The canonical artefact and its cosmological interpretations

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We argue that there is a factor of at least 10^{264} to somehow be accommodated to explain how it is possible that our universe produced a life-form with our advanced level of intelligence. The factor arises from an analytical development of the theme that very intelligent life-forms would give physical expression to a highly distinguished mathematical truth in a canonical way. This structure, which serves to flag the occurrence of a life-form possessing an advanced intelligence, is termed the canonical artefact (TCA).

Using TCA we show that if the laws governing the universe's evolution to the present are indifferent to the emergence of a life-form with advanced intelligence, then the chance of emergence is less than one part in 10^{264}, and, if instead, the laws make the emergence a virtual certainty, then the emergence is greater than a 264 orders of magnitude effect. In the first case, emergence of a life-form with advanced intelligence is exceedingly unlikely, and, in the second case, where such a life-form is inevitable, physics faces the problem of giving even a semblance of a *quantitative* explanation for a 264 orders of magnitude effect. In either case physics is left with a puzzle: explain why our universe contains a life-form with advanced intelligence.

1. Introduction

Suppose that Ψ, the wave function of the universe (Barrow & Tipler 1986, pp. 105, 492–499; Barrow 1991, pp. 63–68; Davies 1982, pp. 122–130) conditioned on our historical epoch of 1 s after the Big Bang, actualizes so that humanity does not come into existence but some form(s) of advanced intelligence does. Is there a kind of minimal artefact for us to construct, bearing incontrovertible evidence of humanity's advanced intelligence, for which we could say, *they*, with their advanced intelligence would also construct a copy? *They* can be regarded as a foil that we use to conceive of an artefact providing material evidence of humanity's advanced intelligence, but otherwise devoid of any peculiarities of our evolutionary or cultural history. Using this foil we will define such an object (equivalence class of objects, to be precise). We call it the canonical artefact (TCA). It is easily constructed and a copy of it can be held in one's hand. TCA can be viewed as a transcendent flag of advanced life. Consequently, the basic question – can a case be made that the collapse of Ψ is apt to include life-forms with an intelligence as advanced as that of humans? – is sharpened and enlivened by consideration of TCA. TCA presents a very special challenge to the explanatory power of contemporary physics, even to be explained 'in principle'.

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Cannot many everyday artefacts be thought of as challenging contemporary physics for a basic explanation for how they came into existence, given the state of the universe at some time shortly after the Big Bang? The point is, that the difficult to explain everyday artefacts, like say a book, can immediately be legitimately dismissed as not being worthwhile to attempt to explain, because their essence is inextricably interwoven with peculiarities of our evolutionary and cultural history. Such peculiarities are far beyond the domain of what theoretical physics is equipped to deal with in any practical and fundamental way. TCA, although also not easy to deal with, is an artefact for which the peculiarity objection cannot be upheld.

While the content here is necessarily wide-ranging, our approach is not difficult. The mathematics that we do is of the simplest kind. Moreover, our definition of TCA allows us to avoid any direct analysis of evolutionary processes. Indeed, we will be able to obtain our results regarding the emergence of advanced intelligence without any direct reference to biology. For our limited goal there is no need to do any analysis involving organic chemistry. Not only do we avoid getting bogged down in intractable calculations that that would entail, but we also avoid being vulnerable to the charge of possible provincialism that could be associated with making the assumption that advanced intelligence is, say, carbon based.

Our approach involves the following. TCA is defined to occupy at least a certain amount of space and time. If the laws of physics are indifferent to the emergence of TCA, a simple counting over-estimate, using all the spacetime that any universe actualization has to offer, shows TCA's chance occurrence to be less than one part in 10^{264}. If, on the other hand, the laws of physics make TCA essentially inevitable, the challenge then is, how do we look at these laws to begin to see hints of a 264 order of magnitude effect? Less extreme degrees of favouring or not favouring TCA, than the two just mentioned, turn out to be just as provocative. Facing these options may make one question the attainability of the ideal of a so-called theory of everything (TOE) (Barrow 1991; Davies & Brown 1988; Hawking 1980; Kaku & Trainer 1987) if it is required to reach so far as to explain material expressions of thought like TCA.

To exhibit the ideas associated with TCA, we tentatively assume the context of a future hypothetical TOE. Exaggerating the reach of contemporary physics simply serves to highlight the challenge to explain TCA. When we will make reference to Ψ, in no way will we be dealing with the arcana of current research into a TOE. It will suffice for the reader to have been exposed to the notion of Ψ in any of the references mentioned in the opening paragraph.

