Wireless Communication Technologies

Lectures 15,16 (March 25, 28)

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1 Diversity – Continued

1.1 Independent Diversity – Continued

1.1.1 Switched Diversity

In a switched diversity system, only one branch at a time is used and the strategy is to remain using the branch until the signal envelope drops below a predetermined threshold value x.

When signal falls below x, two strategies could be used:

- "Switch and Stay" – Select the other signal/branch always.
- "Switch and Examine" – Select the other signal/branch only if above threshold.

Switched diversity has simplicity in implementation as only one receiver (front – end) is required in contrast with selection diversity where all channels need to be monitored continuously.

The following figure shows how this method works when there are a two branches.

Discontinuous signal is obtained at switched instants – amplitude and phase transitions.

$a_1(t)$ and $a_2(t)$ are independent Rayleigh processes. Assume that they have identical mean and autocorrelations.

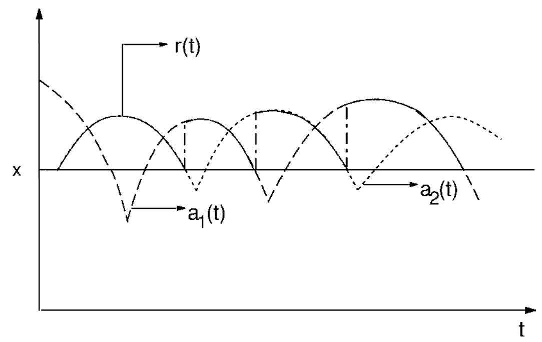

Figure 1: Switched Diversity Waveform

$r(t)$ is a combined process composed of parts of $a_1(t)$ and $a_2(t)$, according to threshold x.

Let γ_x denote SNR corresponding to level x, then the probability that SNR in one branch is below threshold γ_x is:

$$q = P[\gamma_{a_1(or a_2)} < \gamma_x] = 1 - e^{-\frac{\gamma_x}{\gamma_0}}$$

Where γ_0 is the mean SNR in each branch.

The SNR of the combined signal has the PDF:

$$p_c(\gamma) = \begin{cases} (1+q)p_\Gamma(\gamma) & \text{if } \gamma \geq \gamma_x; \\ q p_\Gamma(\gamma) & \text{if } \gamma < \gamma_x. \end{cases}$$

Where $p_\Gamma(\gamma)$ is the exponential PDF in each branch.

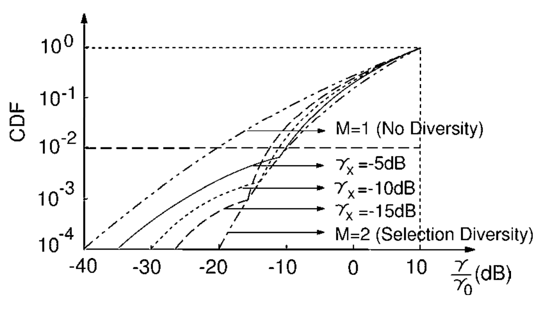

Figure 2: Switched Diversity CDF compared to Selection Diversity ($M = 2$)

Note switched diversity is much worse than selection diversity, but this is a tradeoff between complexity and performance.

1.2 Diversity with Correlated Signals

In practical systems, it may be difficult to achieve uncorrelated branches, for example, when there is limited frequency separation between frequency diversity branches, or closely spaced antennas. Consider the case that $M = 2$.

1.2.1 For Maximum Ratio Combining

The CDF of the SNR of resulting combined signal:

$$P(\gamma) = 1 - \frac{1}{\rho}\left[(1+\rho)\exp\left(\frac{-\gamma}{\gamma_0(1+\rho)}\right) - (1-\rho)\exp\left(\frac{-\gamma}{\gamma_0(1+\rho)}\right)\right]$$

Where ρ^2 is a parameter that captures the normalized covariance of the two signal envelopes:

- $\rho^2 = 1 \rightarrow$ correlated branches, equivalent to no diversity;
- $\rho^2 = 0 \rightarrow$ uncorrelated.

