Write or Radiate?
Inscribed Mass vs. Electromagnetic Channels

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November 1, 2003

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Abstract

We consider information flow via physical transport of inscribed media through space and compare it to information flow via electromagnetic radiation. Somewhat counterintuitively for point to point links, physical transport of inscribed mass is often energetically more efficient by many orders of magnitude than electromagnetic broadcast. And perhaps more surprising, even in a broadcast setting (depending on the receiver density) inscribed mass transport is still energetically more efficient. We discuss the implications of these results for terrestrial telecommunications networks as well as point to point and broadcast communication over great distances with loose delay constraints.

Contents

1 Introduction

At one time or another, every communications theorist has had the following epiphany:

> *Driving a truck filled with storage media (books, cd's, tapes, etc.) across town constitutes a very reliable channel with an extremely large bit rate.*

My own epiphany has occurred a few times over many years, but most recently with the study of short range high data rate channels [1–10] and mobility assisted wireless networks [11, 12] where communications nodes only transfer data to one another when the channel is good – typically at close range. One natural extension of this work is to not radiate electromagnetic energy at all, but rather, to have nodes physically exchange "letters" inscribed on some medium. And from such imaginings comes a simple question: when is it better to write than to radiate?

To begin, consider that one could easily pack ten 60 GByte laptop disk packs in a small box and push it across a table – with a correspondingly impressive data rate of about 4.8 Tb/second. Without much imagination, the idea can be extended to more exotic storage media. Consider a 1 mm^3 "bouillon cube" containing information coded as single stranded RNA (such as the polio virus). At about 1 base per nm^3 [13–17], each cube could store about 1000 Petabits (10^{18} bits) of information. A 10cm^3 volume of such material, if driven from New York to Boston in an automobile would constitute a rate of about $90,000$ Petabits/second (9×10^{19}bps) – dwarfing by about six orders of magnitude the 100 Terabit per second theoretical maximum information rate over optical fiber [18].

Next consider the mass of 1000 Petabits since mass will determine the amount of energy necessary for transport. Again using the virus analogy, single stranded RNA has an average mass of about 330 kDa per kilobase. A Dalton (Da) is the molecular weight of hydrogen and is about 1.67×10^{-24}g [19]. So, the 10^{15} kilobases implied by 1000 Petabits would weigh $330,000 \times 10^{15}$ Da. Conversion to more familiar units shows the total mass of our hypothetical 1000 Petabit message would be 551μg. The *mass information density* would be

$$\rho = 1.82 \times 10^{24} \text{bits/kg} \tag{1}$$

which we will later see is about two orders of magnitude better than rough extrapolations based on the current best micropatterning technology [20].

This impressive figure, however, may leave *some* room at the bottom. That is, there is no published theoretical limit to the amount of information that can be reliably stored as ordered mass. Thus, although Feynman argued a conservative bound of $5 \times 5 \times 5$ atoms per bit [15, 21], and RNA molecules achieve densities on the order of 32 atoms per bit [17], our ≈ 2000 Petabit/gram biological "existence proof" could be overly pessimistic by one or more orders of magnitude. Regardless, the point is that it is not hard to imagine large amounts of information being stored reliably and compactly using very little mass.

So, why hasn't inscribed mass transport been exploited in modern telecommunications networks? There are a number of reasons, but two seemingly obvious answers spring to mind. First, the key problem in telecommunications is *energy efficient* transport of information and delivering inscribed mass from New York to Boston would seem to consume a great deal of energy. Second, modern networks require *rapid* transport of information while the NY-Boston trip requires approximately 3 hours by car – or a few hundred seconds ballistically. These "answers" illustrate the key tensions which concern all telecommunications theorists:

tolerable delay
vs.
tolerable energy
vs.
tolerable throughput

In quantifying these tensions for what we will call *inscribed mass channels*, we will find that under a surprising variety of circumstances they are, bit for bit, much more energy efficient than methods based on electromagnetic radiation. Moreover, from a theoretical perspective, the cost of writing the information into some medium can be made infinitessimally small [22–24], so the energy savings are not necessarily diminished by adding the inscription cost. Thus, something seemingly so primitive as hurling carved pebbles through space can require many orders of magnitude less energy and support dramatically more users than isotropically broadcasting the same information.[1]

And perhaps even more surprising, it is exactly that image which leads to another interesting point. In the regime of very large distances with very loose delay requirements, we will find that mass transport can be many more orders of magnitude more efficient than isotropic radiation. So much so that even if directed radiation methods are used, somewhat heroic engineering, such as very long-lived earth-sized directive apertures, is required to make radiation more efficient than inscribed mass. That is, inscribed mass channels might be a *preferred* way to carry information between specks of matter separated by the vastness of interstellar or intergalactic space.

Though such a conclusion may seem directly at odds with previous work by Cocconi and Morrison [25] which proposed millimeter wave interstellar communications, it is exactly the assumption of loose delay constraints which tips the balance strongly in favor of inscribed mass transport. So, perhaps in addition to scouring the heavens for radio communications from other worlds, we might also wish to more closely examine the seeming detritus which is passing, falling, or has already fallen to earth.

[1] Rolf Landauer mentioned the possibility of inscription and physical transport [22], but specifically in the context of reversible communication and did not calculate the transport energies.

2 Preliminaries

2.1 Definitions and Problem Statement

- ρ: mass information density for inscribed information in bits per kilogram.
- W: bandwidth available for radiated communication in Hertz.
- R: effective receiver aperture radius in meters.
- $A = \pi R^2$: effective receiver aperture in square meters.
- D: distance to target in meters.
- c: speed of light in meters per second.
- N_0: background noise energy in Watts per Hertz (Joules).
- B: message size in bits.
- T: time allowed, in excess of light-speed propagation delay, for the message to arrive.

We compare the energy required to transport B bits over distance D under delay constraint T using electromagnetic radiation with bandwidth W, receiver aperture area A and receiver noise N_0 to that required using inscribed mass with information density ρ

2.2 Empirical Values for Mass Information Density

Detailed consideration of the practicalities of rendering information as inscribed mass and hardening it for transport is provided in separate work [26]. However, it is still useful to examine a few different possible methods of storage to get an empirical feel for "practical" values of mass information density, ρ, based on current technology.

At present, RNA base pair storage seems to be the most compact method for which we have an existence proof with a mass information density as stated in the introduction of

$$\rho_{RNA} = 1.8 \times 10^{24}\text{bits/kg} \tag{2}$$

In comparison, as of this writing a scanning tunneling microscope (STM) can place an equivalent of about 10^{15} bits per square inch using individual Xenon atoms on a nickel substrate [20]. The per bit dimension is then 8Å on a side. By somewhat arbitrarily assuming a 100Å nickel buffer between layers we obtain a bit density of 1.55×10^{20} bits per cm^3. The density of nickel (8.9g per cm^3) will predominate owing to the relatively thick layering so that we have

$$\rho_{\text{stm}} = 1.74 \times 10^{19}\text{bits/g} = 1.74 \times 10^{22}\text{bits/kg} \tag{3}$$

or about two orders of magnitude smaller than RNA storage.

E-beam lithography can achieve feature sizes of 5nm which implies a bit density of 4×10^{12} bits per cm^2. Assuming $100\mathring{A}$ substrate layers we then have 4×10^{18} bits per cm^3. Given silicon density of 2.6g/cm^3 we have a mass information density of

$$\rho_e = (4 \times 10^{18}\text{bits/cm}^3)/(2.6\text{g/cm}^3) = 1.54 \times 10^{21}\text{bits/kg} \quad (4)$$

– about three orders of magnitude smaller than RNA.

Current optical lithographic techniques routinely achieve $0.1\mu m$ feature sizes. Assuming a substrate thickness of 100Å, this corresponds to a density of 10000 bits per $(\mu\text{m})^3$ or 10^{13}bits/mm^3. The silicon density of 2.6g/cm^3 [19] results in

$$\rho_{\text{lith}} = (10000Tb/\text{cm}^3)/(2.6g/\text{cm}^3) = 3.85 \times 10^{18}\text{bits/kg} \quad (5)$$

or approximately six orders of magnitude smaller than RNA.

Magnetic storage density is on the order of $10Gb/\text{cm}^2$ so that again, each bit is about $0.1\mu m$ on a side. Assuming a film thickness of 1000Å and a density similar to FeO_2 (about 5 times that of water [19]) we have

$$\rho_{\text{mag}} = (1000Tb/\text{cm}^3)/(5g/\text{cm}^3) = 200Tb/g = 2 \times 10^{17}\text{bits/kg} \quad (6)$$

which is about seven orders of magnitude smaller than RNA

Finally, we note that volume holographic storage techniques [27] are limited to a volumetric bit density of one bit per λ^3 where λ is the wavelength . Thus, a hologram using $\lambda = 500nm$ blue light could in principle hold 8×10^{12} bits per cubic centimeter and the mass information density, assuming a quartz-like storage medium would be about 3×10^{15} bits/kg, about nine orders of magnitude smaller than biological. For holography using shorter wavelengths, say in the ultraviolet range of $50nm$, the density would scale by a factor of 1000 and in the far ultraviolet ($5nm$) by a factor of 10^6 which is about twice the information mass density of E-beam lithography and three orders of magnitude smaller than RNA.

Regardless, clear limits on the maximum possible density of storage using inscribed mass are unknown. Bounds using simple quantum mechanical arguments are provided in [26].

3 From Here to There: Minimizing Particle Transport Energy Under Delay Bounds

Here we derive lower bounds on the amount of energy necessary to drive a mass m from point A to point B under some deadline τ. We first assume a *free* particle, untroubled by external forces from potential fields (i.e., gravity). Though the results are well known, for continuity we re-derive them here. Also, in keeping with a communication theory flavor, we use only standard communications methods such as basic probability theory and Jensen's inequality. We then consider particle motion through potential fields and derive similar energy bounds using variational calculus.

3.1 Jensen's Inequality

Let $h()$ be a non-negative real-valued function of a single variable and let V be a bounded real random variable with mean $E[V] = \bar{v}$. We also assume that $E[h(V)]$ exists. We first note that

$$\max_v h(v) \geq E[h(V)] \tag{7}$$

and that when V is deterministic

$$\max_v h(v) = E[h(V)] \tag{8}$$

Next we note that for $h()$ convex we have via Jensen's inequality [28, 29]

$$E[h(V)] \geq h(\bar{v}) \tag{9}$$

We now use these relations to derive lower bounds on the amount of energy necessary to move particles under delay constraints.

3.2 Free Particles

We wish to move a mass m over a distance D within time τ where the only external force acting on the particle is what we apply. We will assume an inertial frame for source and destination, an initial mass velocity of zero and that we need not bring the mass to rest at the destination. That is, the mass is "caught" by the destination and the only problem is for the source to deliver it on time with minimum applied energy.

