Characterization of the Cooperative Region in a Cellular Environment

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Abstract—In a wireless network, two or more nodes may cooperate to transmit each others data to a common receiver. The resulting increase in rate region has been studied from an information theoretic framework, for a general baseband communication system [12], [7]. In this work we investigate the benefits of user cooperation for a specific centralized network like the cellular system. We consider a cell with the Base Station (BS) at the center, one mobile (MS) placed at a fixed distance from the BS and obtain the locations of the other mobile where it is beneficial for them to cooperate. We term the set of these locations as the cooperative region and obtain it under various propagation environments for two different metrics. Based on our results we propose a simple geometrical formula to predict the shape of the cooperative regions which is a function of the BS-MS distance and channel propagation parameters.

I. INTRODUCTION

With the advent of next generation communication networks, newer applications have been envisaged which require increasingly higher data rates with more reliability. Traditional systems like Code Division Multiple Access (CDMA) and Global System for Mobile Communications (GSM) based cellular or Wireless Local Area Networks (WLAN) and their extensions are inadequate for this purpose due to inefficient interference management and suboptimal use of available dimensions for transmission. The solution lies in new communications technologies like cooperative communications. The physical layer description of cooperative communication is shown in Figure 1. Mobile MS A which desires to transmit its data to the BS, may enlist the help of one or more partner mobiles MS B and MS C. Since the wireless medium is inherently broadcast in nature, MS B and C overhear A’s transmission and can retransmit the information to the BS thus providing additional spatial paths for MS A’s information to the destination. MS A helps other nodes in a similar way. A cooperative communication system is different from a pure relay system [1], [4] as all nodes in a cooperative network have their own information to send to the BS unlike the relay nodes.

A. Prior Work

The concept of cooperative communications was first investigated for discrete channels in the 1980s. Willems [10] found out the capacity region of a MAC with cooperating encoders. Willems and van der Meulen then considered the limiting case when such cooperating encoders learn about each others transmitted codewords and use this information in their own transmission [11]. An achievable rate region for a MAC with generalized feedback was developed in [12]. It models the system as \((\mathcal{X}_1 \times \mathcal{X}_2, P(y, y_1, y_2|x_1, x_2), \mathcal{Y} \times \mathcal{Y}_1 \times \mathcal{Y}_2)\), where \(\mathcal{X}_i\) are the input alphabets, \(\mathcal{Y}\), the channel output at receiver and \(\mathcal{Y}_i\) the channel output available at MS \(i\). After a gap for about two decades, Sendonaris et al [7], [8] successfully applied the concepts of [12] to a Gaussian multiple access channel where the users cooperate. They derived the rate region, gave achievability strategies and also a practical scheme for realizing cooperation using CDMA. After their work there was a renewed interest in cooperative networks. Kaya et al [3] derived some preliminary results for optimal power allocation policies to achieve the rate region given in [7] and proposed a new and simplified achievability scheme. Instead of achievable rate regions, Laneman et al [5] focused on outage probability for a group of cooperating nodes. They proposed several half-duplex cooperative protocols like Amplify-and-Forward, Decode-and-Forward etc and studied the diversity order of these protocols. The use of outage probability as a metric also spawned the field of coded cooperative communications, where researcher’s tried to integrate channel coding with cooperative communications [2] [9].

In this plethora of research focusing on cooperative regions, strategies and channel codes, many important system level issues of user cooperation seem to have been neglected. A summary of these issues can be found in [7]. A very important question is that how does a mobile, for e.g. MS A in Figure 1,
which sees more than one mobile, like MS B and MS C in its vicinity, decide with whom to cooperate.

B. Our Contribution

Given a circular cell where the BS is at the center, and mobile MS A fixed, our aim is to obtain all locations of a second mobile, MS B with whom cooperation yields mutual benefit. For different locations of the cell, we calculate the rate regions without cooperation and with cooperation [7]. Let \((R_1, R_2)\) be point in the achievable rate region (with or without cooperation) for the MS A and MS B respectively. To compare between the two rate regions, we consider two metrics, \(\max(R_1 + R_2)\) which is an indication of the maximum system throughput and \(\max(R_1 = R_2)\) the maximum equal rate point. The cooperative region (as opposed to the rate region with cooperation) is the set of all locations in the cell for which,

\[
\frac{\max(R_1 + R_2)_{\text{Coop}}}{\max(R_1 + R_2)_{\text{Non-Coop}}} > 1 \text{ or } \frac{\max(R_1 = R_2)_{\text{Coop}}}{\max(R_1 = R_2)_{\text{Non-Coop}}} > 1,
\]

depending on the metric. Note that the two terms in (1) are called the cooperative ratios.

