MUTLIUSER DETECTION

(Lectures 19 and 20)
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Instructor: Dr. Narayan Mandayam
Summary By Shweta Shrivastava (shwetash@winlab.rutgers.edu)

Abstract – This article continues the discussion of multiuser detection. We first analyze the BER performance of a conventional detector. Starting with the results for 2 user case, we extend it the case of K users. We then define certain other performance measures which are simpler to calculate (than BER) and yet give a reasonable insight into the performance of multiuser detectors. Finally, we try to define an optimum detector for multiuser systems.

I. INTRODUCTION

Multiuser Detection deals with the demodulation of mutually interfering digital streams of information. Cellular telephony, satellite communication, high-speed data transmission lines etc. are some of the communication systems subject to multi-access interference. The superposition of transmitted signals may originate from non-ideal characteristics of the transmission medium, or it may be an integral part of the multiplexing method as in the case of CDMA. Multiuser detection exploits the considerable structure of the multiuser interference in order to increase the efficiency with which channel resources are employed.

Until a couple of decades earlier, the conventional wisdom was that it was best to simply neglect the presence of multiaccess interference since its statistical properties would be similar to additive white Gaussian noise, and therefore a single-user matched filter should be near-optimal to combat such interference. This conventional wisdom was proven wrong by the derivation and analysis of the optimal multiuser detector by Sergio Verdu, who showed that there is, in general, a huge gap in performance between the performance of the conventional single-user matched filter and the optimal attainable performance.

In this article, we first analyze the error probability of the conventional receiver for synchronous and asynchronous cases. We then define certain other major performance measures which are used to compare multiuser detectors. Finally, we analyze an optimum detector for multiuser systems.

II. THE MATCHED FILTER IN THE CDMA CHANNEL

This section is concerned with the analysis of the capabilities and performance of multi-user detection using the simplest detector: the single-user matched filter. In the multiuser detection literature, this detector is frequently referred to as the *conventional detector*. We consider the basic synchronous CDMA multiple access *K*-user channel

$$y(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + \sigma n(t) \tag{1}$$

where $A_k \in (0,\infty)$, $b_k \in \{-1,+1\}$ and s_k are the received amplitudes, information bit sequence and unit-energy signature waveform of the kth user respectively, and $n(t)$ is the additive white Gaussian noise. We first continue with the case for 2 users.

Case I: 2 users

In case of 2 users, the probability of error for user 1 was earlier derived to be

$$P_1^c(\sigma) = \frac{1}{2} Q\left(\frac{A_1 - A_2|\rho|}{\sigma}\right) + \frac{1}{2} Q\left(\frac{A_1 + A_2|\rho|}{\sigma}\right) \tag{2}$$

If interference from user 2 is dominant as compared to the received signal from user 1, then the following condition holds.

$$\frac{A_2}{A_1} > \frac{1}{|\rho|} \tag{3}$$

Then the conventional receiver exhibits anomalous behavior. For example, the error probability is not monotonic with σ.

$$\lim_{\sigma \to \infty} P_1^c(\sigma) = \frac{1}{2} \tag{4}$$

$$\lim_{\sigma \to 0} P_1^c(\sigma) = \frac{1}{2} \tag{5}$$

Thus the probability of error tends to ½ in both these cases - when the noise is very high as well as when the noise is negligible. Eq (5) results because, due to (3), as $\sigma \to 0$, the polarity of the output of the matched filter for user 1 is governed by b_2 rather than b_1. We can say that some amount of noise ($\sigma > 0$) is actually good for detection because it leads to $P_1^c(\sigma) < \frac{1}{2}$.

In fact, it can be shown that there is an optimum noise variance that minimizes the BER under condition given by (2), i.e. when interference is dominant. This optimum variance is,

$$\sigma^2 = \frac{A_1 A_2 \rho}{\tanh^{-1}\left(\frac{A_1}{A_2 \rho}\right)} \tag{6}$$

Further, when the contribution from interference is the same as the contribution of the desired user, i.e. when

$$\frac{A_2}{A_1} = \frac{1}{|\rho|} \tag{7}$$

Then, the probability of error is given by,

$$P_1^c(\sigma) = \frac{1}{4} + \frac{1}{2} Q\left(\frac{2A_1}{\sigma}\right) \tag{8}$$

The interpretation of the above is that when condition given by (6) holds, then with probability ½, the signal of user 2 exactly cancels the signal of user 1, which means that we are left only with noise. Also with probability ½, the signal of user 2 doubles the contribution of the desired user.

Figure 1 plots the BER in (2) with ρ = 0.2 as a function of the SNR A_1/σ for several values of the relative amplitude of the interferer.