Let the particle position be $x(t)$ and its velocity $v(t) = \frac{dx(t)}{dt} = \dot{x}$. Let the intrinsic energy of the particle at velocity v be described by a nondecreasing convex function $h(v)$. In order for the particle to be delivered by time τ when moved through distance D, the average velocity must be D/τ. Specifically,

$$E[v(t)] = \frac{1}{\tau}\int_0^\tau v(t)dt = \bar{v} = \frac{D}{\tau} \tag{10}$$

Equation (10) is equivalent to an expectation of $v(t)$ over a random variable t, uniform on $(0, \tau)$.

We seek to minimize the maximum total energy imparted to the particle under the arrival delay constraint. So we seek a trajectory $v(t)$ such that

$$E^* = \min_{v()} \max_t h(v(t)) \tag{11}$$

while requiring $E[v(t)] = \frac{D}{\tau}$. We then note that

$$\min_{v()} \max_{t} h(v(t)) \geq \min_{v()} E[h(v(t))] \tag{12}$$

and that by Jensen's inequality

$$E[h(v(t))] \geq h(\bar{v}) \tag{13}$$

with equality iff $v(t)$ is constant. Since $h()$ and $\bar{v}$ are given, $E[h(v(t))]$ has a lower bound independent of the specific trajectory $v(t)$. Therefore we can absolutely minimize $E[h(v(t))]$ by requiring that the particle move at constant velocity. However, this choice of $v(t)$ also causes equation (12) to be satisfied with equality. This leads to the well known result that minimum energy is expended when the particle is launched from its origin with constant velocity $v(t) = D/\tau$, $t \in (0, \tau]$.

For particles approaching light speed we have

$$h_{\text{total}}(v) = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{14}$$

However, this total energy includes the rest mass energy mc^2. The excess energy owing to velocity is

$$h(v) = mc^2 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) \tag{15}$$

and is convex in v, so that the minimum applied energy is

$$E^* = mc^2 \left(\frac{1}{\sqrt{1 - \left(\frac{\bar{v}}{c}\right)^2}} - 1 \right) \tag{16}$$

For particles traveling much slower than light speed ($\bar{v} \ll c$) we have $h(v) \approx \frac{1}{2}mv^2$ so that

$$E^* \approx \frac{1}{2}m\bar{v}^2 \tag{17}$$

3.3 Particles in a Potential Field

Here we introduce a field which applies force to the particle as a function of position under Newtonian conditions. We will assume conservative (potential) fields such as gravity so that the total energy of the particle is given by

$$\mathcal{E}(t) = h(v(t)) + q(x(t)) \tag{18}$$

where $q(x)$ is the potential energy of the particle at position x. We seek the min max energy $\mathcal{E}(t)$ profile which satisfies the particle arrival deadline.

As before, we form an optimization and bound it from below

$$E^* = \min_{x()} \max_t \mathcal{E}(t) \geq \min_{x()} \frac{1}{\tau} \int_0^\tau \mathcal{E}(t) dt \tag{19}$$

We will then minimize the rightmost expression in equation (19) using the calculus of variations [30]. Euler's equation is

$$\frac{d}{dt}\left(\frac{\partial \mathcal{E}}{\partial v}\right) - \frac{\partial \mathcal{E}}{\partial x} = 0 \tag{20}$$

and application of the definition of $\mathcal{E}(t)$ yields

$$\ddot{x}h''(\dot{x}) - q'(x) = 0 \tag{21}$$

where $\dot{x} = dx/dt = v$ and $\ddot{x} = \dot{v}$.

For low speed motion, $h(v) = mv^2/2$ so that equation (21) becomes

$$m\ddot{x} = q'(x) \tag{22}$$

which implies "free fall" in a potential field since $q'(x)$ is the force on the particle at position x. In turn, free fall implies constant energy over the particle trajectory which leads to equation (19) being satisfied with equality. Thus, the particle should be imparted with enough initial velocity v_0 such that it reaches the destination at time τ.

3.3.1 The Artillery Problem

The value of v_0 depends upon the form of the potential field. For a uniform field where constant force $-\mathbf{F}$ is applied in an inertial frame we have

$$\ddot{\mathbf{x}} = \frac{-\mathbf{F}}{m} \tag{23}$$

If $\mathbf{F} = m\mathbf{g}$, then we have a standard (frictionless) artillery problem as depicted in FIGURE 1

Specifically, let $x(t)$ be the vertical position of the particle and $r(t)$ its horizontal range from launch. Let the initial particle velocity be v_0 and its angle of launch be θ. We then have

$$r(t) = v_0 t \cos\theta \tag{24}$$

With target range D and deadline τ we have $x(0) = 0$, $x(\tau) = 0$, $r(0) = 0$ and $r(\tau) = D$. Therefore by inspection we obtain

$$\frac{D}{\tau} = v_0 \cos\theta \tag{25}$$

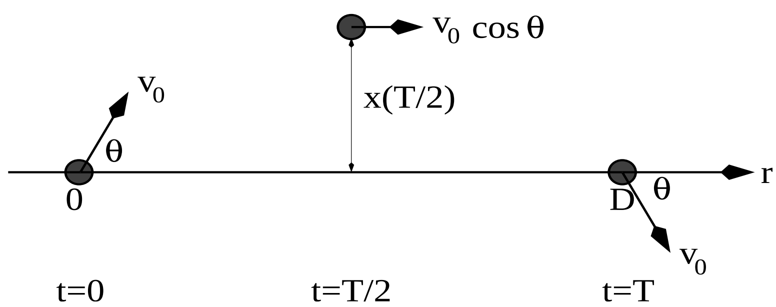

Figure 1: **The Artillery Problem:** a particle is fired from $x = 0$ at $t = 0$ with initial velocity v_0 and angle θ to land at position $x = D$ at time $t = \tau$.

Symmetry also requires that the particle arrive as a mirror image to its launch so

$$2v_0 \sin\theta = g\tau \tag{26}$$

where g is the gravitational acceleration. We then can write

$$v_0^2 = (v_0\cos\theta)^2 + (v_0\sin\theta)^2 = \left(\frac{D}{\tau}\right)^2 + \left(\frac{g\tau}{2}\right)^2 = \bar{v}^2 + \left(\frac{gD}{2\bar{v}}\right)^2 \tag{27}$$

so that the energy required is

$$E = \frac{1}{2}m\left[\bar{v}^2 + \left(\frac{gD}{2\bar{v}}\right)^2\right] \tag{28}$$

to reach a target at range D by deadline τ. We note that one can define a "natural deadline" with minimum transport energy by minimizing equation (28) in τ

$$\frac{dE}{d\tau} = \frac{1}{2}m\left(-\frac{2D^2}{\tau^3} + 2\tau\left(\frac{g}{2}\right)^2\right) = 0 \tag{29}$$

which results in

$$\tau^* = \sqrt{\frac{2D}{g}} \tag{30}$$

and

$$E^* = \frac{1}{2}mgD \tag{31}$$

We then rewrite equation (28) as

$$E = \frac{1}{2}mgD\frac{1}{Dg}\left[\bar{v}^2 + \left(\frac{gD}{2\bar{v}}\right)^2\right] \tag{32}$$

and note that

$$\frac{1}{Dg}\left[\bar{v}^2+\left(\frac{gD}{2\bar{v}}\right)^2\right]=\left[\frac{D}{\tau^2 g}+\frac{\tau^2 g}{4D}\right]\geq 1 \tag{33}$$

for $\forall\tau>0$.

3.3.2 Escape from a Potential Field

For direct escape from a gravitational field we have

$$q'(x)=-\frac{MmG}{(\Delta+x)^2} \tag{34}$$

where Δ is the initial distance from the gravitating point mass M and G is the gravitational constant. We then have

$$\ddot{x}=-\frac{MG}{(\Delta+x)^2} \tag{35}$$

which is not amenable to closed form solution. However, once again the particle is in free fall and $\mathcal{E}(t)$ is constant. Thus, all we require is the initial amount of energy necessary (and therefore an initial velocity v_0) such that the particle reaches the target at the deadline.

The initial particle energy is $\frac{1}{2}mv_0^2$. The potential energy of the particle at position x is

$$Q(x)=\int_0^x \frac{MGm}{(\Delta+z)^2}dz=MGm\left(\frac{1}{\Delta}-\frac{1}{\Delta+x}\right) \tag{36}$$

so the kinetic energy of the particle at position x is

$$\frac{1}{2}mv^2=\frac{1}{2}mv_0^2-MGm\left(\frac{1}{\Delta}-\frac{1}{\Delta+x}\right) \tag{37}$$

or

$$v=\frac{dx}{dt}=\sqrt{v_0^2-2MG\left(\frac{1}{\Delta}-\frac{1}{\Delta+x}\right)} \tag{38}$$

Variable separability leads to

$$\int_0^D \frac{1}{\sqrt{v_0^2-2MG\left(\frac{1}{\Delta}-\frac{1}{\Delta+x}\right)}}dx=\int_0^\tau dt=\tau \tag{39}$$

which we rewrite as

$$\int_0^1 \frac{1}{\sqrt{v_0^2-\frac{2MG}{\Delta}\left(\frac{z}{\frac{\Delta}{D}+z}\right)}}dz=\frac{1}{\bar{v}} \tag{40}$$

The integral in equation (40) can be evaluated [31] but the expression is complicated. We will later plot $E=\frac{1}{2}mv_0^2$ as a function of $\bar{v}$ for comparison to potential-free particles.

4 Energy Bounds on Information Delivery

4.1 Inscribed Mass

Assuming nothing can exceed the speed of light, we define the message receipt deadline, T, as the time allowed *in excess* of the propagation delay with time referenced to the common frames of our two fixed points between which information is sent. The total delay allowed for mass transport is therefore $\tau = (\frac{D}{c} + T)$.

Assuming some value for mass information density ρ, the number of bits transported is $B = m\rho$. The energy necessary to transport mass m with deadline $\tau = \frac{D}{c} + T$ in free space is then via equation (16)

$$E_w = \frac{B}{\rho} c^2 \left(\frac{1}{\sqrt{1 - \left(\frac{\bar{v}}{c}\right)^2}} - 1 \right) \tag{41}$$

where

$$\bar{v} = \frac{D}{\frac{D}{c} + T} \tag{42}$$

For $\bar{v} \ll c$ we have

$$E_w \approx \frac{1}{2} \frac{B}{\rho} \bar{v}^2 \tag{43}$$

In a simple potential field at non-relativistic speeds (the artillery problem) we have via equation (28)

$$E_w = \frac{1}{2} \frac{B}{\rho} m \left(\bar{v}^2 + \left(\frac{gD}{2\bar{v}} \right)^2 \right) \tag{44}$$

and if τ is chosen to be the "natural deadline" of equation (30) we have

$$E_w = \frac{1}{2} \frac{B}{\rho} mgD \tag{45}$$

As mentioned before, the more complex potential escape problem requires numerical calculation of the transport energy. We summarize these results in TABLE 1.

