# Information-theoretic Capacity Analysis in MIMO Distributed Antenna Systems

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***Abstract*** [1] **— Distributed antenna system (DAS) can reduce the access distance and so lower the required transmit power compared with the conventional multiple input multiple out (MIMO) system. This paper investigated the information-theoretic capacity of DAS based on the channel model considering path loss, log-normal shadowing fading and Rayleigh fading. We find that the location of the receiver affects the DAS capacity and the difference between capacity with water-filling power allocation and that with equal-power allocation is significant. Furthermore, simulation results show that DAS offers large capacity gains over the traditional MIMO system.**

## I. INTRODUCTION

Distributed antenna system (DAS) provides macro-diversity to counteract the large-scale fading and reduces the access distance by distributing the antennas geographically. It was first introduced to solve the problem of coverage for indoor wireless communications and later applied to CDMA systems. Power control schemes and user capacity in DAS have been studied in [1][2][3] and results show that DAS can bring significant power saving. However, most existing researches are focused on the receiver performance and few aims at downlink information-theoretic capacity.

Another important technique in the field of wireless communication is multiple input multiple output (MIMO) system which is ignited by the pioneering work by Foschini[4] and Telatar[5]. MIMO has currently drawn considerable attention for its huge capacity potential and link reliability. Its capacity is proportional to the minimum of the number of the transmit (Tx) antennas and receive (Rx) antennas in the presence of rich scattering leading to antenna de-correlation and full channel rank.

Therefore, by combining the advantage of these two techniques together, in this paper, we investigate the information-theoretic capacity of DAS in virtue of MIMO theory based on the channel model considering the Rayleigh fading, log-normal shadowing fading and path loss. To utilize the advantage of short access distance in DAS, we adopt the power control scheme that the overall transmission power grows proportional to the path loss of the nearest transmit antennas.

This information-theoretic capacity study focuses on the downlink single-user (M, N) system, with M Tx and N Rx antennas, utilizing either water-filling power allocation or equal-power allocation. Aside from the capacity comparison of DAS with the traditional MIMO system, we also investigate the impact of the receiver location on the capacity and the water-filling gain in DAS with various numbers of antennas.

In the next section, we present the analytical model, including the DAS topology, channel model, power control scheme and comparison assumptions. In section 3, we review the MIMO channel capacity with two power allocation schemes. Then the numerical results are shown in section 4. Finally, a summary of the paper is presented.

## II. SIMULATION MODELS

### *A. DAS Topology Model*

The involved transmit remote antennas in DAS are distributed uniformly geographically while N receive antennas are co-located at the mobile station. Let each remote antenna be located at the center of a hexagon area, and then we can divide the whole area seamlessly into many hexagon sub-areas. In each frame, the channel conditions of these remote antennas are measured and analyzed by the mobile station and M of the antennas with the best channel gain are selected to transmit the same signals at the next frame.

### *B. Channel Model*

The following notations are used throughout the paper: ' for vector transpose, * for transpose conjugate.

We consider a single-user radio channel with M transmit antennas and N receiver antennas, denoted as (M, N) system. The channel is assumed to be linear and time-invariant, based on the following discrete-time equivalent model:

$$X = [x_1, x_2, x_3, ..., x_M]' \quad (1)$$

[1] This work was partly supported by supported by Chinese National Natural Science Foundation under Grant 90204001 and 863 Project “Research on radio transmission link technology of next generation mobile cellular communication systems”.

$X=[x_1,x_2,x_3,...,x_M]'$ is a $M\times 1$ vector, whose i-th component represents the signal transmitted by the ith Tx antenna. The total average transmit power is assumed to be P regardless of M. Y and Z are both the $N\times 1$ vectors which present the Rx signal and the additive white Gaussian noise (AWGN) with zero mean and unit variance, respectively.