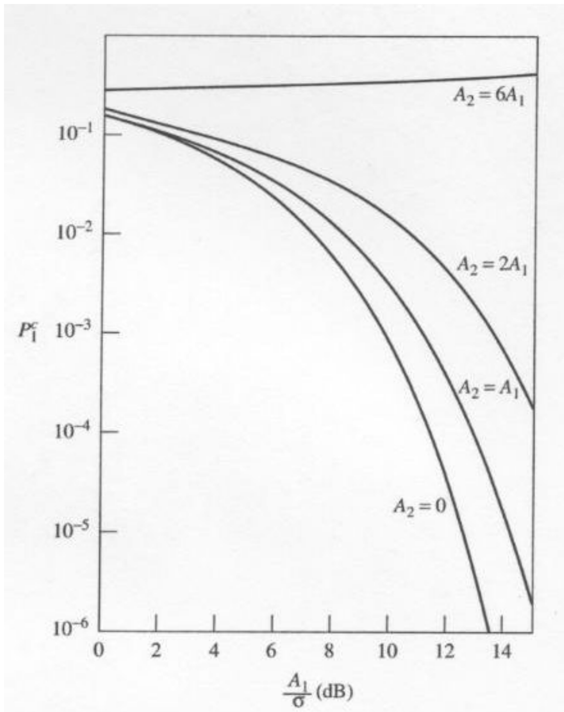

Fig. 1. Bit-error-rate of conventional detector with two synchronous users and ρ = 0.2

We can observe that if the interferer has a stronger signal, it can completely suppress the signal of the desired user (shown with increasing A_2 in the graph). This leads to the *near-far problem*. In other words, the BER degrades rapidly as A_2 increases. An alternate view of the near-far problem is given by the following graph (figure 2).

The graph shows *power tradeoff regions* for the case of 2 users. Power tradeoff regions tell the SNRs required from both users in order to achieve a given BER for a given value of ρ. The graph shows power tradeoff regions to achieve a BER $\leq 3 \times 10^{-5}$ for ρ = 0.2. We have $Q^{-1}(3 \times 10^{-5})$ = 12 dB. When ρ = 0, the signals from the 2 users can't see each other. So, the SNRs required from both need to be greater than 12dB to achieve the above BER.

Note: The graph shows that as ρ increases,

(a) even if amplitudes or power levels of both users are identical, the necessary energy required to achieve a given BER increases rapidly (shown by the points lying on the diagonal).

(b) The sensitivity to imbalance in the received energy from the 2 users increases.

In mobile systems, the received amplitudes may vary over a wide range, which dictates the need for strict power control and also low cross-correlation properties.

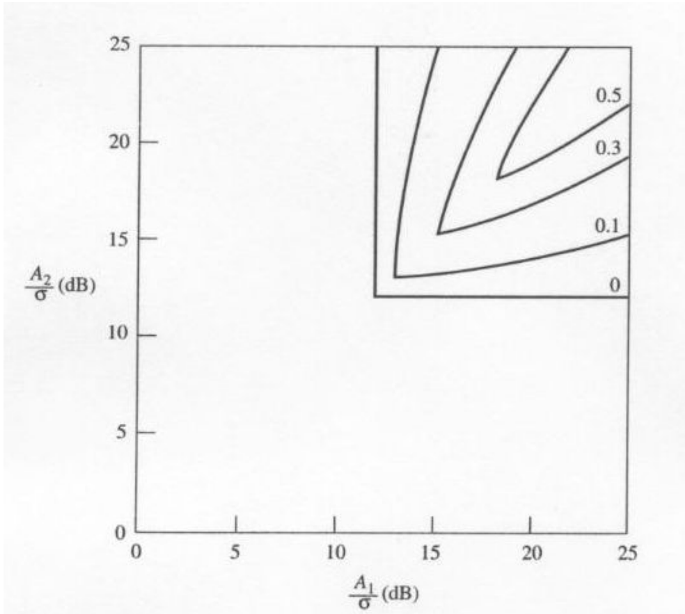

Fig. 2. Signal-to-noise ratios necessary to achieve bit-error-rate not higher than 3×10^{-5} for both users, parameterized by ρ.

To visualize the operation of a conventional detector in signal space, we look at the space y_1-y_2, where y_1 is the output of matched filter 1 and y_2 is the output of matched filter 2.

