig. 6. BER gain $P_k = P_1 = P$, and $P/\sigma_N = 20$ dB.

The gain in BER offered by the approximated decorrelating detector over the matched filter detector is illustrated in Fig. 6. It shows that the approximate decorrelator accommodates 10 – 15 users more than the conventional detector in a system with processing gain equal to 127. In Fig. 7, the number of users that can be supported in the system with a BER $= 10^{-3}$, is shown for both the matched filter receiver and the approximate decorrelator as a function of SNR. It is seen that the approximated decorrelator supports more than twice the number of users that a matched filter receiver can support for the same SNR level. The robustness of the approximate

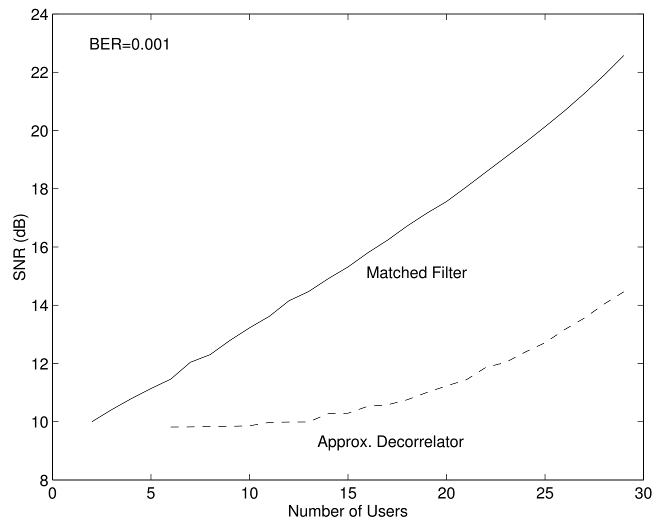

Fig. 7. Required SNR (perfect power control) $N = 127$, BER= 0.001.

decorrelator with respect to imperfections in power control is illustrated in Fig. 8. The power trade-off regions are shown for both the matched filter and the approximate decorrelator for a fixed desired BER. A system with $K = 30$ users is considered. The power trade-off curves are plotted in terms of the SNR's required for user 1 and user 2 so that the users in the system achieve a BER no greater than 10^{-3}. It is seen that the approximate decorrelating detector is tolerant to a wider range of imperfections in power control than the matched filter detector which is sensitive to even slight imbalances in the respective powers of the users in the system.

III. Optimum Linear Multiuser Detectors

A. Maximum Asymptotic Efficiency (MAE)

Consider $\mathbf{y} = \mathbf{R}\mathbf{A}\mathbf{b} + \mathbf{n}$ being the usual normalized vector of MF outputs. Let $\mathbf{V}_k$ be the linear transformation of user k. The decision rule is

$$\hat{b}_k = \mathrm{sgn}(\mathbf{V}_k^T \mathbf{y}), \tag{22}$$

$$\mathbf{V}_k^T \mathbf{y} = \mathbf{V}_k^T (\mathbf{R}\mathbf{A}\mathbf{b} + \mathbf{n}). \tag{23}$$

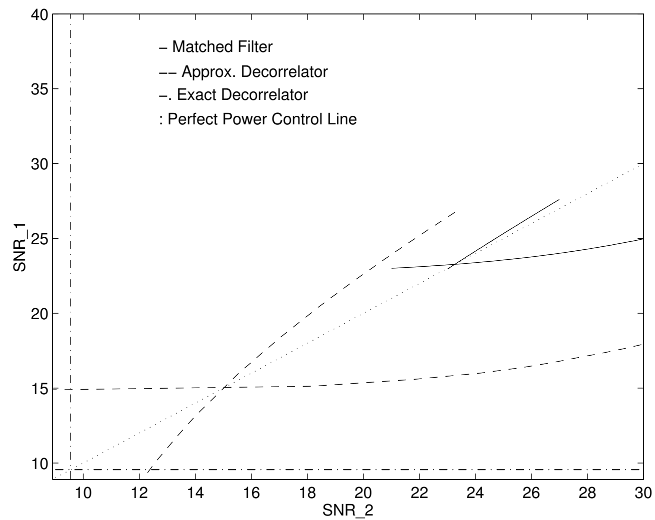

Fig. 8. Power trade-off regions $N = 127$, $K = 30$ and BER≤ 0.001.