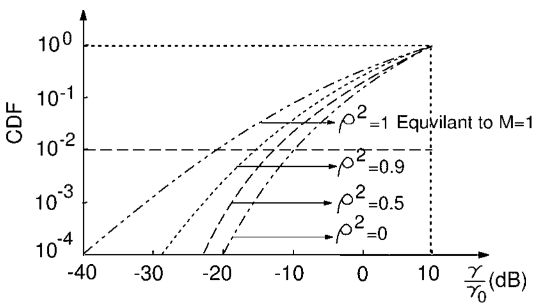

Figure 3: CDF of Correlated Diversity ($M = 2$)

Note that even when $\rho^2 = 0.9$. (i.e. the correlation is high), there are still substantial diversity gain obtained.

1.2.2 For Selection Diversity

$$P(\gamma) = 1 - e^{-\frac{\gamma}{\gamma_0}}[1 - Q(a,b) + Q(b,a)]$$

Where $Q(a,b) = \int_b^{+\infty} e^{-\frac{1}{2}(a^2+x^2)} I_0(ax)x\,dx$, and $b = \sqrt{\frac{2\gamma}{\gamma_0(1-\rho^2)}}$, $a = b\rho$.

For ρ, it has the same meaning as in MRC, and we have the same comment for it.

1.3 Effect of Correlated Noise

Until now, we assume that from branch to branch, the noise are uncorrelated. However, in practical systems, the noise received in one branch may not be uncorrelated with noise components in other branches.

Fact:

- Selection and Switched Diversity are not affected by noise correlation;
- MRC and Equal Gain Combining Diversity are affected by noise correlation.

Denote the noise signal in the i_{th} branch as n_i, assume:

$$E[n_i] = 0, E[n_i^2] = 1, \forall i,$$

$$\rho_{ij} = E[n_i n_j] = \rho, \forall i \neq j$$

The resulting noise power in a system with equal gain combining with M branches is:

$$\begin{aligned} E[n^2] &= E\left[(\sum_{i=1}^{M} n_i)^2\right] = \sum_{i=1}^{M} E[n_i^2] + M(M-1)E[n_i n_j] \\ &= M + M(M-1)\rho = M[1 + (M-1)\rho] \end{aligned}$$

Recall that for uncorrelated noise: $E[n^2] = M$.

Therefore, the resulting noise power in a correlated noise increases by a factor $[1 + (M-1)\rho]$. Sometimes it may be observed that SNR decreases with increasing M.

1.4 BER Performance of Diversity Systems

Bit error rate (BER) is obtained by first evaluating probabilities of error conditioned in SNR, then averaging it with respect to probability of SNR.

$$P_d = \int_0^{+\infty} P_e(\gamma)p(\gamma)\,dx$$

1.4.1 MRC Case

Here we make two assumptions:

1. We use MRC with M branches;
2. We use BPSK.

The detector makes decisions on the output of combiner. The resulting SNR is $\gamma = \sum_{i=1}^{M} \gamma_i = \frac{E_b}{N_0} \sum_{i=1}^{M} a_i^2$. with gains $g_i = ka_i$.

Then conditioning on $\{a_i\}$ or γ, $P_e(\gamma) = Q(\sqrt{2\gamma})$. Recall that

$$p_{MRC}(\gamma) = \frac{1}{(M-1)!}\frac{\gamma^{M-1}}{\gamma_0^M}\exp(-\frac{\gamma}{\gamma_0})$$

we get:

$$\begin{aligned} P_d &= \int_0^{+\infty} P_e(\gamma)p_{MRC}(\gamma)\,dx \\ &= \left(\frac{1-\mu}{2}\right)^M \sum_{i=0}^{M-1} \begin{pmatrix} M-1+i \\ i \end{pmatrix} \left(\frac{1+\mu}{2}\right)^i \end{aligned}$$