Next we will outline our projection of the conventional wisdom that would be associated with the establishment of such a TOE. With this viewpoint, and the ensemble of possible universes clarified (§3), the definitions of both TCA (§4) and advanced life-forms (§5) follow. Analysis of the emergence of advanced life-forms is done in §6. In §7 we summarize what was established regarding emergence. Then in §8 we will discuss some variations on the definition of TCA. In closing, in §9, we mention how consideration of TCA results gives a new insight into a basic computer science issue.

2. Hypothesizing a successful TOE

TOE unites all four forces, with all the equations fitting on a 'tee-shirt', with quantum gravity included. TOE is consistent with a universe that, in its gross respects, is what most physicists take ours to be like. Notably, having about 10^{80} nucleons and lasting well beyond 20 billion years $\approx 10^{18}$ s. Moreover, the large-scale structure implications of TOE are consistent with what astronomical observations tell us about our universe. Also, the intimately related small-scale structure is consistent with what our particle accelerators tell us.

In many realms it is difficult, perhaps in some cases too difficult, to accurately trace TOE's implications. Quantum uncertainties, sensitivity to small influences, incomplete identification of relevant processes, inadequate modelling, and computational limitations may leave many details of the universe's evolution to be explained only 'in principle'. These practical difficulties notwithstanding, the tee-shirt displays the holy grail. All there is to say has been said. Undoubtedly it can be said better and myriad marvellous details remain for generations to articulate, but fundamental theoretical physics is complete. We tentatively assume this context.

3. The possible universes and a very simple spacetime bound

We next carefully specify the actualizations of Ψ that we will be considering. We constrain Ψ to have a heritage that is the same as ours to the end of the age of antimatter annihilation (Davies 1982, p. 30; Rees 1988). We use $\tau_a \approx 1$ s to denote the time duration from the Big Bang to the end of the antimatter annihilation age. When we refer to the class of other potential realizations of the universe, we really mean such potential actualizations of Ψ, satisfying this initial condition (IC), that are elaborated up to the epoch of 20 billion years of age. The mass densities of these actualizations are thereby sufficiently close to the critical density, so that among the predictions that TOE can make, is the virtual certainty of actualizations evolving from the IC, through stellar generations, to form the stable elements in roughly the same relative abundances as in our actualization. Each of these potential universe realizations has, during its maturity (post-antimatter annihilation age), about 10^{80} nucleons.

For the purpose of the very simple combinatorial arguments in this paper it is convenient to just keep count of nucleons and not other matter. Since 20 billion years $\approx 10^{18}$ s, the universe up to the 20 billion year epoch has $\approx 10^{98} = 10^{80+18}$ nucleon-seconds. In what follows that is all that is meant by the very loose statement that the amount of spacetime available is $\approx 10^{98}$ nucleon-seconds. As is well known, for the smallest atom, hydrogen in its ground state, according to the Bohr model, it takes $\approx 10^{-16}$ s for an electron to orbit the proton nucleus. This time is called the atomic year (see Barrow & Tipler 1986, p. 298), and we label it τ_y. We shall find it more convenient to express time in terms of this atomic year rather than seconds. Hence we will say that the amount of spacetime available is $\approx 10^{114}$ nucleon-atomic years. Here is a simple bound using this quantity. If during the 20 billion year history of the universe we consider J objects which existed, the jth of which had η_j nucleons and lasted τ_j atomic years and if no two of these objects simultaneously shared a nucleon, we can say that

$$\sum_{j=1}^{J} \eta_j \tau_j \leqslant 10^{114} \text{ nucleon-atomic years.} \tag{1}$$

Also consider any set of objects each member of which occupies no less than η nucleons and lasts for a duration of no less than τ atomic years. If such a set has the additional property that no two of the objects simultaneously shares a nucleon, then, from the above inequality, one can conclude that there cannot be more than $10^{114}/(\eta\tau)$ objects in the set.

We will find this conclusion useful. The next section describes TCA. This description will provide us with lower bounds on the number of nucleons and the duration of existence for any object that is a possible candidate for TCA. It will turn out that we will be able to use inequality (1) in the way just described to provide us with an upper bound on the number of such objects. From such considerations and some other aspects of TCA it will be a very simple matter to develop the main results that were mentioned in the introduction.