4.2 Electromagnetic Transmission

If a transmitter radiates power P, a receiver at some distance D will capture some fraction of the radiated power $P_r = \nu(D)P$ where $\nu(D)$ is defined as the energy capture coefficient of the

Scenario	**Mass Transport Energy**
Free space	$\frac{B}{\rho}c^2\left(\frac{1}{\sqrt{1-\left(\frac{\bar{v}}{c}\right)^2}}-1\right)$
Free space w/ $\bar{v} \ll c$	$\frac{1}{2}\frac{B}{\rho}\bar{v}^2$
Artillery	$\frac{1}{2}\frac{B}{\rho}m\left(\bar{v}^2+\left(\frac{gD}{2\bar{v}}\right)^2\right)$
Artillery (min)	$\frac{1}{2}\frac{B}{\rho}gD$

Table 1: Minimum energy necessary to deliver B bits in mass to a target at distance D by deadline $\tau = \frac{D}{c} + T$. $\bar{v} = \frac{D}{\tau}$.

receiver. Assuming square law isotropic propagation loss[2] we have

$$\nu(D) = \frac{A}{4\pi D^2} \tag{46}$$

where A is the effective aperture of the receiver. Assuming additive Gaussian receiver noise, the Shannon capacity [29] in bits per second between the transmitter and receiver is

$$C = W \log_2\left(\frac{PA}{4\pi D^2 N_0 W} + 1\right) \tag{47}$$

where N_0 is the background noise spectral intensity and W is the bandwidth of the transmission. If we assume a transmission interval long enough that the usual information theoretic results for long codes can be applied, the number of bits delivered for a transmission of duration T is

$$B = TC = TW \log_2\left(\frac{PA}{4\pi D^2 N_0 W} + 1\right) \tag{48}$$

We note that the time required for arrival of the complete message is $T + \frac{D}{c}$ – identical to the inscribed mass deadline as illustrated in FIGURE 2.

Since $E_r = PT$ we then have

$$E_r = TWN_0\frac{4\pi D^2}{A}\left(2^{\frac{B}{TW}} - 1\right) \tag{49}$$

Long codes imply many channel uses. That is, each bit is coded over multiple “channel uses” where the total number of channel uses is $2TW$ [29]. Thus, we might expect $TW \gg B$. But even if not we can provide a lower bound for equation (49) based on such an asymptotic assumption.

[2]For higher loss exponents such as those seen in terrestrial systems, we can multiply the result by the appropriate power of D.

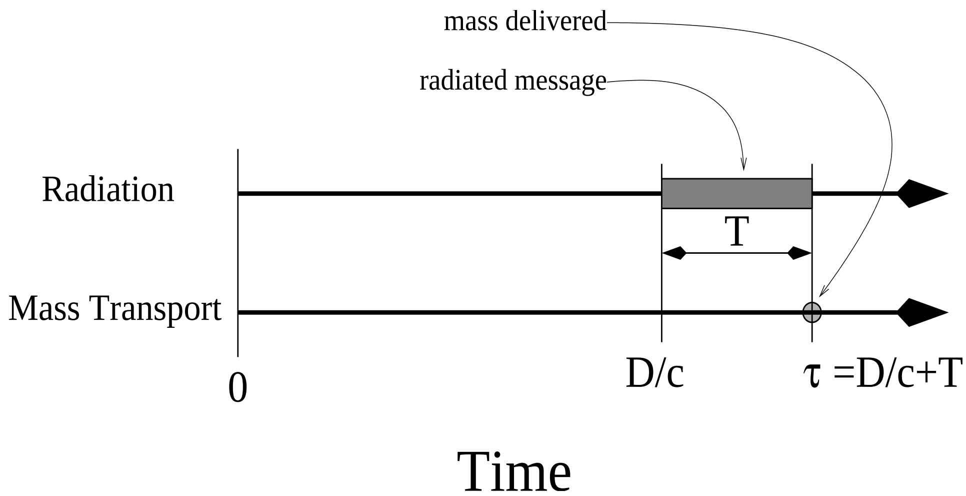

Figure 2: Temporal comparison of message delivery using radiation and mass transport. D: range to target, c: speed of light, T: radiated message duration, τ: message delivery deadline.

First we rewrite equation (49) as

$$E_r = BN_0 \frac{4\pi D^2}{A} \frac{TW}{B} \left(2^{\frac{B}{TW}} - 1 \right) \tag{50}$$

and then since

$$\frac{TW}{B} \left(2^{\frac{B}{TW}} - 1 \right) \geq \lim_{\frac{TW}{B} \to \infty} \frac{TW}{B} \left(2^{\frac{B}{TW}} - 1 \right) = \ln 2 \tag{51}$$

we must have

$$E_r \geq BN_0 \frac{4\pi D^2}{A} \ln 2 \tag{52}$$

It is important to note that although W is often interpreted simply as bandwidth, it is actually a much more general parameter which can be defined to include any number of degrees of freedom one might like – such as polarization, spatial diversity [32] and any others [29, 33, 34]. Thus, in deriving a lower bound on radiated energy based on $\frac{TW}{B} >> 1$ we have essentially allowed infinite (or very large) degrees of freedom by invoking the well known limit

$$\lim_{W \to \infty} W \log \left(\frac{P}{N_0 W} + 1 \right) = \frac{P}{N_0} \tag{53}$$

That is, the minimum radiated energy issue boils down to two parameters: 1) how much radiated power is delivered to the receiver, and 2) the receiver noise temperature.

4.3 The Radiation to Transport Energy Ratio

We define Ω, the *radiation to transport energy ratio*, as

$$\Omega = \frac{E_r}{E_w} \tag{54}$$

and since $A = \pi R^2$ where R is the receiver aperture radius, we find that for free particle relativistic motion we have,

$$\Omega_f \geq \left[(4\ln 2)\rho N_0 \left(\frac{D}{R}\right)^2\right] \frac{1}{c^2} \left(\frac{\sqrt{1-\left(\frac{\bar{v}}{c}\right)^2}}{1-\sqrt{1-\left(\frac{\bar{v}}{c}\right)^2}}\right) \tag{55}$$

For $\bar{v} \ll c$ we then have

$$\Omega_f \geq (8\ln 2)\frac{\rho N_0}{\bar{v}^2}\left(\frac{D}{R}\right)^2 \tag{56}$$

For the artillery problem in general we have

$$\Omega_a \geq \rho N_0 (8\ln 2)\left(\frac{D}{R}\right)^2 \frac{1}{\bar{v}^2 + \left(\frac{gD}{2\bar{v}}\right)^2} \tag{57}$$

and for the "natural deadline" τ^* we have

$$\Omega_a^* \geq \rho N_0 \frac{8\ln 2}{g}\frac{D}{R^2} \tag{58}$$

Finally, we note once again that the radiation to transport energy ratio for the gravity escape problem, defined here as Ω_e, must be calculated numerically. However, we will later plot transport energy as a function of $\bar{v}$ for comparison.

We summarize the results in TABLE 2.

Point to Point Links	
Scenario	**Energy Ratio, Ω**
Free space ($\bar{v} \ll c$)	$\rho N_0\,(8\ln 2)\left(\frac{D}{R}\right)^2\frac{1}{\bar{v}^2}$
Artillery	$\rho N_0(8\ln 2)\left(\frac{D}{R}\right)^2\frac{1}{\bar{v}^2}\frac{1}{1+\left(\frac{gD}{2\bar{v}^2}\right)^2}$
Artillery min	$\rho N_0\,(8\ln 2)\,\frac{1}{g}\frac{D}{R^2}$

Table 2: Energy ratio Ω to deliver B bits to a target at distance D by deadline T with $\bar{v} \equiv \frac{D}{T}$ so long as $\bar{v} \ll c$.

4.4 Network Issues

One benefit (or liability) of electromagnetic transmission is that it can disperse through space to multiple targets. In contrast, inscribed mass transport is intrinsically a point to point method. Whether one is more energetically efficient than the other depends on the density of receivers. Specifically, at range D the radiation to transport energy ratio Ω is the largest number of receivers before radiation becomes more efficient than mass transport. However, there are network information theory issues which require some care.

4.4.1 One Message, Many Receivers

If we want to send the same (shared) message to all receivers, then information theory for broadcast channels suggests we code the message to be received by the most distant receiver since those closer to the transmitter will always receive a stronger signal [29]. So, the amount of energy necessary to disperse the shared message to all receivers with aperture A within a radius U electromagnetically is

$$E_r \geq BN_0 \frac{4\pi U^2}{A} \ln 2 \tag{59}$$

To evaluate the energy required for mass transport, we could hold the deadline T constant – this would favor mass transport since destinations closer to the source could travel more slowly. However, the reduction in energy for variable mass transport velocity is only about 40%, and such velocity variation will complicate evaluation of scenarios where the initial velocity must be larger than some value (i.e., escape velocity). So we opt to use the slightly less optimal fixed particle speed assumption here and thus variable arrival times.

Therefore, the amount of energy necessary to deliver the shared message of B bits to all receivers in a volume of radius U using inscribed mass is the sum of the individual energies necessary for each receiver. The expected energy assuming a Poisson density η of receivers, using the result in TABLE 1, is

$$\bar{E}_w = E_w(U)\left(\eta \frac{4}{3}\pi U^3\right) = 2\eta\pi \frac{B}{\rho}\frac{U^3}{3}\bar{v}^2 \tag{60}$$

where we have assumed the same velocity $\bar{v} \equiv \frac{U}{T}$ for each particle. If we then define $\Omega_{\mathcal{V}}(U)$ as the radiation to mass energy ratio for a spherical volume of radius U and again define the aperture area as $A = \pi R^2$ we have

$$\Omega_{\mathcal{V}}(U) \geq \frac{6 \ln 2}{\pi} \frac{\rho N_0}{\eta} \frac{1}{R^2 U} \frac{1}{\bar{v}^2} \tag{61}$$

For the artillery problem the receivers are distributed in a disc of radius U with Poisson density η'. Here we assume that particle velocity can vary with range to target since, unlike the free space problem, it affords no complications later. We therefore have

$$\bar{E}_w = \frac{1}{2}\eta'\frac{B}{\rho}\int_0^{2\pi} d\theta \int_0^U \left(\frac{r^2}{T^2} + \left(\frac{Tg}{2}\right)^2\right) r dr = \pi\eta'\frac{B}{\rho}U^2\left(\frac{1}{4}\frac{U^2}{T^2} + \frac{1}{2}\left(\frac{Tg}{2}\right)^2\right) \tag{62}$$

so that after defining $\bar{v} = \frac{U}{T}$ as the maximum average forward particle velocity (associated with particles at radius U) we have

$$\Omega_{\mathcal{A}}(U) \geq \frac{16\ln 2}{\pi}\left(\frac{\rho N_0}{\eta'}\right)\frac{1}{R^2}\frac{1}{\bar{v}^2 + \frac{1}{2}\left(\frac{Ug}{\bar{v}}\right)^2} \tag{63}$$

4.4.2 Different Messages, Different Receivers

Now suppose that a different message must be sent to each receiver and that each of these messages is the same size. Let the distance to receiver i be r_i and assume that $r_i \leq r_{i+1}$ $i = 1, 2, ...M$. The transmitter has power budget P which must be split M ways. Let α_i be the fraction of power allocated for receiver i so that $\sum_i \alpha_i = 1$. Applying information theory for the Gaussian broadcast channel [29] provides that the rate $\mathcal{R}_i$ seen by user i satisfies

$$\mathcal{R}_i \leq W\log_2\left(\frac{\alpha_i P\nu(D_i)}{\sum_{j=i+1}^M \alpha_j P\nu(D_i) + WN_0} + 1\right) = W\log_2\left(\frac{\alpha_i P}{\sum_{j=i+1}^M \alpha_j P + N_i} + 1\right) \tag{64}$$

where $N_i = \frac{WN_0}{\nu(D_i)}$ and we note that N_i increases in i.