Then,

$$\mathbf{V}_k^T \mathbf{y} = \sum_{j=1}^{K} (A_j b_j \mathbf{v}_k^T \mathbf{r}_j) + \mathbf{v}_k^T \mathbf{n}, \tag{24}$$

where $\mathbf{r}_j$ is the jth column of normalized cross-correlation matrix $\mathbf{R}$. The asymptotic efficiency of the kth user is

$$n_k(\mathbf{v}_k) = \frac{1}{\mathbf{v}_k^T \mathbf{R} \mathbf{v}_k} \max{}^2 \left\{ 0, \mathbf{v}_k^T \mathbf{r}_k - \sum_{j \neq k} \frac{A_j}{A_k} |\mathbf{v}_k^T \mathbf{r}_j| \right\}. \tag{25}$$

The receiver is optimized (25) with respect to $\mathbf{v}_k$. There is no closed form solution and it is an non-linear optimization problem. Lupus and Verdu suggested an algorithm to solve it [5].

For the two user case $k = 2$, without loss of generality, let $\mathbf{v}_1 = [1 \quad x]^T$, we have

$$n_1(\mathbf{v}_1) = \max{}^2 \left\{ 0, f\left(x, \rho, \frac{A_2}{A_1}\right) \right\} \tag{26}$$

with

$$f\left(x, \rho, \frac{A_2}{A_1}\right) = \frac{1 + x\rho - \frac{A_2}{A_1}|x + \rho|}{\sqrt{1 + 2\rho x + x^2}} \tag{27}$$

The argument x^* that maximizes $f(\cdot)$ is given by

$$x^* = \begin{cases} -\frac{A_2}{A_1}\mathrm{sgn}\rho, & \mathrm{if} A_2/A_1 < |\rho|, \\ -\rho, & \mathrm{otherwise} \end{cases} \tag{28}$$

The result says that if the interferer is strong enough, i.e. $A_2 \geq A_1|\rho|$, then the decorrelator maximizes asymptotic efficiency. Otherwise, the received signal is correlated with

$$s_1(t) - \frac{A_2}{A_1}\mathrm{sgn}(\rho)s_2(t) \tag{29}$$

Therefore, the maximum asymptotic efficiency linear detector is a compromise between the decorrelator and the single-user matched filter, which approaches the latter as the relative power of the interferer decreases. Fig. 9 is the graph depicting asymptotic efficiency for two synchronous users versus the ratio A_2/A_1. We can see that, when the background noise is small relative to the signal strength, if $A_2/A_1 \geq |\rho|$, the optimum linear detector achieves much better asymptotic efficiency than the conventional detect; if $A_2/A_1 < |\rho|$, the optimum linear and non-linear detectors have the same performance.

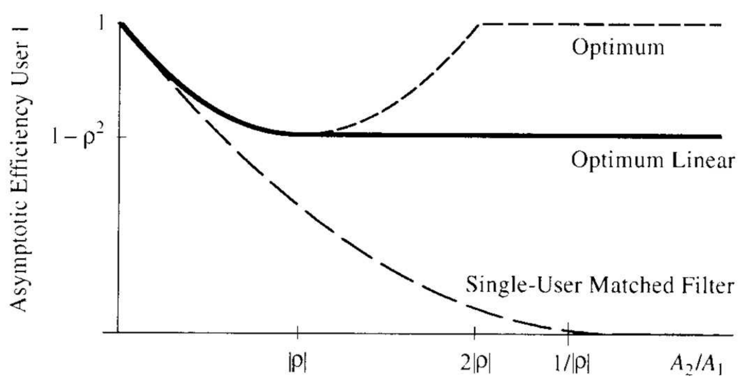

Fig. 9. Asymptotic multiuser efficiencies for two synchronous users.

B. Minimum Probability of Error Receiver

$$\min_{\mathbf{v}_k} P_k^{\mathbf{v}_k} = \min_{\mathbf{v}_k} E\left[Q\left(\frac{A_k \mathbf{v}_k^T \mathbf{r}_k + \sum_{j\neq k} A_j b_j \mathbf{v}_k^T \mathbf{r}_j}{\sigma\sqrt{\mathbf{v}_k^T \mathbf{R} \mathbf{v}_k}}\right)\right] \tag{30}$$

where the expectation is with respect to b_j for $j \neq k$. A stochastic gradient algorithm was proposed in [6] and shown to converge almost surely when eye is open, that is, the near-far resistance is strictly positive.