The channel is assumed to be quasi-static, namely, the channel is constant in a frame and varies from frame to frame. Since the channel in DAS is not point-to-point channel, path loss and lognormal shadowing, as well as Rayleigh fading, are taken into account in our channel model. Particularly, if $h_{ij}$ represents the channel gains between receiver $i$ and transmitter $j$, it can be expressed as

$$h_{ij}=r_{ij}^{-\alpha/2}.\beta_{ij} \tag{2}$$

where, $r_{ij}$ denotes the distance from the $i$-th Rx antenna to the $j$-th Tx antenna. $\alpha$ is the path-loss exponent and set to 4 in this paper. $\beta_{ij}$, a complex random variable, represents the fast fading and log-normal shadowing effects. Its amplitude is Rayleigh distributed, whose mean square value is a log-normal random variable with zero mean and variance $\sigma_s$ [6].

### C. Power Control Scheme

To describe DAS power control scheme more clearly, we firstly review the conventional point-to-point MIMO systems. In MIMO systems, all the M transmit antennas are concentrated as an antenna-set and the distances from the receive antennas to the transmit antennas are the same, which can be denoted as $r_1$. In order to overcome the effect of path-loss, the total transmit power P of the M antennas would be proportional to $r_1^4$:

$$P=P_{0c}\cdot r_1^4 \tag{3}$$

In DAS, however, the distances from each transmit remote antennas to the receivers are not equal. Therefore, the power control scheme of the transmit antennas in DAS should be different from that of the traditional MIMO. There are many possible power control schemes and we adopt one that best utilizes the short access distance advantage of DAS. Suppose the distance between the receiver and the nearest Tx antenna is $r_2$, then the total power P is $P=P_{0d}\cdot r_2^4$. In this scheme, the path loss of the transmit antenna nearest to the receiver can be counteracted; while those of other transmit antennas still influenced the channel capacity.

### D. Comparison Assumptions

In order to compare the capacity of DAS with that of the traditional MIMO fairly, we make the following assumptions. Let $R_m$ and $R_d$ be the radius of remote antenna hexagon in traditional MIMO system and DAS separately. Firstly, the number of transmit antenna in unit area should be equal for DAS and MIMO, that is

$$\frac{M}{R_m^{\ 2}}=\frac{1}{R_d^{\ 2}} \tag{4}$$

Secondly, the total transmit power in unit area of these schemes should be equal too, that is,

$$\frac{\left(\int_0^{2\pi}\int_0^{R_m} P_{\mathrm{m}}\times \mathrm{r}^4\times \mathrm{r}\, dr d\theta\right)}{R_m^2}=\frac{\left(\int_0^{2\pi}\int_0^{R_d} P_{\mathrm{d}}\times \mathrm{r}^4\times \mathrm{r}\, dr d\theta\right)}{R_d^2} \tag{5}$$

$$\text{So,}\quad P_m\cdot R_m^4=P_d\cdot R_d^4\Rightarrow P_m=\frac{1}{M^2}P_d \tag{6}$$

## III. CHANNEL CAPACITY

Channel capacity is defined as the highest rate at which information can be sent with arbitrarily low probability of error [7]. It can be viewed as random variable determined by the specific realization of channel matrix that will change with the location of the receiver, SNR, (M, N) and etc. There are three kinds of commonly used information-theoretic capacity: ergodic capacity, outage capacity and minimum rate capacity [8]. In this paper we focus on the 90% outage capacity with an outage rate 0.10.

In the simulation, the receiver is assumed to know the channel state information perfectly. As to the transmit side, its knowledge of the channel determines the strategy to distribute the overall transmit power over the antennas: if the transmitter has perfect knowledge about the channel, water-filling power allocation would be used to achieve the capacity denoted as $C_w$; however, if the channel is totally unknown at the transmitter, equal power allocation should be performed to get the capacity $C_e$.

From [4], we can get the capacity formula with equal power allocation

$$C_e=\log\det\left(I_N+\frac{P}{M\sigma^2}\mathbf{HH}^*\right)=\sum_{i=1}^{k}\log\left(1+\frac{\rho}{M}\lambda_i\right) \tag{7}$$

, where $k$ is the rank of channel, $\lambda_i$ is the eigenvalue of $\mathbf{HH}^*$ and $\rho=P/\sigma^2$.

[8] gives the capacity formula with water-filling power allocation

$$C_w=\sum_{i=1}^{k}\left(\log(\mu\lambda_i)\right) \tag{8}$$

where $\mu$ satisfies

$$\sum_{i=1}^{k}\left(\mu-\lambda_i^{-1}\right)^+=P \tag{9}$$

and $a^+$ means max {0,a}.