To understand rate bounds we can set $R_i = \frac{B}{T}$ and solve for appropriate α_i

$$B = WT\log_2\left(\frac{\alpha_i P}{\sum_{j=i+1}^M \alpha_j P + N_i} + 1\right) \tag{65}$$

which we rewrite as

$$\frac{N_i}{P} = \frac{1}{\beta}\alpha_i - \sum_{j=i+1}^{M}\alpha_j \tag{66}$$

where $\beta \equiv 2^{\frac{B}{WT}} - 1$. We can then rewrite equation (66) in matrix form as

$$\frac{1}{P}\mathbf{N} = \begin{bmatrix} 1/\beta & -1 & \cdots & \cdots & -1 \\ 0 & 1/\beta & -1 & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 1/\beta & -1 \\ 0 & \cdots & \cdots & 0 & 1/\beta \end{bmatrix} \boldsymbol{\alpha} \tag{67}$$

where $\mathbf{N}^\top = [N_1, \cdots, N_M]$ and $\boldsymbol{\alpha}^\top = [\alpha_1, \cdots, \alpha_M]$. We then have

$$\boldsymbol{\alpha} = \frac{\beta^2}{P}\begin{bmatrix} 1/\beta & 1 & (\beta+1) & \cdots & (\beta+1)^{M-3} & (\beta+1)^{M-2} \\ 0 & \ddots & \ddots & \ddots & & (\beta+1)^{M-3} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \ddots & (\beta+1) \\ \vdots & & & \ddots & \ddots & 1 \\ 0 & \cdots & \cdots & \cdots & 0 & 1/\beta \end{bmatrix}\mathbf{N} \tag{68}$$

Since β and all the N_i are non-negative, all the α_i will also be non-negative. Since we seek the energy E_r necessary to broadcast B bits independently to each location, we must have

$$E_r = PT = (\mathbf{1}^\top P\boldsymbol{\alpha})T = T\beta^2\mathbf{1}_M^\top\begin{bmatrix} 1/\beta & 1 & (\beta+1) & \cdots & (\beta+1)^{M-3} & (\beta+1)^{M-2} \\ 0 & \ddots & \ddots & \ddots & & (\beta+1)^{M-3} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \ddots & (\beta+1) \\ \vdots & & & \ddots & \ddots & 1 \\ 0 & \cdots & \cdots & \cdots & 0 & 1/\beta \end{bmatrix}\mathbf{N} \tag{69}$$

where $\mathbf{1}_M^\top$ is an M-dimensional vector all of whose entries are 1. The reader may wish to verify that equation (69) is equivalent to equation (50) when $M = 1$.

We can rewrite equation (69) as

$$E_r = BN_0\beta^2\frac{TW}{B}\mathbf{1}_M^\top\begin{bmatrix} 1/\beta & 1 & (\beta+1) & \cdots & (\beta+1)^{M-3} & (\beta+1)^{M-2} \\ 0 & \ddots & \ddots & \ddots & & (\beta+1)^{M-3} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \ddots & (\beta+1) \\ \vdots & & & \ddots & \ddots & 1 \\ 0 & \cdots & \cdots & \cdots & 0 & 1/\beta \end{bmatrix}\begin{bmatrix} \frac{1}{\nu(D_1)} \\ \vdots \\ \frac{1}{\nu(D_M)} \end{bmatrix} \tag{70}$$

From equation (51) we know that

$$\beta\frac{TW}{B} = (2^{\frac{B}{TW}} - 1)\frac{TW}{B} \geq \ln 2 \tag{71}$$

So, we can lower bound E_r by taking $\frac{B}{TW} \to 0$ and hence $\beta \to 0$ to obtain

$$E_r \geq BN_0 \ln 2 \mathbf{1}_M^\top \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\nu(D_1)} \\ \vdots \\ \frac{1}{\nu(D_M)} \end{bmatrix} = B \ln 2 N_0 \sum_{i=1}^{M} \frac{1}{\nu(D_i)} \tag{72}$$

Equation (72) implies indirectly that E_r is minimum when each receiver has its own channel, independent of (orthogonal to) the others – essentially a collection of M independent point to point channels. This allows us to calculate the expected minimum energy required for multipoint distinct message radiation as

$$\bar{E}_r \geq 2B\eta \ln 2N_0 \int_0^{\pi/2} \int_0^{2\pi} \int_0^U \frac{4\pi r^2}{A} r^2 dr d\theta sin\phi d\phi = 16\pi \ln 2BN_0\eta \frac{U^5}{5R^2} \tag{73}$$

We can compare this directly to the inherently multipoint mass transport energy of equation (60) to obtain

$$\Omega_{\mathcal{V}} \geq \rho N_0 (\frac{24}{5} \ln 2) \frac{1}{\bar{v}^2} \frac{U^2}{R^2} \tag{74}$$

For the artillery problem, we integrate over the disc of radius U and have

$$\bar{E}_r \geq B\eta' \ln 2N_0 \int_0^{2\pi} \int_0^U \frac{4\pi r^2}{A} r dr d\theta = B(8 \ln 2)\pi\eta' N_0 \frac{U^4}{4R^2} \tag{75}$$

so that using equation (62) we get

$$\Omega_{\mathcal{A}} \geq \rho N_0 (8 \ln 2) \left(\frac{U}{R}\right)^2 \frac{1}{\bar{v}^2} \frac{1}{1 + \frac{1}{2}\left(\frac{Ug}{\bar{v}^2}\right)^2} \tag{76}$$

We summarize the results in TABLE 3.

Multiple Messages		
	Sphere	**Disc**
Shared Message	$\left(\frac{\rho N_0}{\eta}\right) \left(\frac{6 \ln 2}{\pi}\right) \frac{1}{R^2 U} \frac{1}{\bar{v}^2}$	$\left(\frac{\rho N_0}{\eta'}\right) \left(\frac{16 \ln 2}{\pi}\right) \frac{1}{R^2} \frac{1}{\bar{v}^2} \frac{1}{1+\frac{1}{2}\left(\frac{Ug}{\bar{v}^2}\right)^2}$
Distinct Messages	$\rho N_0 (\frac{24}{5} \ln 2) \left(\frac{U}{R}\right)^2 \frac{1}{\bar{v}^2}$	$\rho N_0 (8 \ln 2) \left(\frac{U}{R}\right)^2 \frac{1}{\bar{v}^2} \frac{1}{1+\frac{1}{2}\left(\frac{Ug}{\bar{v}^2}\right)^2}$

Table 3: Radiation to transport energy ratio for delivery of B bits to each of multiple targets in a sphere/disc of radius U. We define $\bar{v} = U/T \ll c$.

4.5 Directed Radiation

We have so far ignored the fact that electromagnetic radiation can be directed toward targets through means of a properly constructed antenna. Here we discuss simple physical limits on such directivity. For continuity, the development is included in APPENDIX A and can be found in any elementary text on electromagnetic propagation such as [35].

So, consider a transmit aperture of radius L, a receive aperture of radius R and a distance between them of D. Further, assume that the radiation has wavelength λ. The fraction of energy captured is given by

$$G = \frac{1}{4}\left(\frac{RL}{D\lambda}\right)^2 \tag{77}$$

where we restrict $G \leq 1$ since the captured power cannot exceed the transmitted power.

We can now ask how the isotropic radiation results compare to directed radiation. For simplicity will only consider point to point links. First, we remove the isotropic energy capture factor of $\frac{R^2}{4D^2}$ from the results of TABLE 1 and add the gain factor G from equation (77) to obtain TABLE 4 for point to point links.

Directed Radiation	
Scenario	**Energy Ratio, Ω**
Free space ($\bar{v} \ll c$)	$\frac{\rho}{G} N_0 \left(2\ln 2\right) \frac{1}{\bar{v}^2}$
Artillery	$\frac{\rho}{G} N_0 (2\ln 2) \frac{1}{\bar{v}^2} \frac{1}{1+\left(\frac{gD}{2\bar{v}^2}\right)^2}$
Artillery min	$\frac{\rho}{G} N_0 \left(2\ln 2\right) \frac{1}{g} \frac{1}{D}$

Table 4: Energy ratio Ω to deliver B bits to a target at distance D by deadline T assuming fraction G (equation (77)) of all radiated energy is captured by the receiver. $\bar{v} \equiv \frac{D}{T}$ so long as $\bar{v} \ll c$.

5 Results

5.1 Gravitational Escape

First we consider particles which must overcome a potential field to reach the target. Since calculating the minimum energy under a given deadline is difficult (see equation (40)) we instead calculate $\bar{v}$ as a function v_0^2 and then plot particle energy $\frac{1}{2}mv_0^2$ as a function of $\bar{v}$. This allows comparison with similar plots for potential-free particles. Furthermore, the differences in energy will allow us to calculate energy ratios (Ω) by direct rescaling of previous results for potential-free particles.

