Sensor Networks for Estimating and Updating the Performance of Cellular Systems

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Abstract—We investigate the use of an auxiliary network of sensors to assist radio resource management in a cellular system. Specifically, we discuss the number and placement of sensors in a given cell for estimating its signal coverage. Here, an "outage" is said to occur at a location if the mobile receiver there has inadequate signal-to-noise ratio (*SNR*-based outage) or, using another criterion, inadequate signal-to-interference ratio (*SIR*based outage); and the "outage probability" is the fraction of the cell area over which outage occurs. A design goal is to confine the number of sensors per cell to an acceptable level while accurately estimating the outage probability.

The investigation uses a generic path loss model incorporating distance effects and spatially correlated shadow fading. Our emphasis is the performance prediction accuracy of the sensor network, rather than cellular system analysis *per se*. Through analysis and simulation, we assess several approaches to estimating the outage probability. Applying the principle of importance sampling to the sensor placement, we show that a cell outage probability of ~ P_o can be accurately estimated using ~ $10/P_o$ power-measuring sensors distributed in a random uniform way over base-mobile distances from 50% to 100% of the cell radius. This result applies to both *SNR*-based and *SIR*-based cases, in both indoor and outdoor environments.

I. INTRODUCTION

We investigate an auxiliary network of sensors which assist radio resource management to improve the capacity and quality of service in cellular systems. Our focus is on new or envisioned cellular system designs in which antenna beams, power per beam and channel sets can be assigned adaptively to accommodate slowly changing conditions of the propagation and user population. The data collected by the sensor network can reduce the measurement demands on the active mobiles; or, it can be augmented by such measurements, to permit more dynamic adapting as individual mobiles change locations, start and end service, and so on.

We envision a network fabric of N sensors per cell ($N \sim 100$) which communicate with each other and, through some sensors, with the cellular system, Fig. 1. Each sensor has an identifying code and a fixed and known location, and it measures received power from pilots sent by its closest base and several bases nearby. As we will show, the collection of data from all the sensors can be used to estimate the percentages of each cell having adequate signal-to-noise ratio (*SNR*) and adequate signal-to-interference ratio (*SIR*).

The key benefit of the sensor network is that it provides round-the-clock measurements from many low-cost sensors Shalini Periyalwar Wireless Technology Labs, Nortel Networks shalinip@nortel.com

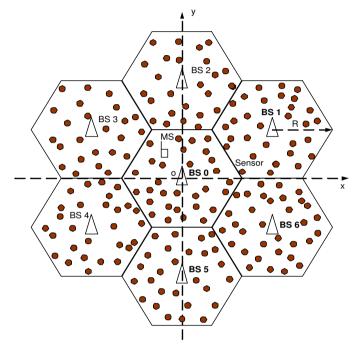


Fig. 1. A 7-cell cluster, with many sensors in each cell. We evaluate "outage" conditions in the center cell, both *actual* and as estimated using the sensors.

per cell, at known locations. The data so obtained can be used not only for medium-term radio resource management, but also for longer-term engineering, e.g., identifying the need for new cell sites. Comparisons with more traditional approaches are presented in Section II. Our calculations are based on a path loss model that incorporates distance effects and spatially correlated shadow fading, as described in Section III.

We will examine downlink outage probabilities based on SNR (Section IV) and on SIR (Section V) and discuss extensions to the uplink (Section VI). We will show, for different N, how accurately sensors can predict outage probability; how variable the predictions are with the specific sensor placements; and how much is gained when the sensors are confined to the region most likely to experience outage (e.g., the outer half of the cell).

A final commentary is in order regarding our study approach. Because predicted outage probability is a variable dependent on the specific realizations of the shadow fading and sensor placement, we will make extensive use of Monte Carlo

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simulations to get useful results. Also, a number of simplifying assumptions will be made to permit easy estimates of both the actual outage probabilities and those predicted by the sensor network. We emphasize that our goal is *not* cellular system analysis *per se* but, rather, an assessment of the performance prediction accuracy of a network of power-measuring sensors.