We find the cooperative regions for different values of MS A to BS distance and the propagation conditions like path loss exponents etc. Based on our observations we propose a simple interpolation method which generates an ellipse that closely approximates the shape of the cooperative region for any value of path loss exponent and MS A to BS distance. We envisage that such knowledge would be useful for future cellular systems, where a BS aware of mobile locations can decide which two mobiles should cooperate. Note that cooperative regions within a cell have also been derived in [6], but the metric used is outage performance for one specific encoding and decoding scheme for all location of MS B. However different locations of MS B gives rise to different sets of direct and interuser channel conditions for which different encoding and decoding schemes might be needed to achieve optimal performance. Our cooperative regions are not based on a fixed coding scheme and thus gives the optimal theoretical bounds.

II. SYSTEM MODEL

In this section we state the results of [7]. The cooperative mobiles are modeled as shown in Figure 2. \(W_i, i = 1, 2\) is the message of MS \(i\), \(X_i\) is the signal it transmits and \(Y_i\) the signal it receives. The base station is indexed by 0. The channel attenuation factor, due to path loss or fading, between entities \(i\) and \(j\) are given by \(K_{ij}\) and these stay constant during the transmission interval. The transmission equations are given by,

\[
Y_0 = K_{10}X_1 + K_{20}X_2 + Z_0
\]

\[
Y_1 = K_{12}X_2 + Z_1
\]

\[
Y_2 = K_{21}X_1 + Z_2,
\]

where \(Z_i\) is \(\mathcal{N}(0, N_i)\) distributed additive white Gaussian noise (AWGN). MS A divides its information into two, \(W_{10}\), which is sent directly to the BS, \(W_{12}\) which is sent to the BS through MS B. Accordingly it transmits \(X_1 = X_{10} + X_{12} + U_1\), where \(X_{10}\) and \(X_{12}\) carry \(W_{10}\) and \(W_{12}\) messages and \(U_1\) is the cooperative information used by the BS in decoding. There is a total power constraint \(P_i\) on MS A’s transmission which is split amongst transmitting \(X_{10}, X_{12}\) and \(U_1\) as \(P_{10}, P_{21}\) and \(P_{U1}\) respectively. Similar expressions hold for MS B. For fixed channel coefficients \(K_{ij}\), the rate region is given by,

\[
R_{12} \leq C\left(\frac{K_{12}^2P_{12}}{K_{12}^2P_{10} + N_1}\right)
\]

\[
R_{21} \leq C\left(\frac{K_{21}^2P_{21}}{K_{21}^2P_{20} + N_2}\right)
\]

\[
R_{10} \leq C\left(\frac{K_{10}^2P_{10}}{N_0}\right)
\]

\[
R_{20} \leq C\left(\frac{K_{20}^2P_{20}}{N_0}\right)
\]

\[
R_{10} + R_{20} \leq C\left(\frac{K_{10}^2P_{10} + K_{20}^2P_{20}}{N_0}\right)
\]

\[
R_{10} + R_{20} + R_{12} + R_{21} \leq C\left(\frac{K_{10}^2P_{10} + K_{20}^2P_{20} + 2K_{10}K_{20}\sqrt{P_{U1}P_{U2}}}{N_0}\right)
\]

where \(C(x) = 1/2 \log(1 + x)\) and \(R_1 = R_{10} + R_{12}\) and \(R_2 = R_{20} + R_{21}\). If \(K_{ij}\) also includes fading then expectations should be taken over the right hand side (RHS) of (5) to (10). The term \(\sqrt{P_{U1}P_{U2}}\) in (10) arises because the mobiles are aware of each others messages due to cooperation and can make use of this information in transmitting their signals so that these coherently combine at the receiver.

III. SIMULATION SETUP

We consider a two dimensional circular cell of arbitrarily chosen radius 5 and place the BS at the center \((0, 0)\) and MS A at \((D, 0)\). In the path loss based channel model, where \(K_{ij} = 1/d_{ij}^{-\alpha}\), where \(d_{ij}\) is the distance between nodes \(i\) and
$j$ and $\alpha$ is the path loss exponent. In this model $K_{12} = K_{21}$. In case of fading $K_{ij} = h_{ij}/d_{ij}^{\alpha/2}$, where $h_{ij}$ is i.i.d. fading in the three paths. We vary the position of the other mobile MS B, over the cell in discrete radial units of $r_{\text{step}}$ and angular units of $\theta_{\text{step}}$. We assume a minimum MS B to BS separation of unity to avoid near field effects. The total power available at the mobiles are $P_1$ and $P_2$. The various parameters are listed in Table I. At all locations of MS B, we compute cooperative regions as defined in (1).