where $\mu = \sqrt{\frac{\gamma_0}{1+\gamma_0}}$

When mean SNR is large, $\gamma_0 \gg 1$, $\frac{1+\mu}{2} \approx 1$, $\frac{1-\mu}{2} \approx \frac{1}{4\gamma_0}$, then

$$\sum_{i=0}^{M-1} \begin{pmatrix} M-1+i \\ i \end{pmatrix} = \begin{pmatrix} 2M-1 \\ M \end{pmatrix}$$

so

$$P_d \approx \begin{pmatrix} 2M-1 \\ M \end{pmatrix} \left(\frac{1}{4\gamma_0}\right)^M$$

P_d decreases with M^{th} power of inverse mean SNR.

$\gamma_b = M\gamma_0 \rightarrow$ for frequency and time diversity.

$\gamma_b = \gamma_0 \rightarrow$ for space diversity, since in space diversity the signal is only transmitted once.

For Frequency and time diversity, at 10^{-5}, $M = 2$ gains $20dB$ over $M = 1$ and $M = 4$ gains $29dB$ over $M = 1$.

For space diversity, at 10^{-5}, $M = 2$ gains $23dB$ over $M = 1$ and $M = 4$ gains $35dB$ over $M = 1$.

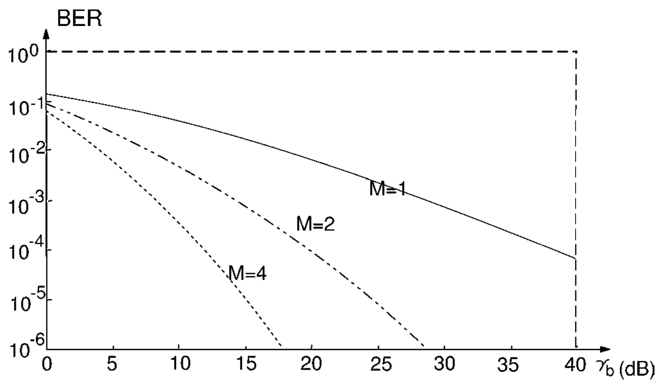

Figure 4: BER of Diversity System (MRC, BPSK)

1.4.2 Other Signaling Modulation Methods with MRC

1. FSK with MRC

$$P_d \approx \begin{pmatrix} 2M-1 \\ M \end{pmatrix} \left(\frac{1}{2\gamma_0}\right)^M$$

2. DPSK with MRC

$$P_d \approx \begin{pmatrix} 2M-1 \\ M \end{pmatrix} \left(\frac{1}{2\gamma_0}\right)^M$$

3. Noncoherent Orthogonal FSK

$$P_d \approx \begin{pmatrix} 2M-1 \\ M \end{pmatrix} \left(\frac{1}{\gamma_0}\right)^M$$

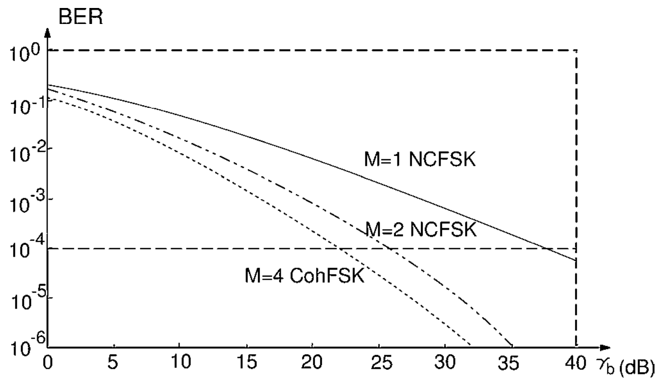

Figure 5: Comparison of BER for Different Modulation

1.4.3 Selection Diversity

For NCFK, $P_d = M \sum_{k=0}^{M-1} (-1)^k \begin{pmatrix} M-1 \\ k \end{pmatrix} \frac{1}{\gamma_0 + 2 + 2k}$

1.5 Macro (Base Station) Diversity Against Shadowing

1. Uplink: Typically Selection used in cellular systems.

2. Downlink: Let S_k be the average received signal strength from the k^{th} base station.

Here we use average received signals, i.e. Rayleigh (short scale) fading effects are averaged out.