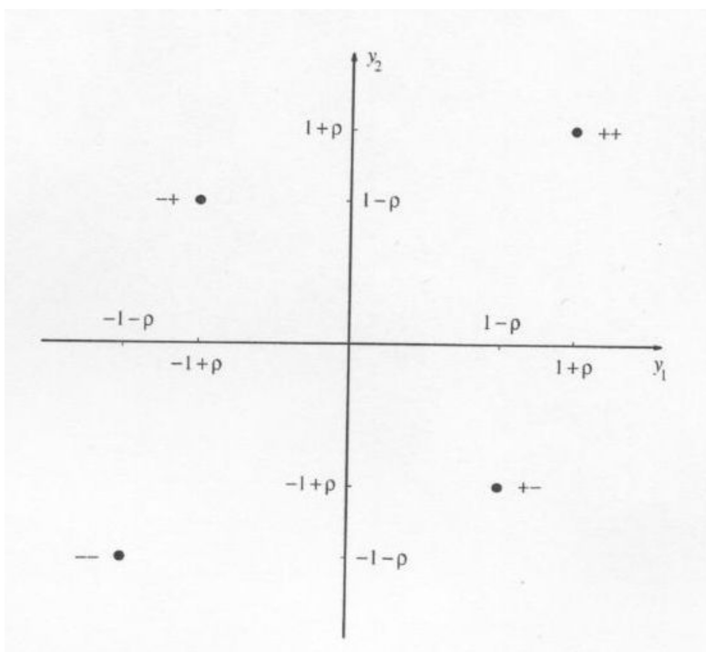

Fig. 3. Decision regions in the two-dimensional space of matched filter outputs

In the above graph, we have assumed that $A_2 = A_1 = 1$. ρ is the cross-correlation between the two signals. (+ +) means that $b_1 > 0$ and $b_2 > 0$, (- +) means $b_1 < 0$ and $b_2 > 0$, and so on. We are looking at mean vectors here. Conditioned on (b_1, b_2) the output vector is a Gaussian vector with mean

$$\begin{bmatrix} A_1 b_1 + A_2 b_2 \rho \\ A_2 b_2 + A_1 b_1 \rho \end{bmatrix}$$

The covariance matrix is

$$\operatorname{cov}(y_1, y_2) = \sigma^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

The received vector can be viewed as the sum of the mean vector and the zero-mean Gaussian vector $\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$.

Therefore, the idea is that if we keep the decision regions fixed, the transmitted vectors change according to the amplitudes and cross-correlations. This may lead to degradation in performance and also anomalous behavior.

Case 2: K Users

Let's generalize the above results to a K-user system. Following the same reasoning as before, the probability of error for the *k*th user is given by,

$$P_k^c(\sigma) = P[b_k = +1]P[y_k < 0 \mid b_k = +1] + P[b_k = -1]P[y_k > 0 \mid b_k = -1] \tag{9}$$

If we adopt the same procedure as in the case for 2 users, we get

$$P_k^c(\sigma) = \frac{1}{2} P\left[n_k > A_k - \sum_{j \neq k} A_j b_j \rho_{jk} \right] + \frac{1}{2} P\left[n_k < -A_k - \sum_{j \neq k} A_j b_j \rho_{jk} \right] \tag{10}$$

The two probabilities (the two terms in the above equation) are symmetric.

$$P_k^c(\sigma) = P\left[n_k > A_k - \sum_{j \neq k} A_j b_j \rho_{jk} \right] \tag{11}$$

Conditioning on all the interfering bits, the above probability becomes

$$P_k^c(\sigma) = \frac{1}{2^{K-1}} \sum_{e_1 \in \{-1,+1\}} \cdots\cdots \sum_{\substack{e_j \in \{-1,+1\} \\ j \neq k}} \cdots\cdots \sum_{e_k \in \{-1,+1\}} Q\left(\frac{A_k}{\sigma} + \sum_{j \neq k} e_j \frac{A_j}{\sigma} \rho_{jk} \right) \tag{12}$$

Observations:

(i) The BER of conventional detector in the CDMA Gaussian channel depends on the shape of the signature waveforms only through the cross-correlations.

(ii) BER depends on received amplitudes and the noise level σ only through the ratios $\frac{A_k}{\sigma}$, as the decisions are invariant to the scaling of the received waveform.

Upper Bound

We can find an upper bound on the probability of error expression of (12) by replacing ρ with $|\rho|$, and considering the largest expression. We get,

$$P_k^c(\sigma) \le Q\left(\frac{A_k}{\sigma} - \sum_{j \ne k} \frac{A_j}{\sigma} \left|\rho_{jk}\right|\right) \tag{13}$$

To observe the near-far behavior, note that (12) → 0 as σ → 0 (i.e. probability of error vanishes when background noise diminishes) iff

$$A_k > \sum_{j \ne k} A_j \left|\rho_{jk}\right| \tag{14}$$

The above condition is known as the *open-eye condition*. It is the condition that the received signal strength of the desired user is greater than the sum of noises due to interference from other users. Thus, if there is no background noise in the system, error free decisions can be made if the open eye condition is met. Under (14), the bound of (13) becomes tight as σ → 0.