## IV. NUMERICAL RESULTS

In this section, we give the simulation parameters and present the numerical results of 90% outage capacity of DAS with different M, N, $\rho$ and power allocation methods.

We set $\sigma_s$ to be 8dB and the radius of hexagon $R_d$ to be 1.The SNR in the figures ranges from 6dB to 16dB. From equation (4), we can get $R_m$ = 2 for the (4, 2) traditional MIMO system in a fair comparison.

We first investigate how the location of the receiver affects the DAS capacity. Simulation results of (2, 2) system in figure 1 indicate that unlike traditional MIMO, the receiver location influences the DAS capacity, where r at the abscissa denotes the distance from the receiver to one remote antenna. When the receiver most approximates the boundary of the hexagon area, that is $r = \cos 30° \approx 0.9$, the capacity reaches its peak. As the distance from the receiver to the nearest remote antenna r rises from 0.1 to 0.9, the capacity increases first steadily then acutely. When r reaches 1, which means the receiver crosses into the hexagon area of another remote antenna, the capacity falls. As we know, the power control scheme only counteracts the path-loss of the nearest remote antenna and that of other antennas still influences the capacity. It is easy to prove that the influence of the path-loss is the least when the receiver is at the boundary of the hexagon areas and it is largest when the receiver is at the hexagon center near one of the remote antenna. This just explains the capacity variance.

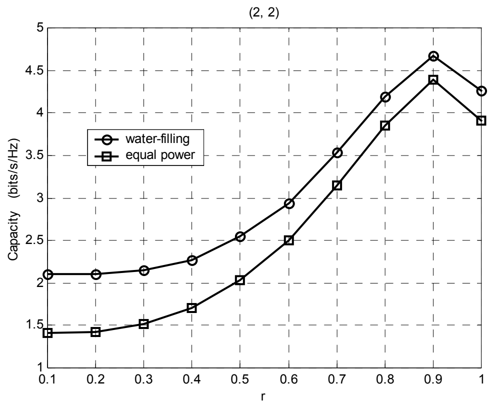
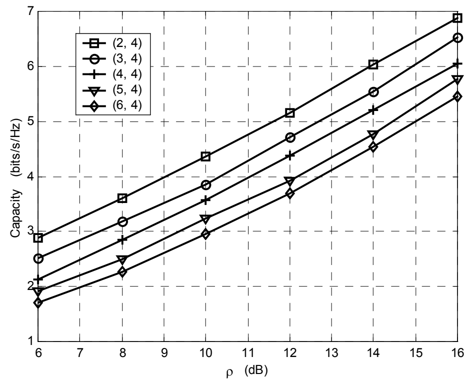
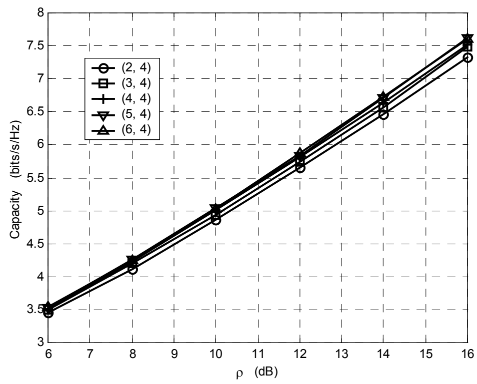



Figure 1. Impact of receiver location on capacity

Secondly, in figure 2, we investigate how the capacity varies with the number of transmit antennas. Figure 2 (a) shows that the capacity with equal-power scheme decreases with the number of transmit antennas rising from 2 to 6, when there are four Rx antennas. It is contrary to the common belief that the MIMO capacity is proportional to the minimum of the number of transmit and receive antennas. To explain this, we should notice that the distributed remote antennas have distinctive path-loss and some of them influence the capacity value. The nearer remote antennas have better sub-channel channel gains than the farther ones. Therefore, increasing the number of the transmit antennas in DAS actually does decrease the element value of the channel matrix while rising its rank. Especially when using the equal power allocation, the capacity gain is very poor by increasing M. Even with the optimized water filling allocation in figure 2 (b), the capacity gain is still not significant, although much better than figure 2 (a).