4. The canonical artefact (TCA)

We stress that advanced life-forms in other potential actualizations of the universe are to be regarded here as foils to catalyse a careful reasoning process aiming to begin to shed some light on the issue of whether the emergence of an advanced life-form is inevitable in a universe described by the hypothesized TOE+IC. TCA helps explore this issue, since, while according with quantum mechanical rules, TCA serves as an indicator of advanced life-forms for all potential actualizations of Ψ. TCA appears because of an advanced life-form, otherwise its appearance is virtually impossible. One often associates divergence with evolutionary history, but, in a sense that we will make clear, advanced life-forms in different potential actualizations *converge* on TCA.

TCA is a material object composed of atoms. It is conceived in the tension between extreme minimalism and incontrovertible evidence of significant intelligence. Here, by extreme minimalism we mean that quality of no ornamentation or digression from the function of flagging a life-form with advanced intelligence. In §4*a* we try to convey the spirit of the blueprint for TCA's construction by giving the two conditions necessary to be in TCA equivalence class (TCAEC), each member of which is designated to be a copy of TCA. In §4*b* we will given an 'example' for clarification. In §4*c* we will show that there is much greater flexibility and robustness associated with concept of TCAEC than may at first be apparent.

(*a*) *TCA properties*

Minimal, yet highly distinguished. This theme facilitates consensus of what TCA is, even among the potential actualizations seeking consensus which have very different histories since time τ_a. Extreme minimalism is used to define its physical form with the great exception that the form is modulated in a very straightforward way by the expression of some very highly distinguished, ordered, non-empty, finite set of positive integers N. The set of number(s) must be from deep within the Platonic realm (Barrow 1991; Penrose 1989), so that we can take for granted that the agency of intelligent life is needed to give it material expression. Making no allowance whatsoever for any possibility of compressing the description of N, let n be the total number of bits needed to express all of the numbers of N. We will require that n be large enough so TCA is easily disassociated from objects admitting a ready explanation by physical processes not involving intelligent life. Ideally, the choice of N should be so distinguished that it is not at all arbitrary. In the following subsection, we shall see that our suggested choice for N is an extraordinarily special sequence, that has such a large n, that we can very easily argue for the aforementioned disassociation.

Distinct presence. This requirement simply gives meaning to TCA having existed in an actualization of Ψ. In accounting for its lifetime, an object that comes in and out of existence is considered to be separate object with each appearance. We also require a region surrounding TCA that is essentially not composed of the same substance as that used for constructing TCA. This is simply to avoid any ambiguity in accounting for the spacetime expended by TCA. The physical object, TCA, is composed of at least of the order of η atoms and lasts for at least a duration of the order of τ. These strictly positive limits will be so unimposing as to permit TCA to be constructed on any scale whatsoever that is convenient for an object composed of atoms. The choices are $\tau = \tau_y$ and $\eta = n$. Regarding the choice of τ_y, the rationale is simply that, after all, the artefact is composed of atoms and it takes at least of the order of $\tau_y = 10^{-16}$ s for an atom to be considered an atom, no matter what kind of atoms are used. The choice of n merely expresses that, at least, on the order of an average of one atom per bit be expended in expressing N. That this requirement on the number of atoms is exceedingly unimposing will be underscored in (*b*).

Once N and hence n is known, we can use the distinct presence requirement and the statement following inequality (1) to get the value of the upper bound on the number of distinct objects that can be contained in an actualization of Ψ that might qualify to be TCA.

(*b*) '*Example*' *of* TCAEC

Our first crude attempt at a definition of TCAEC follows. For it, as the reader probably anticipates, it will be obvious that for humanity a convenient construction would very easily meet the η and τ distinct presence conditions. Indeed, we will see that they can each be met with far over 20 orders of magnitude to spare. After we describe our example we will deal with the specificity of the choices made.

TCA is a material object, not a signal. Yet, TCA idea certainly owes a debt to the philosophy associated with the Search for Extra Terrestrial Intelligence (SETI) project. In Sagan & Drake's (1975) discussion of a possible SETI message we find, 'The first 30 primes for example, would be difficult to ascribe to some natural astrophysical phenomenon.' Even the first 100 primes would not be an appropriate N for us, none the less, the N we do need is in the spirit conveyed by Sagan & Drake.