II. COMPARISON WITH TRADITIONAL APPROACHES

RF planning for wireless systems utilizes both proactive measurements (e.g., path loss) and reactive measurements (e.g., call drops, handovers). For proactive measurements, in most cases, currently used RF planning tools gather propagation information based on the use of a database in a given region, augmented by drive tests conducted during off-peak times. Such static snapshots of RF planning information are suitable for current systems with fixed antenna patterns and limited use of adaptive algorithms. Data collected by mobiles and relayed to base stations may deliver additional timeof-day-specific RF planning information. However, relying on mobiles alone to provide signal and interference power measurements has limited benefit and adds demands to scarce mobile battery resources. Furthermore, a given mobile can measure downlink conditions only, may not be equipped with GPS receivers to help associate its measurements with location, and reports at uncontrolled times and locations.

Sensor-based measurements can react to gradual changes in propagation (e.g., new structures (especially in cities)) or interference (e.g., due to adaptive beamforming). They are not labor-intensive and are available at all times, to accommodate slow adaptive changes in radio resources. The sensors can be more numerous and measurements may be gathered moreor-less uniformly from known locations, facilitating reliable outage evaluations. In fact, the potential exists to accurately pinpoint chronically poor service areas that arise after initial planning, and to identify the need for new or reengineered sites. Additionally, the sensor network could be extended to support multiple air interfaces within overlapping coverage regions (e.g., wireless LAN, DVB-H deployments).

These arguments notwithstanding, a given operator may want to consider a wide range of approaches, including: (1) The traditional combining of site data with drive testing; (2) deploying a dedicated network of sensors; (3) renting service from an existing multipurpose sensor network; (4) using a set of subscriber mobiles, equipped with GPS, to periodically measure and report power measurements; and so on. For those approaches based on sensor or mobile measurements, the rate of measurement-and-report (e.g., hourly, daily, etc) can be tailored to maintain acceptable levels of battery drain. Choosing among candidate approaches would require a cost/performance tradeoff analysis that is beyond the scope of this study; our purpose here is to assess the attainable performance of outage estimation based on distributed power measurements and to minimize the number of such measurements required. It should be kept in mind that the analytical methods and numerical results reported here apply to any distributed-measurement approach, not just dedicated sensor networks.

III. PATH LOSS MODEL

Assuming the model of [1], the path loss (*PL*) from a base station (BS) to a location ξ in the environment is [1]

$$PL(\xi)[dB] = A + 10\gamma \log(d/d_0) + s(\xi); d > d_0 \quad (1)$$

where d is the distance from the BS to ξ and d_o is a reference distance (typically, 1 m indoors and 100 m outdoors). The intercept A is given by $20 \log(4\pi d_o/\lambda)$, where λ is the wavelength. The path loss exponent γ can range from 3 to 6, depending on the environment; the dB shadow fading, $s(\xi)$, is a Gaussian process over space with zero mean and standard deviation σ ; and σ can range from 4 dB to 12 dB. We assume that the autocorrelation of the spatial process $s(\xi)$ depends only on the separation distance, i.e.,

$$E[s_a s_b] = \sigma^2 e^{-d_{ab}/X_c} \tag{2}$$

where d_{ab} is the distance from *a* to *b*; and X_c , the shadow fading correlation distance, can range from several to many tens of meters [2]. We will assume a frequency of 2 GHz and $\gamma = 3.8$ in all our computations; and will consider different combinations of σ , X_c , d_o and cell radius for different cellular environments.

IV. OUTAGE PROBABILITY BASED ON SNR

A. Major Assumptions

We assume each of N sensors in a given cell measures the received power of a downlink pilot signal and compares that power, P_R , to a threshold value. That threshold is the value at which a mobile receiver near the sensor would have just enough signal-to-noise ratio for good reception. The fraction of sensors measuring power below the threshold is the sensor network's estimate of the cell's downlink (*SNR*-based) outage probability. We also assume that the pilot power measurement is over a bandwidth sufficiently wide (5 MHz or more) that multipath fading is averaged out. Thus, the measurement of P_R , combined with knowledge of the downlink transmit power per user and the antenna gains, permits the network to estimate the downlink path loss, *PL*. We note that, due to the averaging over multipath fading, this estimate applies to the uplink path loss as well.

For our purposes, it is safe to assume the antenna gains are independent of sensor position, so that the variation of P_R over the sensors precisely tracks the variation of PL, i.e., $P_R = C - PL$, where C is the same for all sensors. We can thus use the statistical path loss model of Section III to simulate the cell-wide variation of received signal power.