Thus, in FIGURE 3 we plot energy per gigabit versus $\bar{v}$ for a particle launched from the earth's surface toward a distant target and also for a particle launched from earth orbital distance from the sun, but not on the earth's surface. For comparison, we also plot energy per gigabit for a potential free particle on the same graph. We see that the potential and potential-free curves differ

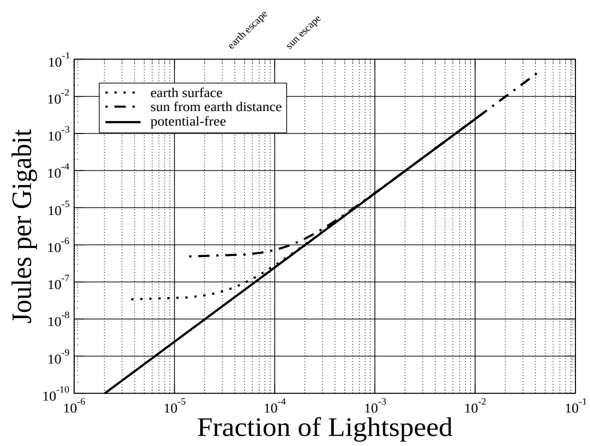

Figure 3: Energy per Gigabit in joules versus $\bar{v}$ for particles which must overcome earth surface gravitation, sun gravitation (from earth orbital distance) and a potential-free particle. Mass information density $\rho = 1.84 \times 10^{24} bits/kg$.

significantly only when $\bar{v}$ is near or below the relevant escape velocity – $\approx 11km/s$ from the earth and $\approx 42km/s$ for the sun from earth orbital distance. Thus, an initial velocity above $10^{-4}c$ will result in roughly the same values of energy ratio Ω as for free particles. For initial velocities of $3 \times 10^{-4}c$ and above, free and gravitationally bound particles will have virtually indistinguishable Ω.

5.2 Point-to-Point Link Comparisons

Here we plot the energy ratio Ω for point to point links first assuming free space propagation over large distances (interstellar) and then terrestrial conditions. The primary differences between these two scenarios are the receiver temperatures and the ratio of target range D to aperture radius R. For terrestrial systems we assume a temperature of 300°K and range to aperture ratios between 100 and 10^5 corresponding to a receiver aperture on the order of $R = 0.1$m and ranges up to 10 kilometers. For interstellar conditions we use a receiver temperature of 3°K and range to aperture ratios above 1.48×10^9, corresponding to a receiver with aperture cross section as large as the earth at one lightyear. For an aperture the size of the Arecibo radio telescope dish ($\approx 150m$ radius), the range to aperture ratio corresponding to one lightyear is 6.3×10^{13}.

In all cases, inscribed mass channels are many orders of magnitude more efficient than radiative channels. For example, using earth-sized apertures, we see in FIGURE 4 that for a mean speed of $\bar{v} = 10^{-3}c$, inscribed mass requires 10^{10} less energy than electromagnetic radiation at a range of one lightyear. At ten thousand lightyears, this gain is 10^{18}. For an Arecibo-sized aperture, the energy gain of mass over radiation is a factor of about 10^{19} at one lightyear and 10^{27} at ten thousand lightyears as may be seen in FIGURE 5. These gains are, for lack of a better word, astronomical.

For terrestrial systems, the gains are not astronomical, but still impressive. In FIGURE 6 we have gains of approximately 4×10^6 at range ten meters, and at ten kilometers, 4×10^9. The delivery delays associated with these distances are 1.4 and 45 seconds, respectively. We also note that if more typical propagation loss characteristics (D^4) were used [35], the gains of inscribed mass over radiation would be much higher. For example, instead of 4×10^9 at ten kilometers we would have 4×10^{17}, and at ten meters, we would gain a factor of one hundred in Ω.

Thus, for reasonable receiver aperture sizes and dense but empirically possible mass information density, inscribed mass transport is *much* more efficient than isotropic radiation over point to point links when some delay can be tolerated.

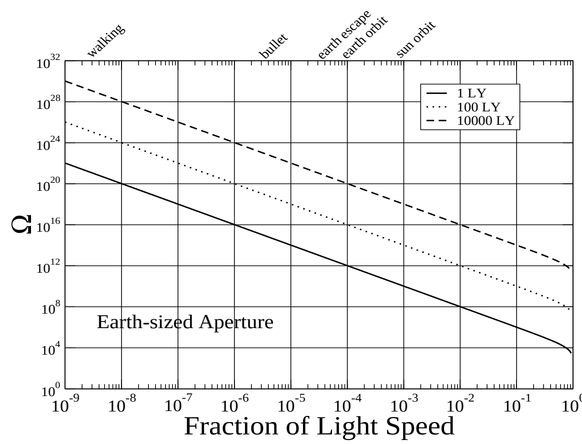

Figure 4: Energy ratio for free space particles versus mean particle speed using equation (55). The receiver is assumed to have an earth-sized aperture so that the range to aperture radius ratio at one lightyear is 1.48×10^9. The bit per mass density is $\rho = 1.8 \times 10^{24}$bit/kg and the receiver temperature $3°$K. Notice that relativistic effects do not emerge until $\bar{v}$ is very close to c.

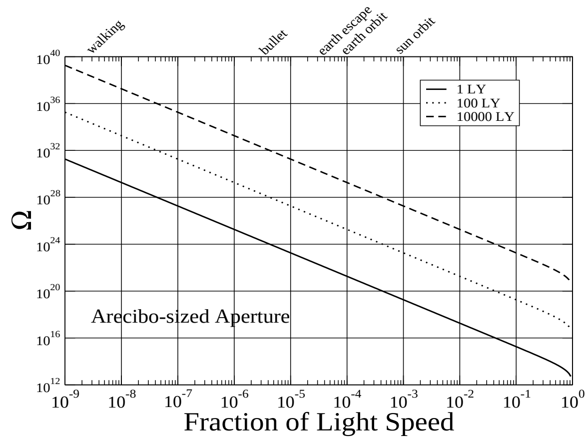

Figure 5: Energy ratio for free space particles versus mean particle speed using equation (55). The receiver is assumed to have an Arecibo-sized aperture ($150m$) so that the range to aperture radius ratio at one lightyear is 6.3×10^{13}. The bit per mass density is $\rho = 1.8 \times 10^{24}$bit/kg and the receiver temperature $3°$K. Notice that relativistic effects do not emerge until $\bar{v}$ is very close to c.

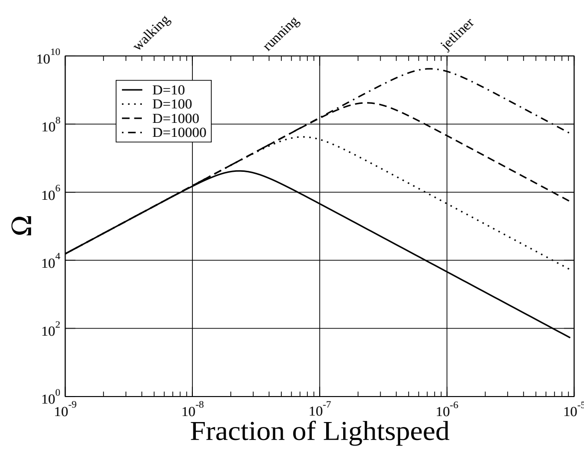

Figure 6: Energy ratio for artillery problem with a receiver aperture of $R = 0.1$m and distance D in meters versus mean particle speed using equation (57). Mass bit density $\rho = 1.8 \times 10^{24}$bit/kg, receiver temperature $300°$K. D^2 propagation loss assumed. Multiply results by D^2 for terrestrial propagation over a ground plane [35].

5.3 Multiple Destinations: Shared and Distinct Message Comparisons

Here we consider the energy ratio Ω when the same message is to be delivered everywhere (shared) and when different messages are intended for different destinations (distinct). We consider interstellar distances with two aperture sizes (Earth-sized and Arecibo-sized) as well as terrestrial distances with an aperture of $R = 0.1m$.

As can be seen by comparing FIGURES 4 and 5 with FIGURES 7 and 8, radiation does confer some benefit for broadcast, but is still insufficient to completely overcome the inscribed mass advantage. Even at very large distances such as ten thousand lightyears where there are many

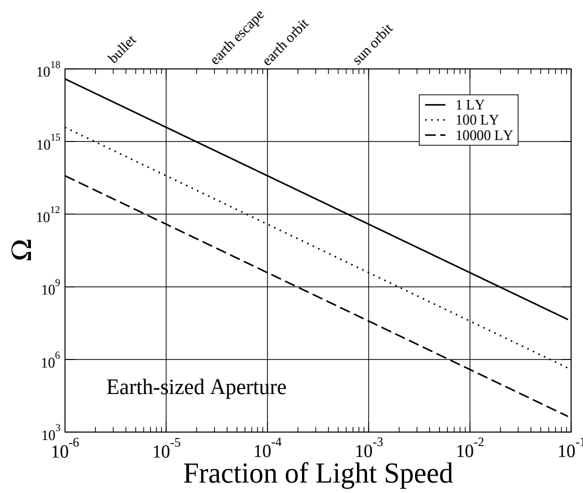

Figure 7: Energy ratio for shared message problem in free space for an earth-sized aperture radius R and interstellar distances U using equation (61). One lightyear corresponds to $\frac{U}{R} = 1.49 \times 10^9$. The bits per mass density is $\rho = 1.8 \times 10^{24}$bit/kg, the receiver temperature is 3°K and the stellar density is assumed to be that of the Milky Way, $\eta = 6.4 \times 10^{-3}/\mathrm{LY}^3$.

potential destinations to be contacted, for $\bar{v} = 10^{-3}c$ and earth-sized apertures, inscribed mass is about 4×10^7 times more efficient than isotropic radiation as compared to a factor of 10^{18} for point to point communication. Likewise, for smaller apertures at the same distance, we have 7×10^{18} versus 10^{27}. Regardless, these are still very large numbers.

For terrestrial systems we see similar behavior in FIGURE 9 which essentially reverses the maximum gain profile as a function of coverage distance seen in FIGURE 6. However, even at ten kilometers, the advantage of inscribed mass over isotropic radiation is still 3.4×10^5, an advantage which would still hold even if the receiver density were one per square meter instead of 1.11×10^{-3} per square meter. As with point to point communications, we note that if the typical radiative loss of D^2 for terrestrial systems were used, the inscribed mass gains would be even higher.

In FIGURE 10 we plot energy ratios for distinct messages and as one might expect, the benefit conferred by radiation when the same message is sent to all receivers all but vanishes. At one lightyear and $\bar{v} = 10^{-3}c$, inscribed mass is about 10^{10} times more efficient than isotropic radiation

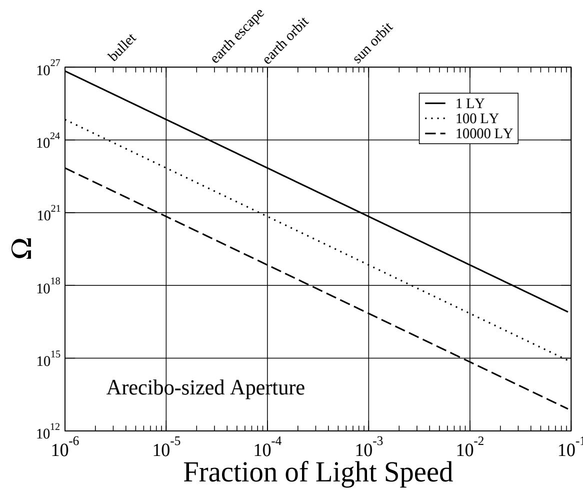

Figure 8: Energy ratio for shared message problem in free space for an Arecibo-sized aperture radius R and interstellar distances U using equation (61). One lightyear corresponds to $\frac{U}{R} = 6.3 \times 10^{13}$. The bits per mass density is $\rho = 1.8 \times 10^{24}$bit/kg, the receiver temperature is 3°K and the stellar density is assumed to be that of the Milky Way, $\eta = 6.4 \times 10^{-3}/\text{LY}^3$.

and at ten thousand lightyears, the advantage is about 10^{18} – both almost identical to the point to point advantage seen in FIGURE 4. Similar behavior is seen by comparing FIGURE 11 and FIGURE 5 and also for terrestrial systems by comparing FIGURE 6 and FIGURE 12. We note again that mass would confer even more advantage had more typical D^4 propagation loss been assumed over the radiative channel.