Our choice for N is the ordered list of the orders of the 26 sporadic simple groups (Gorenstein 1985; Thompson 1983) $N = \{N_i,\ i = 1, 2, \ldots, 26\}$. The sporadic simple groups are highly distinguished among what, in a rough analogy, can be considered to be the 'primes' of finite group theory. To express these 26 numbers, which answer a deep question in mathematics, in a straightforward way, takes about 1245 bits. At the highest descriptive level is the topological form for the material expression of N. This form is simply one closed loop. See figure 1*a*.

In expressing the N_i the question arises as to which base to use. We make the choice with material expression in mind. We would like to randomly code each of the N_i so that if there are long dull patterns, they do not lead to material formations easily confused with natural formations not requiring intelligent life-forms. However, we cannot randomly code, as it is not generally meaningful to communicate the code choice between potential actualizations. Our judgement of the best choice under the circumstances, is to pseudo-randomly code, in an extremely obvious way. Namely, we express each N_i in base m_i, where m_i is the least integer relative prime to N_i. A permissible variation is to express the entire sequence using base 53 which is the smallest prime that is relatively prime to all 26 numbers.

For material expression think of a single loop necklace. We call this TCA construction a mass packet. These terms 'single loop necklace' and 'mass packet' are to be understood to be purely figurative. For the beads, nondescript 'rock shapes'

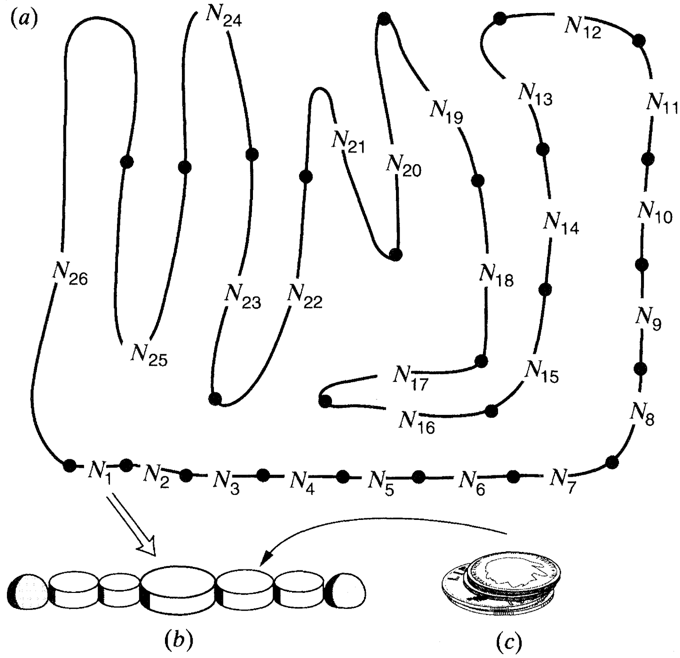

Figure 1. Initial attempt at describing a copy of the canonical artefact. Hypothetical life-forms with mathematics as advanced as that of humanity's would have discovered a certain highly distinguished 26 element sequence, $N = 7920$, 95040, 175560, 443520, 604800, 10200960, 44352000, 50232960, 244823040, 898128000, 4030387200, 145926144000, 448345497600, 460815505920, 495766656000, 42305421312000, 64561751654400, 273030912000000, 51765179004000000, 90745943887872000, 4089470473293004800, 4157776806543360000, 86775571046077562880, 1255205709190661721292800, 4154781481226426191177580544000000, 808017424794512875886459904961710757005754368000000000. As explained in the text, if such a life-form canonically materially expresses N in a certain form, they have composed a copy of the canonical artefact (TCA) and thereby qualify as what we define to be an advanced life-form. A copy of TCA is a member of the equivalence class of objects (defined in the text) that is closely associated with the sequence N. From a cosmological viewpoint, physics faces a very difficult problem in explaining, even 'in principle' the existence of an object in this equivalence class. Each of the ways of explaining it are very provocative. An example of the required form for TCA is a single loop of beads and separators, called a mass packet, that is illustrated above.

(*a*) High level description of mass packet. The 26 numbers are expressed in increasing order. The blackened dots represent separators of the 26 numbers. The numbered curves represent the segments of beads that express the number indicated. The 26 segments cohere in the direction in which they express numbers as one traces around the loop. As represented by the lengths of the segments, there is a considerable range in the number of decimal digits associated with each number: N_1 requires four digits while N_{26} requires 54.