B. The Statistics of Outage Probability

The *true* outage probability in Cell j, denoted by $p_o(j)$, is the fractional area for which a mobile's received power would fall below some threshold P_{Ro} (equivalently, path loss would be above a threshold $PL_o = C - P_{Ro}$). Although the shadow fading spatial distribution, $s(\xi)$, is governed by the same model in every cell, (2), the actual *realization* of $s(\xi)$ will vary, resulting in a cell-to-cell variation in $p_o(j)$ for a given PL_o . Across a large number of cells, then, p_o can be characterized by a *statistical distribution*, with an average P_o and a standard deviation σ_o . The latter represents the natural inter-cell variability of outage probability caused by the randomness of shadow fading.

The *estimated* outage probability $p'_o(j)$ in Cell j contains another form of variability, this one being *intra*-cell. It arises from the fact that one placement of N sensors within a cell will produce a different estimate than another placement. Over a great many placements, the estimates $p'_o(j)$ in Cell j will thus have a distribution of values, with a mean $p_o(j)$ (unbiased estimate) and a standard deviation σ_j . We can expect that σ_j will diminish towards zero as N increases towards infinity. A reasonable design goal is to choose N sufficiently large that $\sigma_j/p_o(j) < 0.25$ for the p_o -value of interest. We will find a simple relationship between N and p_o from our simulations that meets this goal.

C. Simulation Approach

To do a simulation, we first specify a cell radius R and the values of the propagation parameters in (1) and (2). We can then generate a 2-dimensional variation of shadow fading, $s(\xi)$, for Cell 1 that follows the model, using the method described in [3]. The next step is to choose a value for N, a placement for the N sensors within the cell, and a path loss threshold, PL_o . Finally, the path loss at each of the sensors is determined, and $p'_o(1)$ is computed as the fraction of sensors for which $PL > PL_o$.

With $s(\xi)$ fixed, the sensor placement is chosen M times, and with M sufficiently high, the mean and standard deviation, $p_o(1)$ and σ_1 , can be estimated. (As noted, σ_1 is a measure of the variability of the estimate with sensor placement.) This procedure is repeated for a total of N_{sh} generations of the shadow fading variation, $s(\xi)$, corresponding to Cells $1, 2, ..., j, ... N_{sh}$. The mean of $p_o(j)$ over j is the network's estimate of the average outage probability, P_o ; the standard deviation of $p_o(j)$ over j is the network's estimate of the intercell standard deviation, σ_o ; and the mean of σ_j over j, denoted by ϱ , is the average intra-cell standard deviation related to sensor placement. We call the ratio ϱ/P_o the "sharpness" of the estimate, and seek to make it smaller than 0.25.

The baseline values of P_o and σ_o , i.e., those we assume to be the true ones, are obtained by first assigning an extremely large value for N. We have found, by a combination of analysis and simulation (not shown here), that N = 4000 would yield precise estimates in each cell, with negligible variation from one placement of sensors to another. Accordingly, we computed p_o , for each of N_{sh} cells and each of several values of PL_o , by postulating 4000 uniformly located measurements per cell. In this way, we obtained "true" values of P_o and σ_o vs. PL_o and identified the PL_o values producing average outage probabilities of 0.05 and 0.10. Then we applied the procedure of the preceding paragraph for these values, using practical values of N (32, 100 and 200). For these values, we did M = 100 placements of the N sensors, and $N_{sh} = 10$ realizations of the shadow fading distribution.

In all cases, we assumed a randomly uniform placement of the sensors. However, we also considered confining sensor locations to the regions most likely to experience outage. In this way, we reasoned, the estimates from N sensors would be less sensitive to the precise placement, i.e., the "sharpness" of the estimates would be lower. Since outage is more likely at distances farther away from the base station, we considered placements confined to distances from R_{min} to R, with candidate values of R_{min} being 0, 0.5R and 0.7R. In doing this, the estimate of outage probability (fraction of sensors with $PL > PL_{0}$) must be weighted by the ratio of areas, i.e., that of the annular region to that of the entire cell. This approach is the essence of *importance sampling*, in which measurements are focused on the regions where the events of interest are most likely to occur [4]. As an example of the possibilities, Fig. 2 shows a circular cell with N = 4000sensors and $PL_o = 120$ dB. The dark spots are the sensor locations where PL > 120 dB, and they are seen to be concentrated in the outer regions of the cell. Placing sensors close to the center, therefore, can amount to wasting limited resources on predictable "non-events".