Also note that the computation of (12) grows exponentially as the number of users. It is very tempting to use a Gaussian approximation, i.e. replace the binomial random variable $\sum_{j \ne k} A_j b_j \rho_{jk}$ with Gaussian with identical variance. The probability of error for *k*th user with Gaussian approximation becomes,

$$\widetilde{P}_k^c(\sigma) = Q\left(\frac{A_k}{\sqrt{\sigma^2 + \sum_{j \ne k} A_j^2 \rho_{jk}^2}}\right) \tag{15}$$

The above Gaussian approximation is generally good at low SNRs, but may become unreliable at high SNRs. The graphs of figures 4 and 5 draw a comparison between the exact expression of (12) and the Gaussian approximation of (15). The graphs show a plot of BER as a function of SNR for the two cases.

Note from the figures that in the limit as σ → 0, the exact expression and Gaussian approximation behave differently. For example, (14) has non-zero limit even if open-eye condition is satisfied. This difference can be attributed to the fact that the error in replacing binomial random variable with Gaussian random variable is greatest in the tails, which determines the BER (unless background noise is dominant, which is usually not the case for MA systems).

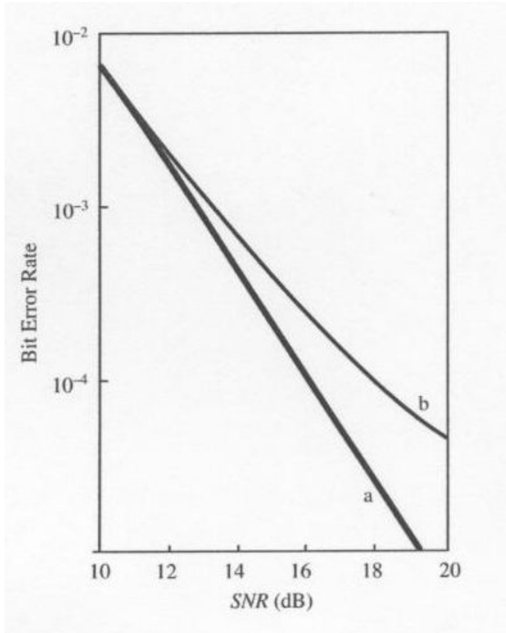

Fig. 4. Bit-error-rate of the single-user matched filter with 10 equal-energy users and identical cross-correlations ρ_{kl} = 0.08; (a) exact, (b) Gaussian approximation.

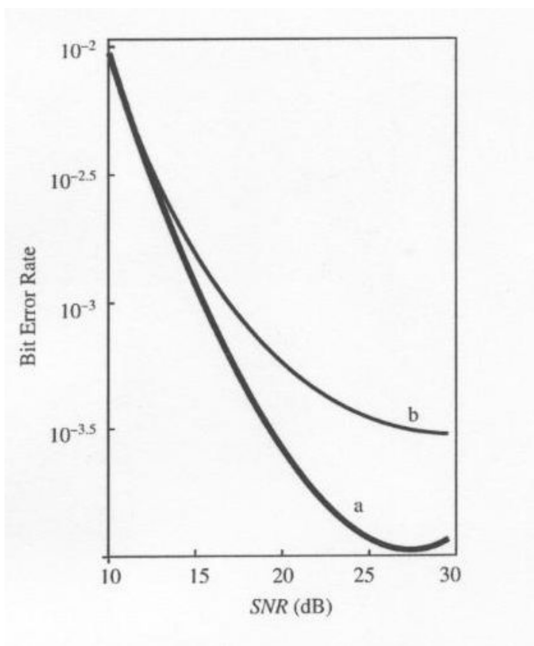

Fig. 5. Bit-error-rate of the single-user matched filter with 14 equal-energy users and identical cross-correlations ρ_{kl} = 0.08; (a) exact, (b) Gaussian approximation.

For the asynchronous case, each bit is affected by 2(k -1) interfering bits (as opposed to k – 1 for the synchronous case). The probability of error for kth user is then given by,

$$P_k^c(\sigma) = \frac{1}{4^{K-1}} \sum_{(e_1,d_1)\in\{-1,+1\}^2} \cdots \sum_{\substack{(e_j,d_j)\in\{-1,+1\}^2 \\ j\neq k}} \cdots \sum_{(e_k,d_k)\in\{-1,+1\}^2} Q\left(\frac{A_k}{\sigma} + \sum_{j\neq k} \frac{A_j}{\sigma}\left(e_j \rho_{jk} + d_j \rho_{kj}\right)\right) \quad (16)$$

Note that $\rho_{jk} \neq \rho_{kj}$. ρ_{jk} and ρ_{kj} are asynchronous cross-correlations; these are same as continuous-time partial cross-correlations, and are given by,

$$\rho_{kl}(\tau) = \int_\tau^T s_k(t) s_l(t-\tau)\,dt \quad (17)$$

$$\rho_{lk}(\tau) = \int_0^\tau s_k(t) s_l(t+T-\tau)\,dt \quad (18)$$

ρ_{kl} is known as the right cross-correlation and ρ_{lk} is known as the left cross-correlation.