C. Minimum Mean Square Error (MMSE) Receiver

The approach here is to turn linear multi-user detection problem into a linear estimation problem.

Idea: Require MSE between the kth user bit b_k and the output of the linear transformation $\mathbf{m}_k^T \mathbf{y}$ to be minimized [7]. This approach does not minimize the bit error rate $\min P[b_k \neq \mathrm{sgn}(\mathbf{m}_k^T \mathbf{y})]$, but still it is a very sensible criterion especially when used in conjunction with soft decisions.

For the kth user solve

$$\min_{\mathbf{m}_k} [E(b_k - \mathbf{m}_k^T \mathbf{y})^2], \quad k = 1, \ldots, K, \quad \mathbf{m}_k \in R^K \tag{31}$$

Combining the K equations

$$\min_{\mathbf{M}\in R^{K\times K}} E[||\mathbf{b} - \mathbf{M}\mathbf{y}||^2] \tag{32}$$

where the expectation is with respect to bits and noise.

Since $||\mathbf{x}||^2 = \mathrm{trace}\{\mathbf{x}\mathbf{x}^T\}$, the above problem is equivalently represented by

$$\min_{\mathbf{M}\in R^{K\times K}} \mathrm{trace}\{E[(\mathbf{b} - \mathbf{M}\mathbf{y})(\mathbf{b} - \mathbf{M}\mathbf{y})^T]\} = \min_{\mathbf{M}\in R^{K\times K}} \mathrm{trace}\{\mathrm{Cov}(\mathbf{b} - \mathbf{M}\mathbf{y})\}. \tag{33}$$

It was shown that $\mathbf{M} = \mathbf{A}^{-1}[\mathbf{R} + \sigma^2 \mathbf{A}^{-2}]^{-1}$ is the optimum transformation and hence the MMSE linear detector output decision is

$$\begin{aligned} \hat{b}_k &= \mathrm{sgn}\left(\frac{1}{A_k}([\mathbf{R} + \sigma^2 \mathbf{A}^{-2}]\mathbf{y})_k\right) \\ &= \mathrm{sgn}\left(([\mathbf{R} + \sigma^2 \mathbf{A}^{-2}]\mathbf{y})_k\right) \end{aligned} \tag{34}$$

where the scaling factor A^{-1} can be dropped without affecting the decision rule. Hence the linear transformation is

$$\mathbf{L} = [\mathbf{R} + \sigma^2 \mathbf{A}^{-2}]^{-1}. \tag{35}$$

MMSE detector is a compromise between the conventional receiver (optimizes to fight only background noise) and the decorrelator (optimizes to fight only interference). It takes into account both the interfering user and the background noise.

In the limiting case, $A_2, A_3, \ldots, A_k \to 0$ with A_1 being fixed, and then the first row of

$$[\mathbf{R} + \sigma^2 \mathbf{A}^{-2}]^{-1} \to \left[\frac{A_1^2}{A_1^2 + \sigma^2}, 0, \ldots, 0\right], \tag{36}$$

which is the same as a conventional receiver (matched-filter) for user 1. As $\sigma \to 0$,

$$[\mathbf{R} + \sigma^2 \mathbf{A}^{-2}]^{-1} \to \mathbf{R}^{-1}.$$

Therefore, the MMSE linear detector converges to the decorrelating detector. That means the MMSE detector and the decorrelating detector have the same asymptotic efficiency and near-far resistance.

IV. Successive Interference Cancellation

Idea: If the decision has been made about an interfering user's bit, then that interfering signal can be recreated at the receiver and subtracted from the received waveform. If the decision on interferer's bit is correct, this perfectly cancels out the interference; if not, it doubles the contribution of interference (interference is propagated). The optimistic view is that the resulting signal contains one fewer user and hence the process can be repeated. The receiver uses the decisions produced by single user matched filter to do successive cancellation [8]. The order in which users are demodulated affects performance. If we simply do it based on received power of the users, we are ignoring about the crosscorrelations. Instead, users are ordered based on matched filter outputs:

$$E\left[\left(\int_0^T y(t) s_k(t) dt\right)^2\right] = \sigma^2 + A_k^2 + \sum_{j \neq k} A_j^2 \rho_{jk}^2 \tag{37}$$

Let us consider the synchronous two-user case and assume user 2 is demodulated first

$$\hat{b}_2 = \text{sgn}\left(\int_0^T y(t)s_k(t)dt\right)^2 = \text{sgn}(y_2) \tag{38}$$