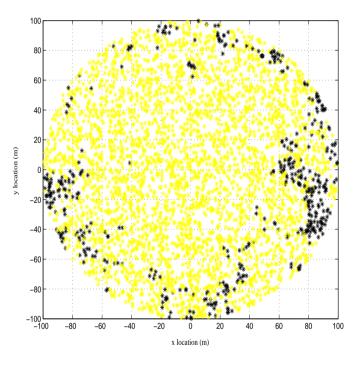


Fig. 2. An "outage" map for a single cell with 4000 uniformly located sensors. In this example, the cell radius is 100 m, $\sigma = 8$ dB, $X_c = 8$ m, and "outage" corresponds to the condition $PL > PL_o = 120$ dB. The dark dots indicate outages, which occur for 11.8% of all sensors, primarily in sensors located towards the cell boundary.

D. Results

First, we investigate an outdoor cell, conveniently assumed to be circular, with radius R = 1000 m. The shadow fading parameters (σ, X_c) are (8 dB, 50 m). We set PL_o at values that yield "true" average outage probabilities, P_o , of 0.05 and 0.10. For each of these two cases, we computed the

network-estimated values of P_o and σ_o and the intra-cell variation parameter ρ . The results are summarized in Table I for N = 32,100 and 200 and, for each N, for full-cell placements of sensors ($R_{min} = 0$) and two candidate partial-cell placements ($R_{min} = 0.5R$ and 0.7R).

The tabulated results show that N = 32 sensors are too few for accurate estimation of outage probabilities of 5% and 10%, if only because the sharpness, ϱ/P_o , is too large. We also see that, for N = 100 and 200, full-cell placement of sensors leads to good estimates of P_o , but that partial-cell placements lead to better sharpness. The case $R_{min} = 0.7R$, however, tends to underestimate P_o because it misses (undercounts) outage events. The best compromise between accurate estimation of P_o and low sharpness occurs consistently for $R_{min} = 0.5R$. Finally, we see that for $P_o = 0.05(0.10)$, the value of N that yields both accurate P_o and low sharpness at this R_{min} is 200 (100). We infer from this that a good rule for the number of sensors per cell is $N \sim 10/P_o$. This result is consistent with binomial statistics.

Next, we examine an indoor environment, where R = 100 m, and we consider three different sets of shadow fading parameters (σ, X_c) : (8 dB, 8 m), (8 dB, 50 m) and (10 dB, 50 m). Results are given in Table II for $P_o = 0.05$. These results, and those for $P_o = 0.1$ as well (not shown), reinforce the findings from the previous example. Moreover, they show that the shadow fading parameters influence σ_o but not the general rules for R_{min}/R and N.

V. OUTAGE PROBABILITY BASED ON SIR

While the above study of outage probability based on *SNR* was generic, the study of *SIR*-based outage probability requires specificity about the radio interface. For this purpose, we assume a CDMA system with a spreading factor of 128 and a required receiver output *SIR* of 5 dB. For simplicity, we assume that the downlink co-channel interference from the six surrounding cells is dominant. Also, we assume that each sensor is able to identify, from downlink pilots, the power from each base (its own plus the six nearest interfering bases) [5]; that each base is transmitting its full rated power; and that an "outage" occurs for a mobile if its serving base runs out of power before it is able to meet that mobile's *SIR* requirement. These assumptions, combined with the above path loss model, enable us to compute outage probability for a given number, K, of active mobiles per cell (or sector).

We note that for K > 1, there is one more layer of randomness, besides those for the shadow fading distribution and the sensor placement, namely, the placement of the K mobiles. Thus, for every combination of $s(\xi)$ -realization and N-sensor placement, the network computes an outage probability for each of M_{mt} random placements of K mobiles over the cell, then averages the M_{mt} values. In our study, we used $M_{mt} = 500$.

The above steps are straightforward for full sensor placement $(R_{min} = 0)$. However, we also considered partial placement, specifically, $R_{min} = 0.5R$. In this case, the N sensors are uniformly distributed over 3/4 of the cell area, but no sensors are in the inner region $(d < 0.5R_{min})$, where, on average, 1/4 of the K mobiles would be located. To address this, the network can estimate outage probability as follows: (1) compute an upper bound by assuming *all* of the K mobiles are in the outer region; (2) compute a lower bound by assuming 3/4 of the mobiles are in the outer region and *none* are in the inner region; and (3) estimate the outage probability as the mean of the two bounds.