(*b*) The first integer $N_1 = 7920$ is shown expressed by a segment of five beads. The two sphere-like objects at the ends of the segment are separators. The number 7 is the smallest number relatively prime to 7920, so 7 is taken to be the natural base in which N_1 is expressed by masses. In base 7 $N_1 = 32043$. The five beads shown have masses in the ratio 3:2:7:4:3.

(*c*) Instead of using mass ratios, another acceptable way of expressing 'digits' in base m is with tokens. Here 4 is represented by using four coins as tokens.

would do fine. All beads have the same composition. For each N_i, beads graded in mass in multiples of the smallest, from one to m_i, are used as needed to express 'digits' in base m_i. To represent zero, m_i is used. The separators that separate the representation of the consecutive N_i are somehow differentiated from all the beads, e.g. in shape or mass or substance. So each bead contacts exactly two other beads or one separator and one bead. Scale is unimportant, except that, at a very small scale, the notion of the beads and separators loses meaning. The N_i appear in increasing order. See figure 1*b* for an illustration of how N_1 can be represented. The sense in which to read the loop is decided by whichever sense gives the sequence.

To underscore how unimposing the η bound mentioned in the distinct presence requirement is, imagine a construction, however impractical, on the very smallest scale using multiples of a single hydrogen atom. It turns out that, about 3000 atoms are needed thus meeting the η bound mentioned in §4*a*. The estimate 3000 comes from a computer calculation converting each of the 26 orders to the required number base (e.g. from figure 1*b*, N_1 needs 19 atoms). On the very smallest scale, the variation that is mentioned using base 53 throughout needs even more atoms.

We have been using graduated mass values to represent number units. This is much too restrictive. We next greatly extend the equivalence class by allowing tokens for unit designations. To best explain some subtle points we proceed by way of illustration. Consider porpoises. We have no idea that porpoises are at all disposed to determine N, much less have any sort of appreciation for the Big Bang. We just want to point out, that if in some potential actualization, an advanced porpoise-like life-form, in a sea like ours, sought to construct TCA, they could comfortably use tokens to compensate for low manipulatory ability. One could accept N in the form of say piles of nominally identical seashells, with a convenient object, say a piece of sea fan, used for each separator. So the notion of a 'bead' (e.g. a pile of seashells) is now interpreted very liberally. For example, a pile with 6 tokens is thereby treated here as acceptable as a basic representation of the 'digit' 6 in, say, the base 7, despite its historically dependent composition. 'Tokenness' is a quality for which detailed composition is irrelevant as tokens are meaningful modulo their detailed composition. The seashells could vary considerably in say, size, but each is a token for exactly one unit. In contrast, a single stone with the character '6' neatly chiselled on it, does not express 'sixness' in a basic way: some historical information is needed to understand the meaning of the symbol (unless it appears in a redundant self-revealing context).

In figure 1*c* tokens suitable for us to use, namely coins, are shown. The essence of TCA is required to be devoid of any historical overtones: the figures on the coins as well as their denominations are irrelevant details as far as their tokenness property is concerned. From the calculation mentioned earlier in this subsection about 3000 coins would be needed. Assuming a lifetime of at least one day, each of the n and τ distinct presence bounds are met by well over 20 orders of magnitude.

(*c*) *Secondary importance of our specific choices*

We understand that what we have put forth here is only an initial attempt whose purpose is to convey the idea of TCA. There could be a more distinguished choice for N than the one we chose. There may be material expressions that are simpler than what we have allowed, and maybe there are better ways to represent N than the way we did it. TCAEC definition aims to be as accommodating as it can be, but not so accommodating that it loses its close identification with N. We proceed as if this mass packet candidate for TCAEC is in fact TCAEC. (The other candidates included, for example, constrictions based on primes, Mersenne primes (Schroeder 1990), as well as the starting representations of π and e and certain physical constants. See also §7.)