Re-modulating user 2 signal with $\hat{b}_2$, we get $A_2\hat{b}_2s_2(t)$ and subtracting it from $y(t)$ yields

$$\hat{y}(t) = y(t) - A_2\hat{b}_2s_2(t) \tag{39}$$

and then

$$\hat{y}(t) = A_1b_1s_1(t) + A_2(b_2 - \hat{b}_2)s_1(t) + n(t). \tag{40}$$

Processing $\hat{y}(t)$ with matched filter for s_1 gives

$$\begin{aligned}
\hat{b}_1 &= \text{sgn}(<\hat{y}_1, s_1>) \\
&= \text{sgn}(y_1 - A_2\hat{b}_2\rho) \\
&= \text{sgn}(y_1 - A_2\rho\text{sgn}(y_2)) \\
&= \text{sgn}(A_1b_1 + A_2(b_2 - \hat{b}_2)\rho + \sigma <n, s_1>)
\end{aligned} \tag{41}$$

General expression for a K-user system,

$$\hat{b}_k = \text{sgn}(y_k - \sum_{j\neq k} A_j\rho_{jk}\hat{b}_j). \tag{42}$$

The power trade-off region is illustrated in Fig. 10. Apparently the power tradeoff regions are asymmetric but depend on the order of demodulations. It is worth noting that equal power allocation to users is not preferred. For example, look at the performance when $\rho = 0.5$. If the received powers are equal, then $\text{SNR}_1 = \text{SNR}_2 \approx 19$ dB. Otherwise, $\text{SNR}_1 = 12$ dB and $\text{SNR}_2 = 15$ dB. In reality this receiver requires highly accurate estimates of amplitude and delay.

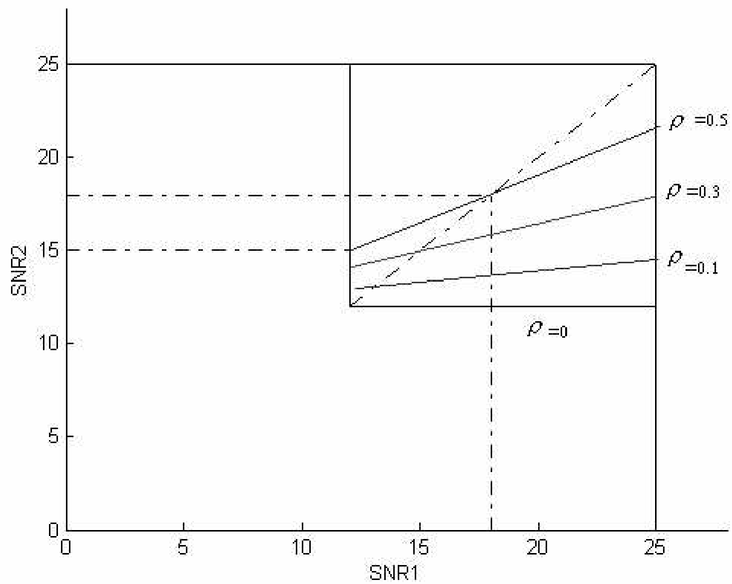

Fig. 10. SNRs to achieve BER$\leq 3 \times 10^{-5}$.

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Radio Resource Management in Wireless Communications

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The goal of a cellular communication provider is to maximize the number of users in the service area by providing acceptable QoS at affordable price. So as to achieve the above goal the service providers deploy countable base stations (as low as possible) in a region for fixed QoS. As the signals are transmitted in air it is very important to study about the power allocation (to reduce interference), channel allocation (to reuse the available frequency), to manage hand-off as the mobile user moves, etc..

In the typical hierarchy of a cellular system, Mobile Switching centers (MSC) are connected to base stations by means of wired link and the base stations are connected to mobile stations by means of radio link.

In this paper we give an overview of subjects related to radio resource management in wireless systems.

Terminology

Service area: Geographical area where the service providers wish to provide mobile users with communication access.

Coverage area: It is the region around a Radio Access Point (RAP) or Base Station (BS) where transmission conditions are favorable enough to maintain a connection of required quality.

Coverage area heavily depends on propagation conditions and current interference caused by other users in the coverage area. The shape of the coverage area is not fixed or is not hexagonal; it changes with the number of users in the coverage area.

Uplink: Transmission of signals from mobile station to RAP or BS.
Downlink: Transmission of signals from RAP or BS to mobile station.