The open-eye condition in case of asynchronous system is,

$$A_k > \sum_{j\neq k} A_j \left(\left|\rho_{jk}\right| + \left|\rho_{kj}\right|\right) \quad (19)$$

Note from the above equations that probability of error is hard to compute even for simple matched filter receivers. Hence we define some other easy to compute performance measures.

III. MUTLIUSER EFFICIENCY AND RELATED MEASURES

While BER is a main performance measure in most communication systems, there are several performance measures derived from it that are useful in design, analysis and understanding of the performance of various detectors.

Signal-to-Interference Ratio (SIR)

SIR is one of the performance measures of multi-user detection. SIR gives the ratio of powers due to the desired user and due to all other components. In the absence of interference, SIR = $\frac{A_k^2}{\sigma^2}$, and single-user performance is achieved, i.e. probability of error is given by,

$$P_k^c(\sigma) = Q\left(\frac{A_k}{\sigma}\right) \quad (20)$$

In the presence of interference,

$$\text{SIR} = \frac{A_k^2}{\sigma^2 + \sum_{j\neq k} A_j^2 \rho_{jk}^2} \quad (21)$$

Therefore, the presence of interference increases the BER.

It is of interest to quantify the multi-user error probability relative to the optimum single-user BER. Hence we define the following metric.

Effective Energy

Effective energy of user k, $e_k(\sigma)$ is the energy that user k would require to achieve a BER equal to $P_k(\sigma)$ in a single-user Gaussian channel with the same background noise, i.e.,

$$P_k(\sigma) = Q\left(\frac{\sqrt{e_k(\sigma)}}{\sigma}\right) \tag{22}$$

Where $P_k(\sigma)$ is the multi-user error probability. Since the multi-user error probability is lower bounded by the single-user error probability, we have,

$$P_k(\sigma) \geq Q\left(\frac{A_k}{\sigma}\right)$$

$$\Rightarrow e_k(\sigma) \leq A_k^2 \tag{23}$$

(23) tells us that the effective energy for user k is upper bounded by the actual energy. If we normalize the effective energy by σ^2 (noise variance), we obtain,

$$e_k(\sigma) = \sigma^2 \left(Q^{-1}(P_k(\sigma))\right)^2 \tag{24}$$

The power tradeoff region can then be characterized in terms of the effective energy as follows. The power tradeoff region for a given permissible BER P (same for all users) is the set of SNRs $\left(\frac{A_1^2}{\sigma^2}, \frac{A_2^2}{\sigma^2}, \ldots\ldots, \frac{A_k^2}{\sigma^2}\right)$, such that $\max_k P_k(\sigma) \leq \mathrm{P}$, or equivalently,

$$\min_k \frac{e_k(\sigma)}{\sigma^2} \geq \left[Q^{-1}(\mathrm{P})\right]^2 \tag{25}$$

Multi-user Efficiency

Multi-user efficiency is defined as the ratio of effective and actual energies. It is given by $e_k(\sigma)/A_k^2$, and quantifies the performance loss due to the existence of other users in the channel. Multi-user efficiency depends on the signature waveforms, the received amplitudes (SNRs) and on the detector employed. It follows from (23) that multiuser efficiency belongs to the interval [0,1] (or [-∞, 0] in dB).

Asymptotic Multi-user Efficiency

Asymptotic multi-user efficiency characterizes the performance loss (in effective signal-to-noise ratio (SNR)) of multi-user detector as the background noise vanishes. It is defined for user k as,

$$\eta_k = \lim_{\sigma \to 0} \frac{e_k(\sigma)}{A_k^2} \tag{26}$$

An alternate and more formal definition of multi-user efficiency is given as,

$$\begin{aligned} \eta_k &= \sup_r \left\{ 0 \le r \le 1; \lim_{\sigma \to 0} \frac{P_k(\sigma)}{Q\left(\frac{\sqrt{r}A_k}{\sigma}\right)} = 0 \right\} \\ &= \frac{2}{A_k^2} \lim_{\sigma \to 0} \sigma^2 \log\left[\frac{1}{P_k(\sigma)}\right] \quad . \end{aligned} \tag{27}$$

Therefore, we see that when the "eye is closed" (i.e. BER does not vanish as $\sigma \to 0$), the asymptotic multiuser efficiency is equal to 0. If $\eta_k > 0$, then BER $\to 0$ as $\sigma \to 0$ and moreover, it vanishes exponentially in the SNR. η_k also measures the slope with which $P_k(\sigma) \to 0$ on a log scale in high SNR region. Typically, the asymptotic multiuser efficiency is very close to multiuser efficiency (except at low SNRs).

