1. Read the following statements and state if they are “true” or “false”. State appropriate reasons to justify your answer. (20)

(a) Samples (in time) taken from a white noise process are independent.
(b) Raised cosine pulse shaping with a roll-off factor $\alpha = 0.2$ requires lesser transmission bandwidth than ideal Nyquist pulse shaping.
(c) An ideal raised cosine pulse shape with $\alpha = 1$ has the property that the pulse width measured at half amplitude is exactly the bit duration.
(d) A decision-feedback equalizer is an example of a linear equalizer.
(e) QPSK modulation supports a higher data rate than BPSK modulation, but at a higher bit-error-rate.
(f) Noncoherent modulation improves error performance over coherent modulation.
(g) BPSK modulation with spreading improves the bit-error rate performance over BPSK modulation without spreading on an AWGN channel.
(h) BPSK modulation with spreading improves the bit-error rate performance over BPSK modulation without spreading on a multipath channel.
(i) Pseudo-Noise (PN) sequences of length $N$ can be closely approximated by random binary sequences for large values of $N$.
(j) The period of a PN sequence produced by a linear feedback shift register with $m$ flip-flops cannot exceed $2^m - 1$.

2. Binary data are transmitted over a radio link using coherent binary phase shift keying (15) at a data rate of $10^6$ bits per second. The power spectral density of the additive white Gaussian noise at the receiver is $10^{-10}$ Watts per Hertz.

(a) Find the average carrier power required to maintain an average probability of error of $10^{-4}$.

*Hint:* You may assume that $\text{erfc}(2.65) = 0.0002$, where the complimentary error function $\text{erfc}(x)$ which is given as

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-z^2)dz$$
3. Consider a binary communication system where a bit 1 is transmitted by sending a signal $s_1(t)$ and a bit 0 is transmitted by sending a signal $s_2(t)$. A modified correlator is designed as shown in Figure 1 to decide if a 0 or 1 was transmitted. The signals $s_1(t)$ and $s_2(t)$ are of unit energy and their correlation coefficient is given as

$$\rho = \int_0^T s_1(t)s_2(t)dt$$

![Figure 1: Modified Correlator](image)

(a) Assuming equiprobable hypotheses and transmission over an AWGN channel ($w(t)$ is a Gaussian random process of zero mean and power spectral density 1/2), find the probability of error of the above receiver as a function of the correlation coefficient $\rho$. Your answer should be in terms of the complimentary error function erfc(x) which is given as

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-z^2)dz$$

(b) Using the result in part (a), determine if orthogonal signaling ($s_1(t)$ and $s_2(t)$ are orthogonal to each other) is better than antipodal signaling ($s_1(t) = -s_2(t)$)

4. It is desired to transmit an analog signal with a maximum frequency of 5 KHz over a channel of bandwidth 2 MHz using direct-sequence spread spectrum modulation with spreading codes from an alphabet $\{+1, -1\}$. Assuming ideal sampling followed by using 2-bit quantization, answer the following questions:

(a) What is the processing gain of the system?
(b) What is the chip duration of the direct-sequence code?
(c) How many synchronous users can be supported in the system if it is desired to obtain a bit-error performance similar to that in a single user system? Justify your answer.
5. In a direct-sequence CDMA system with BPSK modulation, the feedback shift register used to generate the PN sequence has length $m = 7$. In the presence of an externally generated jamming signal, the system is required to have an average probability of bit error that does not exceed $10^{-4}$. Calculate the following parameters:

(a) Processing gain
(b) Antijam margin

*Hint:* Use the *Hint* from question 2 (a)

6. Consider a fiber-optic communication system where a bit 1 is represented by transmitting a laser of intensity $\lambda_1$ and a bit 0 is represented by sending a laser of intensity $\lambda_0$ (usually $\lambda_1 > \lambda_0$). The optical signals that are transmitted over an optical fiber are detected using a photo-detector receiver as shown in Figure 2 to decide if a 0 or 1 was transmitted.

The transmitted laser excites the photodetector device to produce photoelectrons $n$ at the output of the detector. The photoelectrons generated at the output are Poisson random variables distributed according to the intensity of the laser (either $\lambda_1$ or $\lambda_0$) as follows:

$$P(n = r) = \frac{\exp(-\lambda_i)\lambda_i^r}{r!}, \ i = 0, 1$$

The number of photoelectrons $n$ at the output of the photodetector is compared to a threshold $\gamma$ to decide if a 1 or a 0 was transmitted. Determine the optimum threshold $\gamma$ that minimizes the probability of making an error assuming equiprobable bit transmission.

*Good luck and Happy Holidays!*