In Frequency Division Duplex (FDD) the propagation conditions and the interference are different for both Uplink & Downlink, where as in Time Division Duplex (TDD) they are close in both the directions.

Range limited systems: Systems where range of a RAP is smaller than inter RAP distance.

Bandwidth or interference-limited systems: Systems where number of transmitters is large compared to available bandwidth.

Radio Resource Management (RRM): Radio resource management is the study of interference-limited systems where the number of simultaneous connections is larger than the number of orthogonal signals that the available bandwidth may produce. This involves the following issues:

1) Transmit Power Control: Implementing power control algorithms in downlink to limit attenuation of transmitted signal and interference seen by other users.
2) Frequency/ Channel Allocation: Selection of frequency or channel for transmission.
3) BS Assignment: Assigning BS or RAP based on mobile user location.

Reuse distance and its impact on capacity: -
Consider two transmitters s_1 and s_2 separated by a distance $d = D_{12} + r_2$ as shown in the figure1. M_1 & M_2 are intended receivers for s_1 and s_2 respectively. The transmitters use identical modulation schemes and transmitter powers (P_1 and P_2). Lets assume the SIR in each case be $\gamma \geq$ 10 dB.

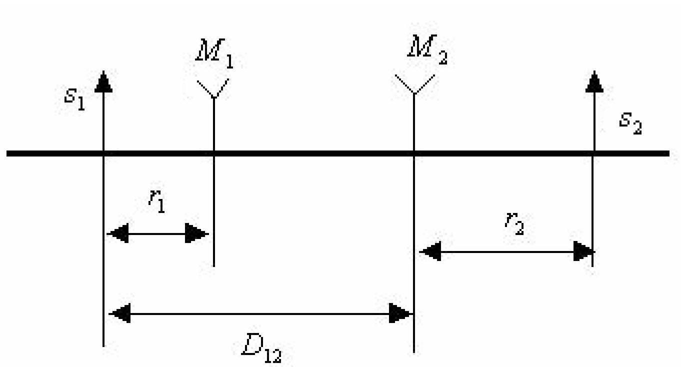

Figure 1: Used to calculate reuse distance.

The reuse distance in the presence & absence of shadow fading are explained below:

No Shadow Fading: -
Lets assume the path loss gain H_{11} and H_{12} from transmitter s_1 to mobile M_1 and M_2 respectively. Similarly assume H_{21} and H_{22} from transmitter s_2 to mobile M_1 and M_2 respectively as shown figure 2.

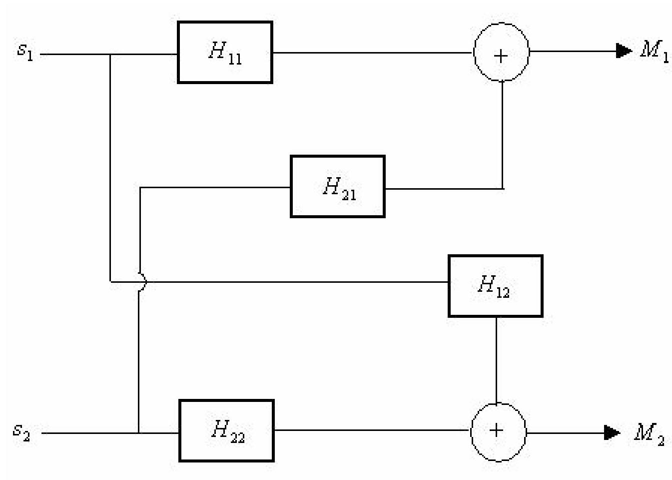

Figure 2

The received power is given by $P_{rx} = P_{tx} c D^{-\alpha}$

SIR at receiver M_2 is given by

$$\gamma_2 = \frac{cP_2 r_2^{-\alpha}}{cP_1 D_{12}^{-\alpha}} = \frac{P_2}{P_1}\left(\frac{D_{12}}{r_2}\right)^{\alpha}$$

for equal transmit powers $P_1 = P_2$ we have $\gamma_2 = \left(\frac{D_{12}}{r_2}\right)^{\alpha}$ and for $\gamma \geq$ 10 dB we have

$$\left(\frac{D_{12}}{r_2}\right)^{\alpha} \geq 10 \Rightarrow D_{12} \geq r_2 10^{1/\alpha}$$

$$\therefore D_{12} \approx \begin{cases} 3.2 r_2 & \text{if } \alpha = 2 \\ 1.8 r_2 & \text{if } \alpha = 4 \end{cases}$$

In presence of Shadow fading: -
The received power is $P_{rx} = P_{tx} c D^{-\alpha} G$ where G is a lognormal random variable and assume G is zero mean and $\sigma = 6dB$.