Results are given in Table III for different combinations of N, K and R_{min}/R . The increase in P_o with K, due to the dividing of transmit power among more mobiles, is evident. We also see, as before, that partial placement with $R_{min} = 0.5R$ and $N \sim 10/P_o$ yields good accuracy and sharpness.

VI. CONCLUSION

We have postulated a sensor network approach to estimating downlink outage probabilities in a cellular system. Using stochastic simulation, we investigated ways to minimize the number (N) of sensors needed, including the principle of importance sampling. Minimizing N can have a substantial payoff, in terms of both the drain on sensor batteries and the information bandwidth needed by the sensor network.

Extensions of this study might take into account issues such as base station selection (e.g., *SIR*-based as opposed to distance-based, as considered here), other kinds of interference (outer rings, intra-cell, etc), location information and uplink performance. Regarding the latter, we note that each sensor is assumed to be able to estimate its path loss (which is essentially the same in both directions) to the seven nearest bases. This information, plus some additional computing, would allow the sensor network to estimate *uplink* outage probabilities, as well. The availability of location information can be used to map the variations in outage regions with adaptive algorithms to assist in the calibration of these algorithms to provide ubiquitous coverage. In addition to outage, data rate coverage regions can also be identified.

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| Case | N | 4000 | 200 | | | 100 | | | 32 | | |
|------|-----------------------|------|------|------|------|------|------|------|------|------|------|
| | $R_{min}(\mathbf{m})$ | 0 | 0 | 500 | 700 | 0 | 500 | 700 | 0 | 500 | 700 |
| 1 | $P_o(\%)$ | 5 | 5.07 | 4.99 | 4.51 | 5.02 | 5.06 | 4.45 | 5.1 | 4.88 | 4.31 |
| | $\sigma_o(\%)$ | 0.92 | 1 | 0.97 | 0.94 | 1.07 | 0.96 | 0.95 | 1.09 | 1.07 | 0.95 |
| | $\varrho(\%)$ | 0 | 1.53 | 1.30 | 1.02 | 2.21 | 1.85 | 1.4 | 3.93 | 3.22 | 2.52 |
| | ϱ/P_o | 0 | 0.30 | 0.26 | 0.23 | 0.44 | 0.37 | 0.31 | 0.77 | 0.66 | 0.58 |
| 2 | $P_o(\%)$ | 10 | 10 | 9.97 | 8.56 | 9.99 | 9.79 | 8.62 | 9.95 | 9.72 | 8.66 |
| | $\sigma_o(\%)$ | 1.68 | 1.71 | 1.67 | 1.59 | 1.78 | 1.84 | 1.74 | 1.85 | 1.79 | 1.82 |
| | $\varrho(\%)$ | 0 | 2.07 | 1.81 | 1.30 | 3 | 2.41 | 1.92 | 5.25 | 4.42 | 3.16 |
| | ϱ/P_o | 0 | 0.21 | 0.18 | 0.15 | 0.30 | 0.25 | 0.22 | 0.53 | 0.45 | 0.36 |

TABLE I Simulation results for $P_o=5\%$ and 10%, R=1000 m, and $(X_c,\sigma)=(50$ m, 8 dB).

TABLE II

Simulation results for $P_o = 5\%$, R = 100 m, and three cases of (X_c, σ) : (8 m, 8 dB), (50 m, 8 dB), (50 m, 10 dB), in ascending order.