Near-Far Resistance

The near-far resistance is the asymptotic multiuser efficiency minimized over the received energies of all the other users. It measures the robustness of the detector to varying levels of interference. It is given by,

$$\overline{\eta}_k = \inf_{\substack{A_j > 0 \\ j \ne k}} \eta_k \tag{28}$$

The near-far resistance depends on the signature waveforms and on the demodulator.

It is sometimes easier to compute these measures (multiuser efficiency and near-far resistance) than the probability of error.

We now analyze the robustness of conventional receiver for multi-user detection by calculating its multi-user efficiency and near-far resistance.

Conventional Receiver

We consider the case for 2 users, i.e. $K = 2$ with synchronous CDMA model.

Asymptotic multiuser efficiency:

The probability of error for user 1 as given by (2) is,

$$P_1^c(\sigma) = \frac{1}{2} Q\left(\frac{A_1 - A_2|\rho|}{\sigma}\right) + \frac{1}{2} Q\left(\frac{A_1 + A_2|\rho|}{\sigma}\right)$$

When the eye is closed, $A_2|\rho| > A_1$, then as σ → 0, $P_1^c(\sigma)$ does not vanish (recall that it goes to ½).

$\therefore \eta_1^c = 0$

If eye is open, $A_1 > A_2|\rho|$, then

$$\lim_{\sigma\to 0}\frac{P_1^c(\sigma)}{Q\left(\frac{\sqrt{r}A_1}{\sigma}\right)} = \lim_{\sigma\to 0}\frac{\frac{1}{2}Q\left(\frac{A_1 - A_2|\rho|}{\sigma}\right)+\frac{1}{2}Q\left(\frac{A_1 + A_2|\rho|}{\sigma}\right)}{Q\left(\frac{\sqrt{r}A_1}{\sigma}\right)}$$

$$= \begin{cases} 0, \sqrt{r}A_1 < A_1 - A_2|\rho| \\ \infty, \sqrt{r}A_1 > A_1 - A_2|\rho| \end{cases}$$

Using the above equation and (27), we get,

$$\eta_1^c = \left(1 - \frac{A_2}{A_1}|\rho|\right)^2 \tag{29}$$

Thus the overall asymptotic multiuser efficiency (using both eye-open and eye-closed conditions) is given as,

$$\eta_1^c = \max{}^2\left\{0, 1 - \frac{A_2}{A_1}|\rho|\right\} \tag{30}$$

The asymptotic multiuser efficiency is plotted as a function of the relative amplitude for ρ = 0.2 in figure 6.

Evaluating similarly for the K-user case, we get,

$$\eta_k^c = \max{}^2\left\{0, 1 - \sum_{j\neq k}\frac{A_j}{A_k}|\rho_{jk}|\right\} \tag{31}$$

Near-far resistance:

The near-far resistance is obtained by minimizing (31) with respect to A_j, $j \neq k$.

We observe that $\overline{\eta}_k^c = 0$, unless $\rho_{jk} = 0$ for all $j \neq k$.

We can thus conclude that the matched filter or conventional receiver is not near-far resistant unless the signature waveform of the *k*th user is orthogonal to each of the partially overlapping waveforms from all other users.

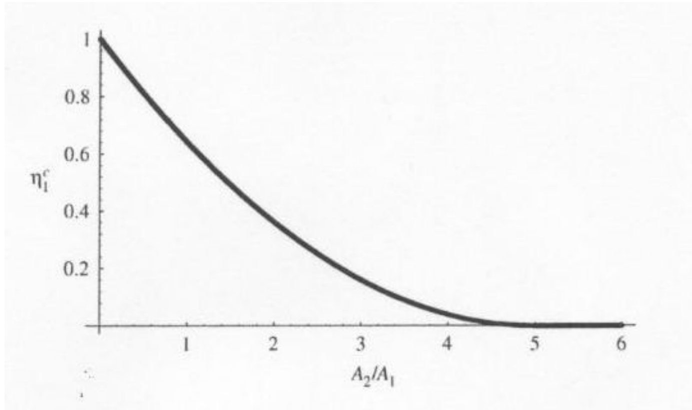

Fig. 6. Asymptotic multiuser efficiency of conventional detector as a function of the amplitude of the interferer; ρ = 0.2 (linear plot).