### IV. SIMULATION RESULTS FOR PATH LOSS MODEL

In this section we vary the distance between MS A and BS and the channel parameters to obtain the cooperative regions. Before proceeding to the specific cases we make the following general observation, which holds true for all of them,

**Observation 1:** The cooperation ratio depends upon the quality of the inter-user channel relative to the direct channels. This is because for cooperation an user spends part of his transmit power to communicate to his partner over the interuser channel. If the interuser channel is of an inferior quality than the direct links, then there is no benefit in cooperation and the cooperative case degenerates to the non cooperation. In general if $K_{ij} \ll \min \{K_{10},K_{20}\}$, which for the path loss based channel model becomes $d_{12} = d_{21} \gg \max\{d_{10},d_{20}\}$, then there is no benefit in cooperation.

**A. $D = 1.5$ and $\alpha = 2$**

Figure 3 depicts the nature of the cooperative region when MS A is placed at a distance of $D = 1.5$. In addition to Observation 1 we also see that,

**Observation 2:** The cooperative area for $\max(R_1 = R_2)$ is more than $\max(R_1 + R_2)$. More precisely the former contains most of the area of the latter.

To understand this consider the points where the two regions are different. This corresponds to those regions where the two MS have widely dissimilar channels. As seen from the right subfigure of Figure 4, the $\max(R_1 = R_2)$ point for cooperation, $P_{NC}^C$ increases over the equivalent point for non cooperation, $P_{NC}^E$, while the corresponding points for $\max(R_1 + R_2)$ namely $P_{SC}^C$ and $P_{SC}^E$ are close to each other. This is because if the users have dissimilar channels $\max(R_1 + R_2)$ is achieved by letting the user with the good channel transmit close to his maximum capacity for both cooperation and non cooperation, while for $\max(R_1 = R_2)$, the rate of the user with good channel is pulled down for both cooperation and non cooperation, but the former achieves higher rate due to coherent combining.

The apparent discontinuities at the edge of the cooperative regions are because we have not considered all points in the circle, but as mentioned in Table I, discretized the regions in $r_{\text{step}}$ and $\theta_{\text{step}}$. Taking a smaller step sizes would increase the accuracy at the border regions, but our aim is to get an accurate characterization of the interior which forms the bulk of the region. For this purpose our chosen values of $r_{\text{step}}$ and $\theta_{\text{step}}$ suffice. Also recall that since we have assumed a lower bound of unity on the MS B to BS separation, there are ’white spaces’ in the center of our graphs.

We have commented upon the fact that $\max(R_1 = R_2)$ pulls down the rate of the MS with the good user. Figure 5 shows the ratio of $\max(R_1 + R_2)$ to $2 \times \max(R_1 = R_2)$ over the circular region for $D = 1.5$. As expected we see that when both MS have similar channel to the BS, the ratio is close to unity but it increases to 4 when MS have dis-similar channels.

**B. Effect of Increasing MS A to BS Distance**

In Figure 6 we plot the $\max(R_1 + R_2)$ cooperative regions for $D = 2.5, 3.5$ and $\alpha = 2$.

**Observation 3:** The cooperative regions expand and also exhibit a slight shift toward the right if the distance between MS A to BS is increased.

These could be explained along the lines of Observation 1.

**C. Effect of increasing Path Loss Exponent**

In this section we increase the path loss exponent from $\alpha = 2$ to $\alpha = 4$. Figure 7 shows the ratio of $\max(R_1 + R_2)$ with cooperation for $\alpha = 2$ to that for $\alpha = 4$ for $D = 2.5$. 