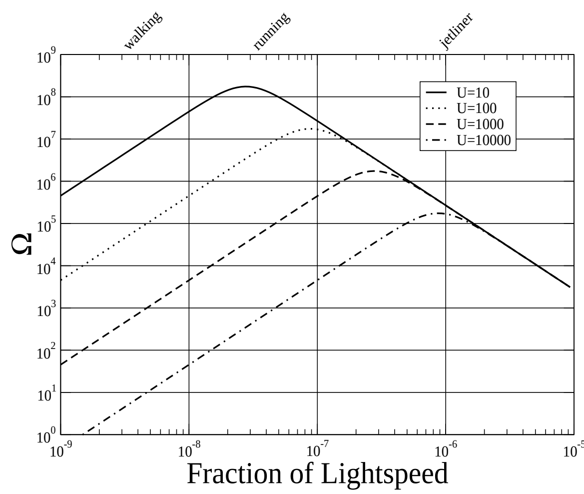

Figure 9: Energy ratio for shared message artillery problem with a receiver aperture of $R = 0.1$m and coverage distance U in meters versus mean particle speed using equation (63). The bits per mass density is $\rho = 1.8 \times 10^{24}$bit/kg, the receiver temperature $300°$K, and the receiver density was taken as $1.11 \times 10^{-3}/\text{m}^2$, or one user every $(30m)^2$. U^2 propagation loss assumed. Multiply results by U^2 for terrestrial propagation over a ground plane [35].

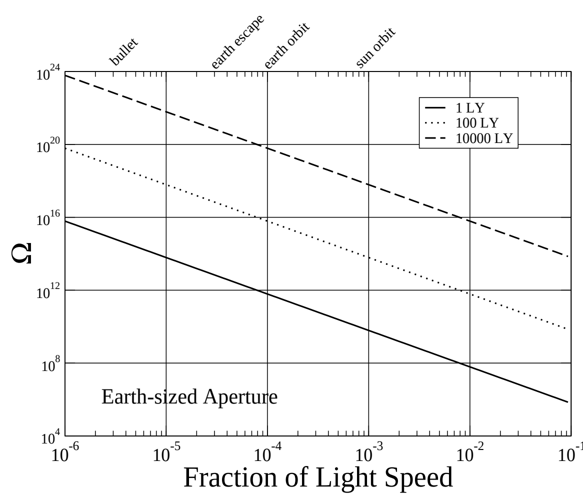

Figure 10: Energy ratio for distinct message problem in free space for an earth-sized aperture radius R and interstellar distances U using equation (74). One lightyear corresponds to $\frac{U}{R} = 1.49 \times 10^9$. The bits per mass density is $\rho = 1.8 \times 10^{24}$bit/kg, the receiver temperature is $3°$K and the stellar density is assumed to be that of the Milky Way, $\eta = 6.4 \times 10^{-3}/\text{LY}^3$.

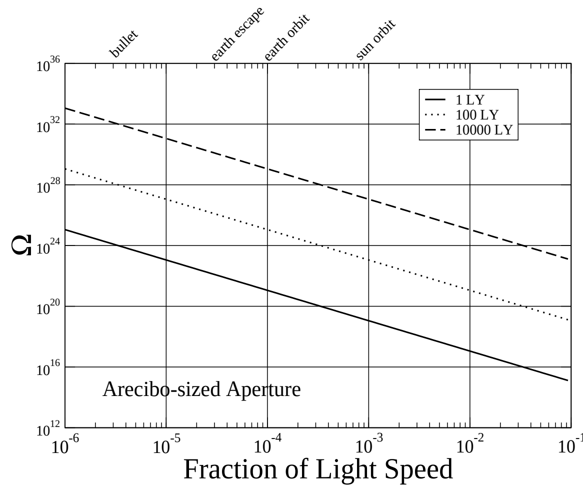

Figure 11: Energy ratio for distinct message problem in free space for an Arecibo-sized aperture radius R and interstellar distances U using equation (74). One lightyear corresponds to $\frac{U}{R} = 6.3 \times 10^{13}$. The bits per mass density is $\rho = 1.8 \times 10^{24}$bit/kg, the receiver temperature is $3°$K and the stellar density is assumed to be that of the Milky Way, $\eta = 6.4 \times 10^{-3}/\mathrm{LY}^3$.

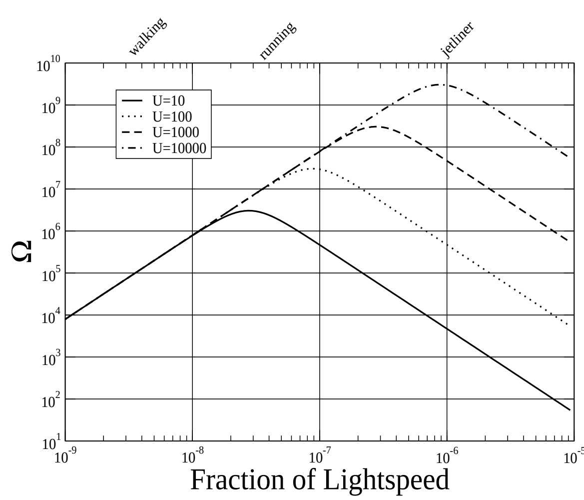

Figure 12: Energy ratio for the distinct message artillery problem with a receiver aperture of $R = 0.1$m and coverage distance U in meters versus mean particle speed using equation (76). The bits per mass density is $\rho = 1.8 \times 10^{24}$bit/kg, the receiver temperature $300°$K, and the receiver density was taken as $1.11 \times 10^{-3}/\mathrm{m}^2$, or one user every $(30m)^2$. U^2 propagation loss assumed. Multiply results by U^2 for terrestrial propagation over a ground plane [35].

5.4 Directed Radiation and the Critical ρ

Suppose all the radiated energy is captured by the receiver so that $G = 1$ in equation (77). We can then ask what value of mass information density ρ makes mass transport more efficient – the *critical value* of ρ such that $\Omega \geq 1$. We note that the result will be a max min bound for ρ since it may not be possible to achieve $G = 1$ for given wavelengths, aperture sizes and distances. As before, for the terrestrial system we will assume a receiver temperature of $300°$K and for the interstellar receiver, $3°$K. We then have

$$\rho_{\text{terr}} \underset{\text{radiate}}{\overset{\text{write}}{\gtrless}} 1.71 \times 10^{21} D \tag{78}$$

and

$$\rho_{\text{stel}} \underset{\text{radiate}}{\overset{\text{write}}{\gtrless}} 1.74 \times 10^{22} \bar{v}^2 \tag{79}$$

For $D = 1\text{m}$, $\rho \geq 1.7 \times 10^{21}$ will suffice to make mass more efficient. For $D = 100\text{m}$ we have $\rho \geq 1.7 \times 10^{23}$bits/kg. Both these figures lie within our empirical “existence proofs” for mass information densities. Thus, for $D < 100\text{m}$ we can say that terrestrial inscribed mass channels are *always* more efficient than radiative channels for empirically observed mass information densities, and that this advantage grows inversely with the distance between source and destination. The delay to be tolerated is $\sqrt{2D/g} = 0.45\sqrt{D}$ which seems not too onerous for most terrestrial distances.

In contrast, for interstellar transmission at reasonable speeds ($\bar{v} \geq 10^{-4}c$), we must have $\rho \geq 1.57 \times 10^{31}$bits/kg which is about seven orders of magnitude larger than our largest empirical ρ. So, if the radiated energy can be perfectly focused at the intended destination, radiative channels are much more efficient than inscribed mass channels even using the most dense storage medium we know.

However, it may not always be possible to focus all radiated energy on the destination receiver. So let us refine these ρ bounds by considering what is possible given receiver and transmitter apertures. We multiply equation (78) and equation (79) by G from equation (77) (assuming $G \leq 1$) to obtain

$$\rho_{\text{terr}} \underset{\text{radiate}}{\overset{\text{write}}{\gtrless}} 4.27 \times 10^{20} \frac{R^2 L^2}{D \lambda^2} \tag{80}$$

and

$$\rho_{\text{stel}} \underset{\text{radiate}}{\overset{\text{write}}{\gtrless}} 4.36 \times 10^{21} \bar{v}^2 \frac{R^2 L^2}{D^2 \lambda^2} \tag{81}$$

For the terrestrial system we will assume receive and transmit apertures of radius 0.1m (handheld devices) and 1m (typical of base station transmitters) and a wavelength of 5.66cm corresponding to a transmission frequency of 5.3GHz (middle U-NII band [36]). For interstellar systems we will consider Arecibo-sized (150m radius) and Earth-sized (6.38×10^6m radius) apertures at the receiver and transmitter and a somewhat arbitrary wavelength of $\lambda = 1\mu$m in the infrared range. Corresponding critical ρ values are given in TABLE 5.

We see that terrestrial inscribed mass channels can be readily made more efficient than radiative channels for all values of D assuming reasonable apertures and mass information densities greater than 1.33×10^{22}bits/kg. Likewise we see that for large (earth-sized) apertures it would be difficult to make inscribed mass channels more efficient than radiative ones up to $D = 10^4$LY since mass escape from the solar system requires average speeds on the order of $\bar{v} = 10^{-3}c$. However, for less heroic apertures (Arecibo-sized), inscribed mass channels can readily be made more efficient than radiative channels.

Critical ρ Values

	$D = 10$m		$D = 1000$m	
	$R = 0.1$m	$R = 1$m	$R = 0.1$m	$R = 1$m
$L = 0.1$m	1.33×10^{18}	1.33×10^{20}	1.33×10^{15}	1.33×10^{17}
$L = 1$m	1.33×10^{20}	1.33×10^{22}	1.33×10^{17}	1.33×10^{19}

	$D = 1$LY		$D = 10^4$LY	
	$R = 150$m	$R = 6.38 \times 10^6$m	$R = 150$m	$R = 6.38 \times 10^6$m
$L = 150$m	$2.46 \times 10^{10}\bar{v}^2$	$4.46 \times 10^{19}\bar{v}^2$	$246\bar{v}^2$	$4.46 \times 10^{11}\bar{v}^2$
$L = 6.38 \times 10^6$m	$4.46 \times 10^{19}\bar{v}^2$	$1.74 \times 10^{22}\bar{v}^2$	$4.46 \times 10^{11}\bar{v}^2$	$8.06 \times 10^{20}\bar{v}^2$

Table 5: **TOP:** Critical ρ for $\lambda = 5.66$cm (5.3GHz U-NII band) and receiver/transmitter apertures of $L, R = 0.1m, 1m$. **BOTTOM:** Critical ρ for $D = 1\text{LY}, 10^4\text{LY}$, $\lambda = 10^{-6}m$ and receiver/transmitter apertures of $L, R = 150m, 6.38 \times 10^6 m$. Source to destination distance D as shown.