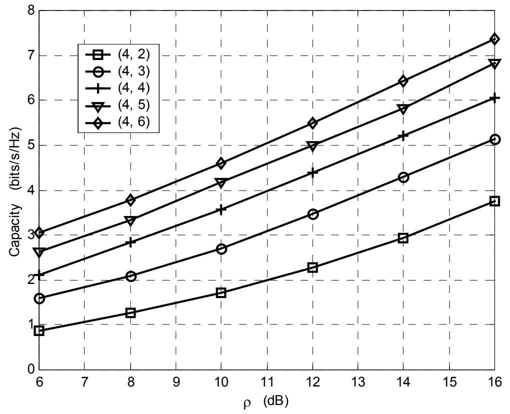

(a). Capacity with equal power allocation



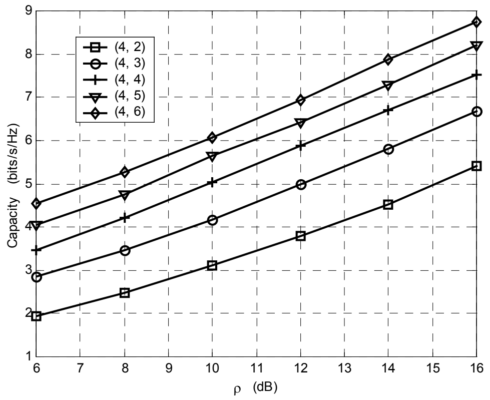

(b). Capacity with water-filling

Figure 2. Capacity with four receive antennas

Similarly, we study the relationship of the channel capacity and the number of the receive antennas in figure 3, with equal-power in fig (a) and water-filling in fig (b). It is indicated that the DAS capacity can be improved with the number of the receive antennas in both cases, just similar to that of the traditional MIMO.




(a). Capacity with equal power allocation




(b). Capacity with water-filling

Figure 3. Capacity with four transmit antennas

Considering both cases above, another thing we can see is that the capacity gain of water-filling over equal-power is significant in DAS, especially with large number of transmit antennas. E.g. the (6, 4) systems obtains more than 80% water-filling gain.

Furthermore, the outage capacity of DAS is compared with that of the conventional point-to-point MIMO system. From figure 4 we can see that the capacity of (4, 2) DAS system is 30-60% percent higher than that of the conventional (4, 2) MIMO system. In another word, the DAS system has about 3dB benefit than traditional MIMO. As is mentioned before, DAS that distributes the transmit antennas reduces the access distance, so it reduces the required transmit power, leading to the improvement of the channel capacity.

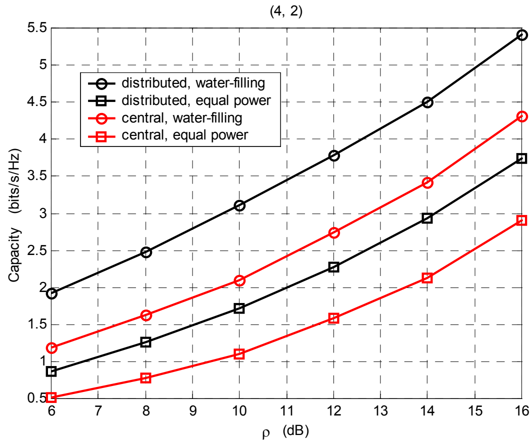



Figure 4. DAS vs. conventional MIMO in capacity

## V. CONCLUSIONS

In this paper, we investigated the channel capacity of DAS in virtue of traditional MIMO theory. Through simulation, we found that in DAS, more capacity potential can be gained at the border of two remote receive antenna areas than that near the center of one remote antenna area. Secondly, it was shown that the water-filling gain is significant in DAS. Because the access distance and path-loss are not equal for the transmit antennas, it is difficult to find a power control scheme to counteract the path-loss effect completely. This path attenuation left explains these capacity characters. Finally, we found that DAS leads to a significantly capacity improvement, e.g. in the (4, 2) system, DAS has approximate 3dB capacity gain over conventional MIMO system.

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