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1,P_2$</td>
<td>2</td>
</tr>
<tr>
<td>$D$</td>
<td>1.5,2,3.5</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>2.4</td>
</tr>
<tr>
<td>$r_{\text{step}}$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\theta_{\text{step}}$</td>
<td>10$^\circ$</td>
</tr>
<tr>
<td>$N_0,N_1,N_2$</td>
<td>1</td>
</tr>
</tbody>
</table>

**TABLE I**

SIMULATION PARAMETERS
As expected, the ratio increases rapidly with distance. This is because the cooperative gain depends on the various channel gains, especially the inter-user channel, which decreases with distance faster for $\alpha = 4$. However, when compared to the non-cooperative case for $\alpha = 4$, cooperation still has benefits as shown in Figure 8. We make the following observation:

**Observation 4:** The cooperative region shrinks and exhibits a shift due to increased channel attenuation.

V. ELLIPTICAL INTERPOLATION

In this section, we want to quantify Observations 3 and 4 and give an set of equations that describe the geometry of the cooperative regions in Figures 3, 6, and 8. We state some comments in this regard,

1) This is important as given any value of $D$ and $\alpha$, the BS can calculate the cooperative region around MS A and communicate it to the mobile it should cooperate. This solves the partner choice problem as mentioned in Section I-B.

2) We are concerned only in accurately characterizing the interior of the region as mentioned in Section IV. We allow for inaccuracies at the boundary regions because the BS would find a partner mobile in the interior of the region mostly.

3) Interpolation would not have been necessary if (5) to (10) could have been solved to obtain the exact analytical expressions for $\{P_{10}, P_{12}, P_{U1}\}$ and $\{P_{20}, P_{21}, P_{U2}\}$ as a function of channel variables $\{K_{10}, K_{20}, K_{12}, K_{21}, N_1, N_2, N_0\}$ and total powers $P_1$ and $P_2$. However, only preliminary results in this direction are reported in [3] and a complete characterization does not exist till date because of the non-convex nature of the problem.
Based on visual inspection of Figures 3, 6 and 8 we assume that the cooperative region is an ellipse. We assume that the semi-major axis, semi-minor axis and center are functions of $D$ and $\alpha$. We first determine the values of these three quantities for given $(D, \alpha)$ pairs by trial and error and then apply a simple linear interpolation. We present the results of our interpolations below.

**Proposition 1:** If MS A is placed at $(D,0)$ and path loss exponent is $\alpha$, then the coop region can be approximated by an ellipse with semi-major axis $a$ along the $y$-axis, semi-minor axis $b$ along the $x$-axis and center at $(c, 0)$ s.t.

\[
    a = D - 0.3\alpha + 1.6 \tag{11}
\]
\[
    b = D - 0.25\alpha + 1.2 \tag{12}
\]
\[
    c = 1.4D - 0.15\alpha + 0.4 \tag{13}
\]

In Figure 9 we plot the cooperative regions for different $(D, \alpha)$ pairs and also the corresponding ellipses given by (11) to (13). We see that the ellipses closely approximate the interior of the cooperative regions. Observations 3 and 4 are verified by the magnitude and sign of the coefficients of $D$ and $\alpha$ respectively in (11) to (13). We infer that better representation of the boundary of the cooperative region could have been obtained by fine tuning the coefficients of (11) to (13) and/or choosing a higher order interpolation.

**VI. RESULTS FOR FADING**

We model the fading in all the paths by i.i.d. rayleigh fading with unit variance (to keep the average received SNR same for fading as compared to the no fading case). For a given instance of fading for all paths, the cooperative regions can shrink or expand than the region with no fading. To study the average behavior we average over 30 instances of fading. Figure 10 we compare the cooperative regions without fading and fading for $D = 1.5$ and $\gamma = 2$. We make the following observation.

**Observation 5:** The cooperative region is enhanced with fading for the same average received SNR. This is because in order for the cooperative region for fading to be smaller than no fading, the fading realizations should be such that both the MS A-BS and MS B-BS channels have to be stronger than the interuser channel, while for cooperative region for fading to be larger just one MS-BS channel has to be weaker than the interuser channel.

**VII. CONCLUSIONS AND FUTURE WORK**

In this paper we consider a circular cellular region with the BS at the center, one mobile fixed at a distance $D$ and evaluate geographical positions of a second mobile where it is beneficial to cooperate. We evaluate the achievable rate regions and compare the $\max(R_1 + R_2)$ and $\max(R_1 = R_2)$ points for cooperation versus no cooperation. Based on our regions we conclude that the former region is contained in the latter and
hence it suffices to study the former as maximizing it would also maximize the latter. We show that the cooperative region expands with increasing $D$ and fading parameter and shrinks with increasing path exponent. We further quantify them by proposing an elliptical interpolation of the geometry of the cooperative regions. We also show that a cooperative scheme exploits fading to achieve a larger cooperative region.

VIII. ACKNOWLEDGMENTS

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REFERENCES