| (X_c, σ) | N | 4000 | 200 | | 100 | | | 32 | | | |
|-----------------|----------------|------|------|------|------|------|------|------|------|------|------|
| | $R_{min}(m)$ | 0 | 0 | 50 | 70 | 0 | 50 | 70 | 0 | 50 | 70 |
| (8 m, 8 dB) | $P_o(\%)$ | 5.0 | 4.96 | 5.02 | 4.59 | 5.00 | 4.96 | 4.53 | 4.92 | 4.91 | 4.50 |
| | $\sigma_o(\%)$ | 1.42 | 1.57 | 1.52 | 1.37 | 1.56 | 1.42 | 1.44 | 1.32 | 1.58 | 1.44 |
| | $\varrho(\%)$ | 0 | 1.51 | 1.30 | 1.03 | 2.08 | 1.83 | 1.47 | 3.75 | 3.30 | 2.53 |
| | ϱ/P_o | 0 | 0.30 | 0.26 | 0.22 | 0.42 | 0.37 | 0.32 | 0.76 | 0.67 | 0.56 |
| (50 m, 8 dB) | $P_o(\%)$ | 5.0 | 5.04 | 5.05 | 4.65 | 4.97 | 5.07 | 4.63 | 4.91 | 5.15 | 4.60 |
| | $\sigma_o(\%)$ | 4.27 | 4.86 | 4.93 | 4.54 | 4.80 | 4.89 | 4.58 | 4.57 | 5.14 | 4.39 |
| | $\varrho(\%)$ | 0 | 1.36 | 1.15 | 0.90 | 1.89 | 1.65 | 1.28 | 3.53 | 2.96 | 2.24 |
| | ϱ/P_o | 0 | 0.27 | 0.23 | 0.19 | 0.38 | 0.33 | 0.28 | 0.72 | 0.57 | 0.49 |
| (50 m, 10 dB) | $P_o(\%)$ | 5.0 | 5.08 | 4.99 | 4.56 | 5.06 | 4.98 | 4.53 | 5.15 | 5.11 | 4.46 |
| | $\sigma_o(\%)$ | 5.17 | 4.90 | 4.94 | 4.50 | 5.03 | 4.81 | 4.52 | 5.09 | 4.93 | 4.45 |
| | $\varrho(\%)$ | 0 | 1.35 | 1.15 | 0.91 | 1.91 | 1.64 | 1.24 | 3.33 | 2.88 | 2.27 |
| | ϱ/P_o | 0 | 0.27 | 0.23 | 0.20 | 0.38 | 0.33 | 0.27 | 0.65 | 0.56 | 0.51 |

TABLE III Simulation results for R=100 m, $X_c=8$ m, $\sigma=8$ dB, and four values of K: 1, 4, 8, 12.

| K | N | 4000 | 200 | | 1(| 00 | 32 | | |
|----|----------------|------|------|------|------|------|-------|-------|--|
| | $R_{min}(m)$ | 0 | 0 | 50 | 0 | 50 | 0 | 50 | |
| 1 | $P_o(\%)$ | 6.02 | 6.00 | 5.98 | 5.98 | 5.94 | 6.09 | 5.93 | |
| | $\sigma_o(\%)$ | 1.32 | 1.34 | 1.28 | 1.22 | 1.32 | 1.28 | 1.25 | |
| | <i>ρ</i> (%) | 0 | 1.65 | 1.42 | 2.34 | 1.99 | 4.20 | 3.59 | |
| | ϱ/P_o | 0 | 0.28 | 0.24 | 0.39 | 0.34 | 0.69 | 0.61 | |
| 4 | $P_o(\%)$ | 6.90 | 7.00 | 6.96 | 7.08 | 6.97 | 7.11 | 7.04 | |
| | $\sigma_o(\%)$ | 1.39 | 1.44 | 1.47 | 1.45 | 1.42 | 1.48 | 1.35 | |
| | $\varrho(\%)$ | 0 | 1.78 | 1.55 | 2.47 | 2.09 | 4.32 | 3.66 | |
| | ϱ/P_o | 0 | 0.25 | 0.22 | 0.35 | 0.30 | 0.61 | 0.52 | |
| 8 | $P_o(\%)$ | 8.33 | 8.36 | 8.46 | 8.38 | 8.44 | 8.67 | 8.57 | |
| | $\sigma_o(\%)$ | 1.61 | 1.60 | 1.58 | 1.57 | 1.56 | 1.69 | 1.52 | |
| | $\varrho(\%)$ | 0 | 1.87 | 1.58 | 2.55 | 2.22 | 4.57 | 3.82 | |
| | ϱ/P_o | 0 | 0.22 | 0.19 | 0.30 | 0.26 | 0.53 | 0.45 | |
| 12 | $P_o(\%)$ | 9.66 | 9.72 | 9.89 | 9.85 | 9.95 | 10.01 | 10.23 | |
| | $\sigma_o(\%)$ | 1.81 | 1.78 | 1.75 | 1.79 | 1.76 | 1.74 | 1.72 | |
| | $\varrho(\%)$ | 0 | 1.98 | 1.70 | 2.83 | 2.29 | 4.94 | 4.22 | |
| | ϱ/P_o | 0 | 0.20 | 0.17 | 0.29 | 0.23 | 0.49 | 0.41 | |