Let the requirement be $P[\gamma_2 > 10dB] > 0.9$. In this case the SIR at receiver 2 is

$$\gamma_2 = \frac{cP_2 r_2^{-\alpha} G_{22}}{cP_1 D_{12}^{-\alpha} G_{12}} = \frac{G_{22}}{G_{12}} \frac{P_2}{P_1}\left(\frac{D_{12}}{r_2}\right)^{\alpha}$$

For equal powers $P_1 = P_2$ we have

$$\gamma_2 = \frac{G_{22}}{G_{12}}\left(\frac{D_{12}}{r_2}\right)^{\alpha} \underline{\Delta} G\left(\frac{D_{12}}{r_2}\right)^{\alpha}$$

G is lognormal with mean zero and $\sigma_G = 8.5dB$.

Then $P[\gamma_2 > 10dB] = \Pr\left[10\log_{10} G > 10 - 10\alpha\log_{10}\left(\frac{D_{12}}{r_2}\right)\right]$

$$\Rightarrow Q\left(\frac{10 - 10\alpha\log_{10}\left(\frac{D_{12}}{r_2}\right)}{\sigma_G}\right) > 0.9$$

$$\Rightarrow D_{12} \geq r_2 10^{2/\alpha}$$

$$\therefore D_{12} \approx \begin{cases} 11r_2 & \text{if } \alpha = 2 \\ 3.3r_2 & \text{if } \alpha = 4 \end{cases}$$

Observations: -

1) SIR increases rapidly with D_{12}.
2) SIR depends only on the ratio $\frac{D_{12}}{r_2}$ and not on absolute distances i.e. the system is scalable.
3) Increase in α allows smaller reuse distances.
4) If the powers are unequal $(P_1 \neq P_2)$ then the relative power affects the system performance.

General Formulations of Resource Allocation Problem: -

1) Interference is mapped to SIR & Performance is mapped to achieved $\gamma \geq \gamma_0$ (min. requirement).
2) Signal quality depends on local SIR and slow flat fading is assumed.
3) All transmitters can choose orthogonal signals from the available set.

Let $M = \{1, 2, 3, ...\text{M}\}$ be the set of active mobiles in the coverage area because the number of mobile users changes with time typically M is a random variable. Let $B = \{1, 2, 3, ...\text{B}\}$ be the set of RAPs and is a constant and let $C = \{1, 2, 3, ...\text{C}\}$ be the set of orthogonal channels available for establishing links between BS and mobiles.

Resource Allocation Algorithm: -
Every mobile in the coverage area is assigned the following

A) A RAP from set B.
B) Channel Pair from the set C.
C) Transmit power for the RAPs & all the mobiles such that all the links meet their min SIR requirement.

The link gain matrix describes about the link gain between every M active mobiles & B base station and is characterized by

$$G = \begin{pmatrix} G_{11} & G_{12} & \dots\dots\dots & \dots\dots & G_{1M} \\ G_{21} & G_{22} & \dots\dots\dots & \dots\dots & G_{2M} \\ . & . & \dots\dots\dots & \dots\dots & . \\ . & . & \dots\dots\dots & \dots\dots & . \\ G_{B1} & G_{B2} & \dots\dots\dots & \dots\dots & G_{BM} \end{pmatrix}$$

Given that a mobile *j* has been assigned to a BS *i* on a channel pair *c*, the following must hold for constant transmit power.

$$\gamma_j^{th} = \frac{PG_{ij}}{\sum_{m \in M^{(c)}} PG_{im} + N_b} \geq \gamma_o \quad \{\text{Uplink}\}$$

$$\gamma_j = \frac{PG_{ij}}{\sum_{b \in B^{(c)}} PG_{bj} + N_m} \geq \gamma_o \quad \{\text{Downlink}\}$$

$M^{(c)} = \{m : \textit{mobile}$ m has been assigned to channel c.
$B^{(c)} = \{b : \textit{base station}$ b has been assigned to channel c.
N_b, N_m is thermal noise at the BS & MS respectively.

Refrences

1.) Lecture notes, Wireless Communication Technologies, Fall 2005 – Dr. Narayan B. Mandayam