IV. OPTIMUM DETECTOR

A very simple demodulator for the CDMA channel was analyzed in section II. We turn our attention now to the derivation and analysis of optimum strategies. The analysis of optimum multiuser detectors yields the minimum achievable probability of error (and optimum asymptotic multiuser efficiency, as well as optimum near-far resistance) in CDMA channels. This serves as a baseline of comparison for suboptimum multiuser detectors.

The conventional receiver requires no knowledge beyond the signature waveforms and timing of users it wants to demodulate. In the derivation of an optimum receiver, we will assume that the receiver not only knows the signature waveform and timing of every active user, but it also knows (or can estimate) the received amplitudes of all users and the noise level.

Consider the *K*-user basic synchronous CDMA channel:

$$y(t) = \sum_{k=1}^{K} A_k b_k s_k + \sigma . n(t)\,, \qquad t \in [0, T] \tag{32}$$

The optimum decision rule in this case is the maximum a posteriori probability rule (MAP). However, two optimum decision strategies using MAP can be employed, and they need not result in the same decision. They are the following.

Individually optimum:

$$\max_{b_k} P\left[b_k \mid y(t), 0 \le t \le T\right], \qquad k = 1,....., K \tag{33}$$

Jointly optimum:

$$\max_{b_1, b_2, ..., b_k} P\left[(b_1, b_2, ..., b_K) \mid y(t), 0 \le t \le T\right] \tag{34}$$

The following example illustrates that the two criteria are indeed different. Consider *K* = 2. Let the noise realizations are such that the a posteriori probabilities take the following values:

$$P[(+1,+1) \mid y(t)] = 0.26$$
$$P[(-1,+1) \mid y(t)] = 0.26$$
$$P[(+1,-1) \mid y(t)] = 0.27$$
$$P[(-1,-1) \mid y(t)] = 0.21$$

Then the jointly optimum decisions are the one with highest a posteriori probability, i.e., $(b_1, b_2) = (+1, -1)$.

The individually optimum decisions are given by evaluating

$$P[b_1 \mid y(t)] = P[(b_1,+1) \mid y(t)] + P[(b_1,-1) \mid y(t)]$$
$$P[b_2 \mid y(t)] = P[(+1,b_2) \mid y(t)] + P[(-1,b_2) \mid y(t)]$$

Thus the individually optimum decisions are $(b_1, b_2) = (+1, +1)$.

However, we usually expect the two results to be the same with very high probability if the probability of error is low. Hence, either criteria is acceptable.

Let us consider the jointly optimum demodulation of

$$\underline{b} = [b_1, \ldots\ldots, b_K]^T$$

For the case of *n(t)* in (32) being AWGN, the optimum receiver is the maximum likelihood receiver and also the minimum probability of error receiver.

$$\hat{\underline{b}}_{ML} = \arg\max_{\underline{b}} P[y(t), t \in [0,T] \mid \underline{b}] \tag{35}$$

It can be shown that a sufficient statistic for ML detection is $\underline{y} = [y_1, y_2, \ldots\ldots, y_K]^T$, which is the vector of matched filter outputs.

$$y_k = \int_0^T y(t) s_k(t) dt = A_k b_k + \sum_{j \neq k} A_j b_j \rho_{jk} + n_k \tag{36}$$

And $\underline{n} = [n_1, n_2, \ldots\ldots, n_K]^T$. $\underline{n}$ is jointly Gaussian random vector.

$$E[\underline{n}] = \underline{0}$$
$$E[n_j n_l] = \sigma^2 \rho_{jl}$$

We can write

$$\underline{y} = \underline{R}\,\underline{A}\,\underline{b} + \underline{n} \tag{37}$$

where $\underline{R} = [\rho_{ij}]$ is the normalized cross-correlation matrix whose diagonal elements are equal to 1 and whose (i, j) element is equal to the cross-correlation ρ_{ij}.

A is $K \times K$ diagonal matrix of received amplitudes,

$$\underline{A} = diag\{A_1, \ldots\ldots, A_K\} \tag{38}$$

$$p(\underline{y} \mid \underline{b}) = \exp\left\{ \frac{-\frac{1}{2}(\underline{y} - \underline{R}\underline{A}\underline{b})^T (\sigma^2 \underline{R})^{-1} (\underline{y} - \underline{R}\underline{A}\underline{b})}{\sqrt{(2\pi)^K \sigma^2 |R|}} \right\} \tag{39}$$

Therefore, the ML rule is,

$$\hat{\underline{b}}_{ML} = \arg\max_{\underline{b}} \Omega(\underline{b}) \tag{40}$$

Where,

$$\Omega(\underline{b}) = 2\underline{b}^T \underline{A}\underline{y} - \underline{b}^T \underline{H}\underline{b} \tag{41}$$

$$\underline{H} = \underline{A}\underline{R}\underline{A} \tag{42}$$

The above maximization is a combinatorial optimization problem, which implies that the complexity grows exponentially in the number of users (need to search over 2^K choices).