6 Discussion and Conclusion

In the previous sections we have seen that inscribed mass channels can be many many orders of magnitude more efficient than channels which use electromagnetic radiation – even when assumptions are made which favor the radiative channel such as large bandwidth ($\frac{WT}{B} \gg 1$) as well as best case D^2 propagation loss in terrestrial systems. The only situation where it might be difficult to make inscribed mass transport more efficient are for what seem heroically large (earth-sized) receive and transmit apertures. Furthermore, from a theoretical perspective, the energy cost of transferring local information to inscribed mass can be made as small as necessary so that no energy penalty need be paid for the inscription process [22–24]. So, in what follows, we will simply assume that inscribed mass is the more efficient method of information transport when delay can be tolerated, and pursue some ramifications of that assumption.

6.1 Size Limits for Radiated Messages Under Bandwidth Constraints

Following equation (51), let us define a radiative energy penalty

$$\chi = \frac{x}{\ln 2}\left(2^{\frac{1}{x}} - 1\right) \tag{82}$$

which is plotted in FIGURE 13. We see immediatelv that in order for the bound of equation (51)

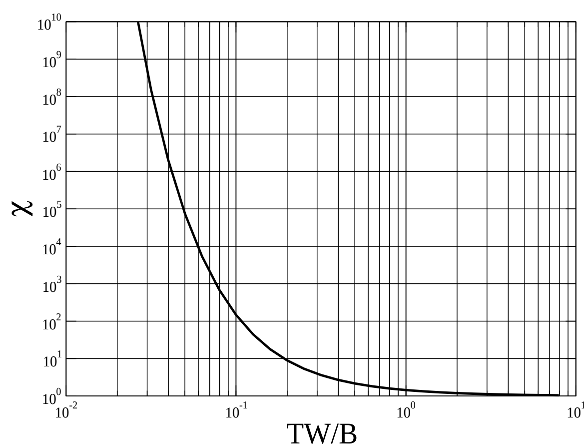

Figure 13: Energy penalty χ plotted as a function of $x = \frac{TW}{B}$. $\chi(\infty) = 1$.

to be tight, we must have $\frac{TW}{B} > 1$.

For terrestrial transport with delays associated with minimum energy mass transport ($\tau = \sqrt{2D/g} \approx 0.45\sqrt{D}$) we must therefore require that

$$B \leq 0.45W\sqrt{D} \tag{83}$$

For $W = 100$MHz, the width of each U-NII [36] band in the 5GHz range and distances of one and ten thousand meters, we have $B \leq 45$Mbit and $B \leq 4.5$Gbit respectively. Radiated messages much larger than this will incur an exponentially large energy penalty as seen in FIGURE 13. For instance, with $\frac{TW}{B} = 0.1$ and 0.01, the radiated energy penalties are $\chi = 147.6$ and $\chi = 1.83 \times 10^{28}$ respectively.

For interstellar and intergalactic systems, even for small bandwidths W, the associated delays make it difficult to imagine that $\frac{TW}{B}$ will *not* be greater than 1. However, following through on the analysis, we see that for a delay of 100M years (3.15×10^{15} seconds) and a bandwidth of 100MHz, radiated messages much larger than 3.15×10^{23} bits would incur an energy penalty.

6.2 Inscribed Mass and Wireless Ad Hoc Networks

As mentioned in the introduction, this study of inscribed mass channels was stimulated by recent work on wireless mobile ad hoc networks where nodes communicate only when near one another [11] and therefore wait until good channels arise [12]. Similarly, Gupta and Kumar [37] have shown that in fixed wireless ad hoc networks, the connection graph should be planar – which also implies nearest neighbor communication. Regardless, for all such networks, mobile and fixed, the nearest neighbor rule is a good balance between the strong interference but reduced multihop delay produced by long range transmission, and the lower interference but increased multihop delay produced by short range transmissions. That is, wireless ad hoc networks are generally interference limited structures and since interference limits throughput, short range isotropic radiation is favored.

Now harkening back to the introduction, suppose nodes simply exchanged messages inscribed on some medium when they were in proximity to one another. We have already seen that such transport, surprisingly enough, can be much more energy efficient than radiated messages. But in addition, owing to the compactness of inscribed mass, one could also argue that such communications could be essentially interference-free with at most moderate particle launch scheduling. So, following this line of thought, let us obtain operating characteristics for such a network as a function of the number of nodes N, the message size B and the delay τ. We define Γ as the throughput per node and S as the mean message delay.

In regular planar networks under uniform load, the throughput varies inversely with $\sqrt{N}$, where N is the number of nodes since the mean number of hops (mean internodal distance) a message must take between source and destination goes as $\sqrt{N}$ [38]. So, for simplicity let us assume a uniform network such as a square or hexagonal grid.

Now, ignoring all the practical concerns associated with mass transport such as the process by which mass packets are read and written, the requirement for ballistic paths between nodes and the fact that minimum energies were calculated for particles in vacuum, suppose nodes are separated by some typical distance D such that the minimum energy mass transport delay for inscribed mass is τ. Assuming incoming mass packets could be read/written by nodes in a small fraction of τ, the mean delay would be $S \propto \tau\sqrt{N}$. The throughput seen by each node would then be $\Gamma \propto \frac{B}{\tau\sqrt{N}}$.

Now suppose, as is typical in ad hoc network problems, we want to hold throughput fixed as N increases. We see that B need only increase as $\sqrt{N}$. And since $B \propto \rho\ell^3$ where ℓ is the typical dimension of the inscribed mass packet, we see that $\ell \propto N^{\frac{1}{6}}$ so mass packet sizes would grow very slowly with network size.

As a specific example, consider a network with $N = 100$ nodes arranged in a cartesian grid as shown in FIGURE 14 with a 1km separation between adjacent nodes. For maximum Ω transport,

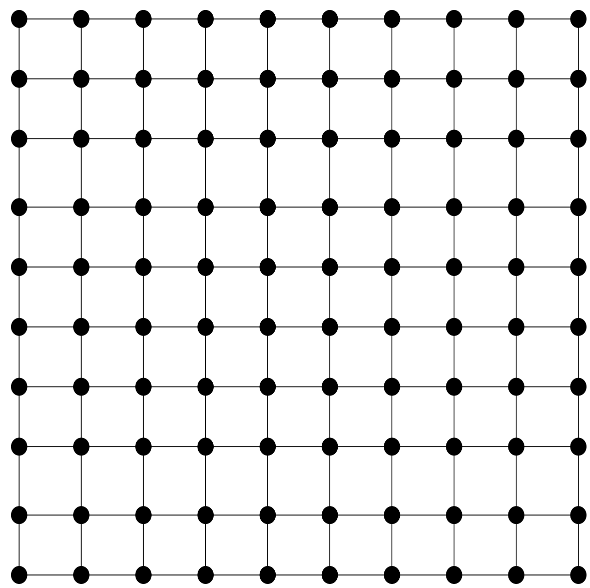

Figure 14: Manhattan street network with $N = 100$ nodes. Mean internodal distance (graph theoretic sense) is $\bar{h} = 6.6$ hops.

the delay between nodes would be 14.3sec and the energy efficiency would be $\Omega \approx 3 \times 10^8$ for D^2 propagation and 3×10^14 for more typical D^4 propagation. The mean number of hops between nodes is $\bar{h} = 6.6$ so that the average message delay is about 94sec. With a 1mm^3 mass packet at a density of one bit per nm^3, the per user throughput would be 10^{18}bits$/(14.3$sec $\times$ $6.6) =$ 1.06×10^{16}bits/sec. This rate is about 100 times the theoretical capacity of optical fiber [18].

If we increased N by a factor of 100 to $N = 10^4$ by increasing the density of nodes, the nodes would be 100m apart, the internodal delivery delay between nearest neighbors would be 4.52sec and the efficiency $\Omega \approx 3 \times 10^7$ (or 3×10^{11} with D^4 propagation). The mean number of hops between nodes is $\bar{h} = 66.6$ so that the average message delay is about 300sec and the per user throughput 10^{18}bits$/(4.52$sec $\times$ $66.6) = 3.3 \times 10^{15}$bits/sec – 33 times the capacity of fiber.

We note that in both cases, substantial mean delay improvements could be had by simply increasing the particle speed, albeit at some loss in energy efficiency relative radiation. However, particle speeds well in excess of typical rifle muzzle velocities (10^3m/s) still provide many orders of magnitude energy savings over radiation for the distance range considered, so increasing average particle speed by a factor of 10 would be not be unthinkable.

In addition, planar networks are well known to have poor multihop performance as compared to networks which allow longer range links or even random networks [38–40]. For example, the mean number of hops in our network of $N = 10^4$ nodes decreases from $\bar{h} = 66.6$ to $\bar{h} = 6.6$ if links between nodes are made randomly. One can also randomly augment regular planar graphs with "tunnels" [40]. Adding a randomly directed link to each node with a probability of 0.05 yields $\bar{h} \approx 17$ for a grid network of 10^4. The per user throughput would thus go up by a factor of about 4 and were link transit times held constant by increasing the speed of particles on longer distance links (at some tolerable penalty in Ω), the mean delay would decrease by a factor of 4 as well, or to 75sec for the 10^4 node network. The effect of such random networks and tunnels is more prominent in larger networks since randomly directed links rapidly push the mean internodal distance down toward the Moore bound [41]

$$\bar{h} \geq \sum_{k=0}^{L} kp^k \Big/ \sum_{\ell=0}^{L} p^k = \frac{p-1}{p^{L+1}-1} \sum_{k=0}^{L} kp^k \tag{84}$$

where p is the average number of links per node and the number of nodes is $N = \frac{p^{L+1}-1}{p-1}$.

For example, in a 400×400 grid network, without random links $\bar{h} = 264$ while with 5% random links $\bar{h} = 25$. For a 1000×1000 grid $\bar{h}$ goes from 666 to 30 with a similar addition of random links. The comparable completely random networks of 160000 and 10^6 nodes have $\bar{h} \approx 9$ (Moore bound ≤ 7.66) and $\bar{h} \approx 10$ (Moore bound ≤ 8.66) respectively. [3]

We again note that a number of critical technical issues have clearly been ignored, including rapid message insertion/extraction for mass packets, ballistic line of site between transceivers and transport frictional losses. However, the large energy efficiencies over radiation suggest that compensating for frictional losses and possible provision for alternate (non-ballistic) transport would not change the answers very much. Likewise, the potential to achieve throughputs per user which far exceed the theoretical capacity of optical fiber – through the exchange of what amounts to grains of sand (or sugar cubes for less dense media) – might merit a more thorough and practical investigation of rapid mass incription and readout.