$S_k \sim$ independent log-normal random variables, with mean $m_s dB$, standard deviation σdB. Let $\log_{10} X = S_k$, we have

$$p(x) = \frac{1}{x\sigma\sqrt{\pi}} e^{-\frac{1}{2\sigma^2}(\log_{10} x - m_s)^2}, x > 0$$

Note: M BSs, only 1 transmits to mobile at any time. The BS is determined by looking at strongest received signal from the mobile on the uplink, then the strongest BS transmits back to it.

From Macro Diversity systems with M BS, the probability that the average signal is below some threshold γ is:

$$P(S < \gamma) = \sum_{k=1}^{M} P(S_k < \gamma)$$

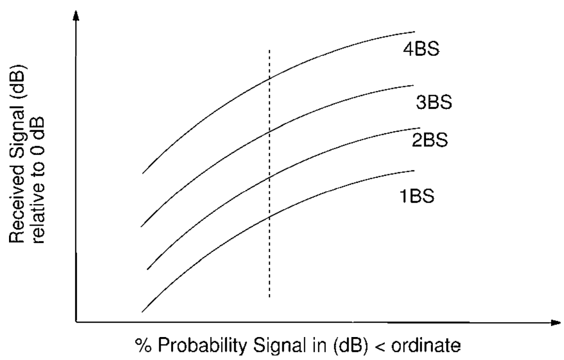

Figure 6: Macro Diversity

2 Spread Spectrum Modulation – CDMA

Definition: "Spreading" refers to expansion of bandwidth well beyond what is required to transmit digital data.

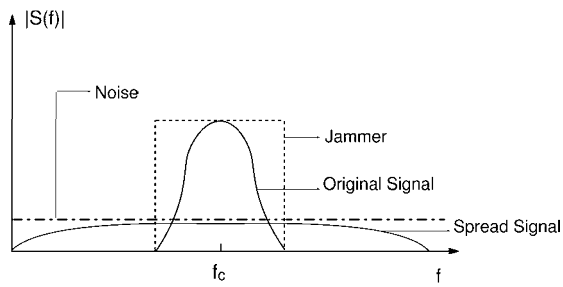

Figure 7: Comparison of Original and Spread PSD

Narrow band jammer has "less" impact on spreaded waveform.

1. Direct sequence spread spectrum:

$$s(t) = \sqrt{2p}b(t)c(t)\cos(\omega_c t + \phi)$$

   where $c(t)$ is spreading waveform and $b(t)$ is bit waveform.

2. Frequency Hopping

   Hop to new frequency every once in a while. Hedy Lamar (an actress) first proposed this idea.

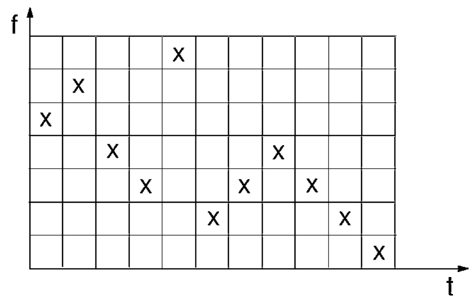

Figure 8: Frequency Hopping Pattern

2.1 DS–CDMA

Transmitting BPSK:

$$b(t) = \sum_{i=-\infty}^{+\infty} b_i p_T(t - iT), b_i \in \{-1, +1\}$$

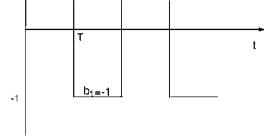

Figure 9: $b(t)$

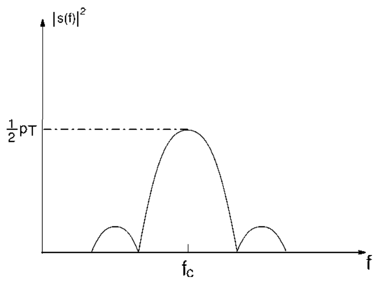

Figure 10: PSD of unspread signal $s(t) = \sqrt{2p}b(t)\cos(\omega_c t + \phi)$

TX signal:

$$s(t) = \sqrt{2p}b(t)c(t)\cos(\omega_c t + \phi)$$

where $c(t) = \sum_{i=-\infty}^{+\infty} c_n \psi(t - nT_c), c_n \in \{-1, +1\}$, and $\psi(t)$ is the chip waveform – time limited to $[0, T_c]$, which satisfies $\frac{1}{T_c}\int_0^{T_c} \psi(t)\,dt = 1$.

In practice, we usually choose $\psi(t) = p_{T_c}(t)$ or $\psi(t) = \sqrt{2}\sin\left(\frac{\pi t}{T_c}\right) p_{T_c}(t)$.

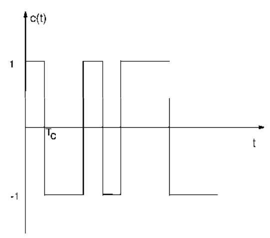

Figure 11: $c(t)$

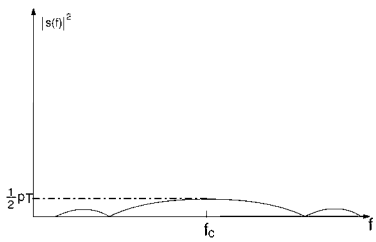

Figure 12: PSD of spread signal

Receiver for BPSK The received signal is:

$$r(t) = \sqrt{2p}b(t)c(t)\cos(\omega_c t) + \eta(t)$$

where $\eta(t)$ is AWGN.

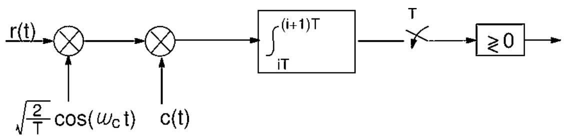

Figure 13: Matched Filter Receiver

$$\begin{aligned} Z &= \int_{iT}^{(i+1)T} \sqrt{\frac{2}{T}}c(t)\cos(\omega_c t)\gamma(t)\,dt \\ &= \sqrt{E_b}b_i + \eta' + \text{double frequency term} \end{aligned}$$

where double frequency term will become 0 after integration.

$$\eta' = \int \eta(t)\sqrt{\frac{2}{T}}\cos(\omega_c t)c(t)\,dt \sim N\left(0, \frac{N_0}{2}\right)$$

and $Z \sim N\left(\pm\sqrt{E_b}, \frac{N_0}{2}\right)$, where ' $\pm$ ' depends on 1 or -1 is transmitted.

So

$$P_b = P[Z > 0|b_i = -1] = \frac{1}{2}erfc\left(\sqrt{\frac{E_b}{N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

from which we can see that the performance doesn't change due to spread.

2.2 Multiple Access Capability of Spread Spectrum Advantage of CDMA

- Graceful degradation in performance with number of users;
- Universal frequency reuse;
- Voice activity can be exploited to increase capacity;
- Soft handoffs;
- Robustness to fading.

References

[1] Prof. Narayan B. Mandayam, class notes

[2] John G. Proakis, Digital Communications, 3^{rd} Edition, McGraw-Hill Inc., 1996

[3] Theodore Rappaport, Wireless Communications: Principles and Practice, Prentice Hall, 1996