Wireless Communications Technologies
Course No: 16332:559 - (Spring 2002)

Solution to Homework 2

1. The series expansion for the Bessel function of order $m$ of the first kind, $J_m(z)$ is

$$ J_m(z) = \left( \frac{z}{2} \right)^m \sum_{k=0}^{\infty} \frac{(-1)^k}{k! \Gamma(m + k + 1)} \left( \frac{z}{2} \right)^{2k} $$

2. In general,

$$ J_m(z) = \frac{1}{\pi} \int_0^\pi \cos(mt - z\sin(t)) \, dt, \quad m = 0, 1, 2, \cdots $$

![Bessel Functions](image)

**Figure 1: Bessel Functions**

3. The modified Bessel function of order $m$, $I_m(z)$ is given as the solution to the following differential equation:

$$ \frac{d^2 w}{dz^2} + \frac{1}{z} \frac{dw}{dz} - \left( 1 + \frac{m^2}{z^2} \right) w = 0 $$

The relation to the Bessel function of the first kind is given as

$$ I_m(z) = i^{-m} J_m(iz) $$

4. (a)

Let $r(t) = r_I(t) + r_Q$. Then it follows by definition that

$$ E[ r(t) \, r(t+\tau) \, r(t+\tau)^* ] = E[r_I^2(t) r_I^2(t+\tau)] + E[r_Q^2(t) r_Q^2(t+\tau)] + E[r_I^2(t) r_Q^2(t+\tau)] + E[r_Q^2(t) r_I^2(t+\tau)] $$

Note the following two facts:
• If \( X \) and \( Y \) are Gaussian random variables with zero mean (see Papoulis or any basic probability book for details):

\[
\]

• Since \( r(t) \) is WSS, we have \( \phi_{\text{c},\text{c}}(t) = \phi_{\text{r},\text{r}}(t) \) and \( \phi_{\text{r},\text{r}}(t) = -\phi_{\text{c},\text{c}}(t) \)

Using the above two facts it follows that

\[
E[\left| r(t) \right|^2 | r(t + \tau) |^2] = 4\phi_{\text{r},\text{r}}^2(0) + 4\phi_{\text{r},\text{r}}^2(t) + 4\phi_{\text{c},\text{c}}^2(t)
\]

For isotropic scattering, \( \phi_{\text{r},\text{r}}(t) = \frac{\Omega}{2} J_0(2\pi f_m t) \) and \( \phi_{\text{c},\text{c}}(t) = 0 \). Therefore

\[
E[\left| r(t) \right|^2 | r(t + \tau) |^2] = \Omega^2 [1 + J_0^2(2\pi f_m t)]
\]

(b)


Then

\[
E[\left| r(t) \right|^2 | r(t + \tau) |^2] = 4\phi_{\text{r},\text{r}}^2(0) + 4\phi_{\text{r},\text{r}}^2(t) + 4\phi_{\text{c},\text{c}}^2(t) - 4S^2(\phi_{\text{r},\text{r}}(0) + \phi_{\text{r},\text{r}}(t)) + 2S^4,
\]

where \( S^2 = E^2[X] + E^2[Y] \)

5. For Ricean fading, we have the power of the desired user to be distributed with the following non-central chi-square distribution with 2 degrees of freedom:

\[
p_x(x) = \frac{K + 1}{\Omega_o} \exp(-K - \frac{K + 1}{\Omega_o} x) J_0(2\sqrt{\frac{K(K + 1)}{\Omega_o}} x),
\]

where \( \sigma^2 = \frac{\Omega}{K+1} \) is the diffused signal component and \( s^2 = \frac{K\Omega_0}{K+1} \) is the direct LOS signal component. In the above \( \Omega_0 \) is the mean signal power.

Note that a single cochannel Rayleigh interferer’s power has density function (with mean \( \Omega_1 \))

\[
p_y(y) = \frac{1}{\Omega_1} \exp(-\frac{y}{\Omega_1})
\]

\[
O = 1 - Pr\left(\frac{x}{y} > \lambda_{th}\right) = 1 - \int_0^{\infty} p_x dx \int_0^{\frac{x}{\lambda_{th}}} p_y(y) dy = \int_0^{\infty} \exp(-\frac{x}{\lambda_{th}\Omega_1}) p_x(x) dx
\]

\[
\Rightarrow \quad O = E[\exp(-\frac{x}{\lambda_{th}\Omega_1})]
\]

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It can be shown that (for non-central \( \chi^2 \)), the moment generating function is:

\[
E[\exp(rx)] = \frac{1}{1 - r\sigma^2} \exp \left( \frac{r s^2}{1 - r\sigma^2} \right)
\]

\[
\Rightarrow
\]

\[
O = \frac{\lambda}{\lambda_h + b} \exp \left( - \frac{Kb}{\lambda_h + b} \right),
\]

where \( b = \frac{\Omega}{(K+1)\Omega_1} \).