For $K = 2$, the optimum receiver's asymptotic multiuser efficiency is given by,

$$\eta_1^{opt} = \min\left\{ 1, 1 + \frac{A_2^2}{A_1^2} - 2|\rho| \frac{A_2}{A_1} \right\} \tag{43}$$

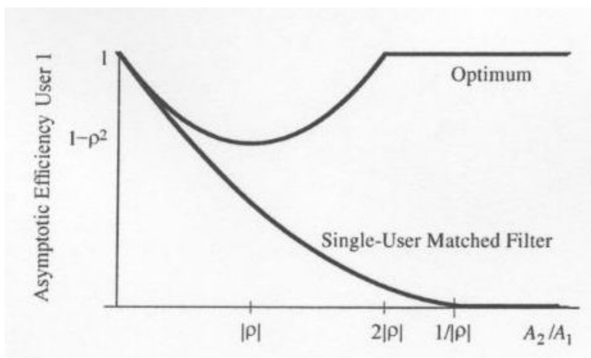

Fig. 7. Optimum and single-user asymptotic multiuser efficiencies for two synchronous users.

Figure 7 shows the asymptotic efficiency of user 1 for both optimum and conventional receiver. It can be seen that for optimum receiver asymptotic efficiency is not monotonic in A_2/A_1. Actually, if

$$\frac{A_2}{A_1} > 2|\rho|$$

then $\eta 1 = 1$. Therefore, as long as the energy of user 2 exceeds the threshold given by above equation the asymptotic bit-error-rate of user 1 is equivalent to the single-user case where user 2 is not active The explanation of this behavior of the optimum receiver is that if the interfering user is sufficiently powerful, then the primary source of errors committed in the optimum demodulation of user 1 is the background Gaussian noise, rather than the randomness of the information carried by the interfering signal. This fact could be explained using the successive decoding technique.

The near-far resistance is obtained by minimizing equation 10 over $A_2/A_1 \geq 0$.

The least favorable relative amplitude of user 2 is

$$\frac{A_2}{A_1} = |\rho|$$

which yields the near-far resistance for either user:

$$\overline{\eta}_k = 1 - \rho^2$$

Figure 8 shows the two-user power-tradeoff region so that the optimum bit-error rate of both the users is not higher than 3×10^{-5}, for $|\rho|$=0.8, 0.9 and 0.95. If we compare this figure with the one for conventional receiver, we can conclude that the permissible signal-to-noise ratios are indistinguishable as long as the cross-correlation satisfies $\rho \leq 0.5$. Also for high cross-correlations values, equal powers for users are detrimental. The reason is that if both signature waveforms are very much alike, then the similar amplitudes complicate the task of the optimum receiver.

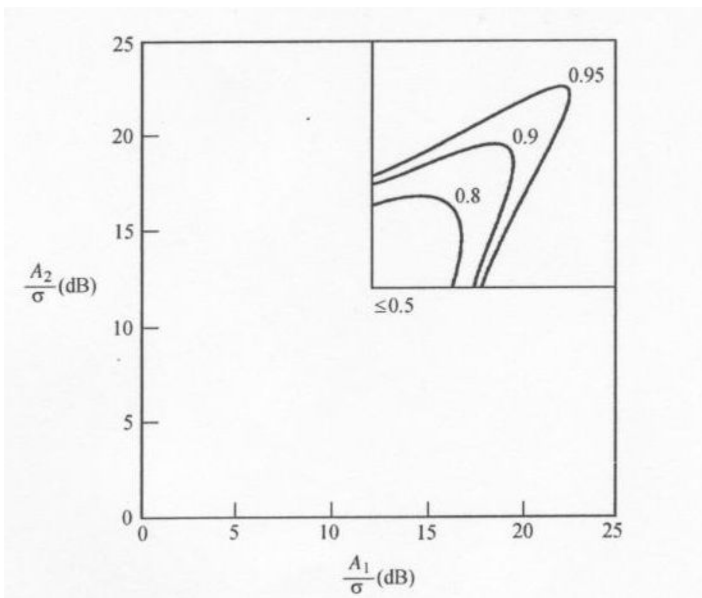

Fig. 8. Signal-to-noise ratios necessary to achieve optimum bit-error-rate not higher than 3×10^{-5} for both users.

The complexity of optimum multi-user detector requires one to come-up with other suboptimum multiuser detectors that exhibit good performance and complexity tradeoffs.

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