6.3 Transport Delays for Terrestrial, Interstellar and Intergalactic Mass Transport

For terrestrial systems, the optimum delay using inscribed mass transport is given by equation (30): $\tau^* \approx 0.45\sqrt{D}$. For D ranging from one meter to ten kilometers we have a range of 450msec to 45sec which seem reasonable if delay can be tolerated.

For interstellar mass delivery even at high speed, transport delays can easily be geological in scale. Thus, in FIGURE 15 we provide a plot of message transport delay versus distance in lightyears for different fractions of light speed and place it explicitly in a geological context. We note that the solar system is on the order of four billion years old and the visible universe is on the order of thirteen billion years old. So, limiting delivery delays to some fraction of the earth's age

[3]The use of such long range random links is only possible owing to the assumption of non-interference between particles in transit and that the transport energy increases only linearly in D. Where possible, such addition of random long range links would be equally useful in raising the throughput of RF ad hoc networks.

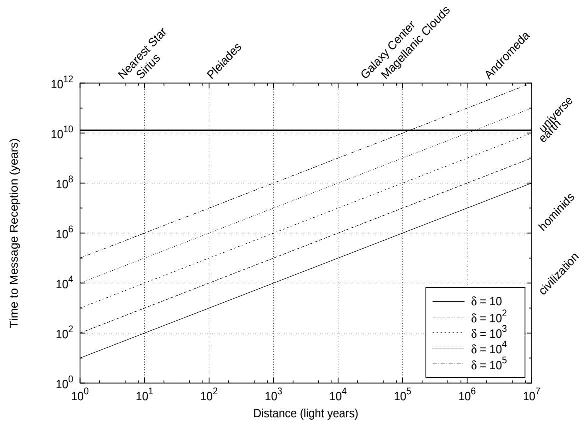

Figure 15: Message transport delay versus distance for different transport velocities. Approximate ages of human civilization, earth and the visible universe shown for reference.

(say 1 billion years) seems reasonable. For distances within the Milky Way ($100K$LY diameter) a transport speed of $\bar{v} = 10^{-3}c$ allows delivery within 100M years. Local extra-galactic messaging ($\approx 10M$LY) at $\bar{v} = 10^{-3}c$ would result in delays comparable to the age of the visible universe, so perhaps we would have to assume $\bar{v} = 0.01c$. We note that although increased velocity reduces the efficiency of inscribed mass transport, the greatly increased distance more than makes up for this loss. Specifically, by equation (81), increasing D by a factor of 1000 while increasing $\bar{v}$ by a factor of 10 results in an overall gain for inscribed mass efficiency by $\times 10^4$.

6.4 An Epiphany Revisited

It is interesting (or at least amusing) to revisit the "media in a truck" scenario mentioned in the introduction to see where intuition breaks down. Burning a gallon of gasoline liberates about 1.2×10^8Joules [42, pp.317]. A not terribly fuel efficient automobile gets 20 miles to the gallon, so the hypothetical N.Y.C.–Boston trip would consume about 10 gallons or 1.2×10^9Joules. Isotropic radiation of information over the 200mile $= 320$km distance consumes at least $N_0 \frac{4\pi D^2}{A} \ln 2$ by

$$E_r \geq BN_0 \frac{4\pi D^2}{A} \ln 2 \quad . \tag{85}$$

which assuming a 1m^2 aperture and a $300°$K receiver temperature is 3.7×10^{-9}Joules/bit. Allowing for a more realistic D^4 propagation loss, the energy necessary per bit is 378Joules. Thus, with an energy budget of 1.2×10^9 joules, either 3.24×10^{17} bits or 3.17Mbits could be delivered via radiation depending on the propagation constant in force. The critical payload in kilograms for inscribed mass transport to equal that possible with radiation would then be $3.24 \times 10^{17}/\tilde{\rho}$ kg or $3.17 \times 10^6/\tilde{\rho}$ kg. Thus, for any of the empirical mass information densities described in section 2.2, driving inscribed mass between New York City and Boston is indeed much more energy efficient for a variety of storage media.

As specific examples, for D^2 propagation with the most dense storage media, we would need only $3.24 \times 10^{17}/1.84 \times 10^{24} = 176\mu$grams to equal the energy efficiency of radiation. However, the same amount of information using media such as CD's, diskpacks, tapes and the like would be far too massive to transport. With D^4 propagation, very crude media could be used such as 20-pound paper with a 1000 dot per inch black and white laser printer. Specifically, a 500 sheet ream of 20-pound paper has dimensions $8.5'' \times 11'' \times 2''$ and mass of roughly 2.32kg. One sheet therefore weighs about 4.64×10^{-3}kg. A single sheet can hold 6.03×10^{8} dots (bits) for a mass information density of

$$\tilde{\rho}_{\text{paper}} \approx 2.0 \times 10^{10} \text{bits/kg} \tag{86}$$

Thus, only 1.59×10^{-4} kg – a small fraction of a sheet – would need to be transported to be more efficient than radiation. Repeating the same calculations for a distance 100 times closer (2 miles) we have $3.24 \times 10^{19}/\tilde{\rho}$ kg or $3.17 \times 10^{12}/\tilde{\rho}$ kg. To be as efficient as D^2 propagation radiation would require very dense storage media, and with D^4 tapes or disk packs as opposed to paper would be necessary.

Thus, for D^4 propagation, the epiphany is real – driving a truck across town is more efficient than electromagnetic broadcast. However, for D^2 or guided wave communications, physical transport using typical storage media such as disks and tape is much less efficient than radiation.

6.5 Open Issues for Interstellar Channels

We have completely ignored the channel characteristics for inscribed mass by essentially assuming that what is sent arrives intact. For terrestrial systems, this is probably not a bad assumption. However, for interstellar transport, a mass packet would be subject to a variety of high energy insults for a long period of time. This issue is important and the subject of ongoing work [26]. However, we note that the relative efficiency of inscribed mass can be at times so enormous, that incredibly high error rates could be tolerated using simple redundancy codes, by sending large numbers of separate messages, or even by encasing the message in a hardened transport carrier. Nonetheless, the effect of insults to the information integrity of mass packets needs investigating.

We have also skirted the issue of what sort of messages one might want to send, how they might be detected or where they might be sent [43–46]. The large delays associated with interstellar travel and the seeming fragility of species to cosmic insults suggests that messages should be constructed "for posterity" as opposed to for initiating a chat. One might also think of "colonization" as a goal as well [47]. In both regards, one ostensible virtue of inscribed mass channels is that once the message arrives, it is persistent as compared to electromagnetic radiation which is transient and thus must be sent repeatedly in order to assure the message is received. Of course, constructing mass packets to be hearty, easily detected and/or self replicative seems well outside our current engineering ken, but does offer interesting questions for SETI-like lines of inquiry.

A Antenna Directivity

Consider a transmit aperture of size L as in FIGURE 16. Let the field intensity and phase in the aperture be given by the complex quantity $E(x)$ and assume a frequency of $2\pi f_c$. Modeling each point in space as a point source we can obtain the field distant from the aperture as approximately the fourier transform of the aperture field.

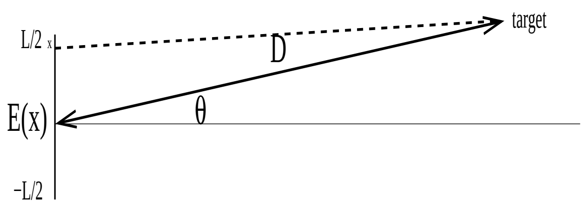

Figure 16: Calculation of far fi eld from aperture of size L.

The distance from point x in the aperture to the far field target is $\left[(D\cos\theta)^2 + (x - D\sin\theta)^2\right]^{1/2}$. Assuming a wavelength $\lambda = c/f_c$ we then have

$$E(\theta) = \int_{-L/2}^{L/2} \frac{E(x)}{\left[(D\cos\theta)^2 + (x - D\sin\theta)^2\right]^{1/2}} e^{j\frac{2\pi}{\lambda}\left[(D\cos\theta)^2 + (x - D\sin\theta)^2\right]^{1/2}} dx \tag{87}$$

We have

$$\left[(D\cos\theta)^2 + (x - D\sin\theta)^2\right]^{1/2} = D\left[1 - 2\frac{x}{D}\sin\theta + \left(\frac{x}{D}\right)^2\right]^{1/2} \approx D\left[1 - 2\frac{x}{D}\sin\theta\right]^{1/2} \tag{88}$$

since we assume $R \gg x$. We then have

$$E(\theta) \approx \int_{-L/2}^{L/2} \frac{E(x)}{D\left[1 - 2\frac{x}{D}\sin\theta\right]^{1/2}} e^{j\frac{2\pi D}{\lambda}\left[1 - 2\frac{x}{D}\sin\theta\right]^{1/2}} dx \approx \frac{1}{D} e^{j\frac{2\pi D}{\lambda}} \int_{-L/2}^{L/2} E(x) e^{-j\frac{2\pi}{\lambda}x\sin\theta} dx \tag{89}$$

which is the Fourier transform of $E(x)$ evaluated at $\frac{\sin\theta}{\lambda}$.

Now let us consider the far field resolution by setting $E(x) = 1$ in the aperture (plane wave). We then have

$$E(\theta) = \frac{1}{D} e^{j\frac{2\pi D}{\lambda}} L \frac{\sin\frac{\pi L\sin\theta}{\lambda}}{\frac{\pi L\sin\theta}{\lambda}} \tag{90}$$

A rough measure of the resolution is the distance between the first zero crossings of $E(\theta)$ where 90% of the beam energy is concentrated. So first we set

$$\frac{L}{\lambda}\sin\theta = 1 \tag{91}$$

to obtain

$$\sin\theta = \frac{\lambda}{L} \tag{92}$$

The beam width at the target is then

$$Q = 2D\sin\theta = \frac{2D\lambda}{L} \tag{93}$$

We can now compute the fraction of power captured by a receive aperture of radius R at distance D as

$$G = \frac{\pi R^2}{\pi(\frac{2D\lambda}{L})^2} = \frac{1}{4}\left(\frac{RL}{D\lambda}\right)^2 \tag{94}$$

and note that we must restrict $G \leq 1$.

B Acknowledgments

I wish to thank G.J. Foschini, a kindred spirit, for many useful discussions and Gregory Wright for both his enthusiasm and for tolerating my ignorance of sophomore level physics. I would also like to thank Peter Shor for pointing me toward Rolf Landauer's study of reversible computation and communication.

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