In §6 we will do computations using our above definition of TCAEC. However, just to set the stage to later demonstrate the great robustness of TCAEC concept, we briefly give a different definition. We very strongly suggest, that this other equivalence class, which we label TCAEC^{+}, is unnecessarily broad, none the less, it is quite acceptable for what we aim to achieve in this paper. To begin with, TCAEC^{+} could allow τ in the presence requirement to be the characteristic nuclear time, $\tau_N = 10^{-24}$ s, in the place of τ_y. The duration τ_N represents the time it takes to traverse the 'diameter' of a proton when travelling at the speed of light (Davies 1982, p. 9). Using τ_N is eight orders of magnitude more lax than using τ_y. We could allow many other simple high-level structures beside the single loop choice in which the 26 N_i representations appear coherently expressed and in increasing order. For example, if we did not insist on order or coherent sense of direction there would be about $10^{33} = 25!2^{25}$ as many associations with N. We could allow curves (open loops). We could also permit any 26 edge, undirected, graph that has the N_i associated with its edges in a way that is invariant to permutations. All these possibilities increase the total number of allowable graphs to about 10^{34}. Other select pseudorandom coding choices would also be acceptable. For these other choices we will extravagantly allow a factor of 10^{12} even though it may be difficult to conceive of more than a few other choices. With all these openings of the membership qualifications allowed together, the simple probabilistic bounds that we develop later in §6*a* change, but not in a way that would thwart us from developing the very same points we are aiming to develop with TCAEC. We will clarify this matter in §6*c*.

5. A definition of an advanced life-form

We define an advanced life-form as one that constructs TCA: their constructed attempt at TCA turns out to be in our TCAEC. Thereby, we make it inevitable that TCA be constructed by advanced life-forms. We argue that this is not an unfair circularity. The idea is, that to qualify as an advanced life-form, the life-form needs to have attained our level of understanding of mathematics and physics, and to have an extremely keen interest in the likelihood of advanced life occurring during the course of the universe actualization process. Moreover, it should have sensibilities to the extent of an appreciation for a sense of minimalism and what 'highly distinguished' means. They should also have at their disposal the manipulatory ability to translate the thought of TCA into material expression. They construct TCA because they have the capacity to do so and it is more interesting for them to affirm TCA's existence than its non-existence. Certainly such intellectual and manipulatory capabilities and curiosity about the likelihood of advanced life are legitimate qualities to use in the definition of advance life-forms. With TCA construction behind it, an advanced life-form can meaningfully approach the following question: What does physics have to say about Ψ actualizing to include a member of the class of advanced life-forms to which we belong, or equivalently, to include TCA?

We emphasize the extreme robustness of our definition. In the defining of TCAEC there was no requirement for perfect reciprocity among potential actualizations, although that is the consensus that is the targeted ideal. With our definition of TCAEC, we are figuratively casting a virtually perfect net for what *we* define as an advanced life-form. This definition may not be exactly the same as another potential actualization's definition. However, that does not mean that we miss that other actualization with our definition. For example, suppose that their definition is the same as ours, except that their presence requirement requires only τ_N instead of τ_y. They might still construct their artefact so that it obeys the τ_y requirement and therefore, according to our definition, we would consider them to be an advanced life-form. We aim in our definition to be extremely open, but life-forms whose TCA construction is more ephemeral than τ_y would not be acknowledged by us as candidates for advanced life-forms.

Can we say that humanity has already constructed TCA? Just about, but strictly speaking, some very minor work remains to be done in order to say yes. We could have said yes, if the requirements for membership in TCAEC allowed that the tables of N, that are printed in references such as Gorenstein (1985) and Thompson (1983), qualify as copies of TCA. However, it is difficult to cast an appropriate TCAEC definition, of such a wide scope as to include such printed tables. Such an approach to defining TCAEC involves a theme of maximal openness to diverse modes of expressing N that do not compromise the tight association with N. The openness in the definition would need to allow for modal peculiarities with strong evolutionary or culturally historic dependencies and even the definition of TCAEC^+ is not aimed at being adequate for that. In contrast, the mass packet approach had a minimalism theme. To eliminate hesitation about saying that TCA has been constructed, it is so much easier to construct TCA in accord with the mass packet form that we have already described.

6. On the probability of the event [the universe constructs TCA]

Contrast the event [the universe constructs TCA] with [the universe constructs the classic novel *Crime and punishment*]. What is the point of asking about the probability of the latter event (except to engage an avowed determinist)? The probability would seem to be so infinitesimal and so obviously far, far beyond the practical reach of TOE to estimate. Any attempt to estimate it would be expected to degenerate into a muddle holding no value for illuminating the prospect for advanced life-forms. The question, what is Pr[the universe constructs TCA]?, while also an extraordinarily difficult one, cannot be so dismissed. One cannot say that the question is unworthy of consideration because TCA expresses fine details and peculiarities of an overwhelmingly complex history. Indeed, TCA does not express such things. As we now underscore, the existence of TCA demands explanation by any theory aspiring to be a TOE, unless advanced life is to be regarded as less probable than a one part in 10^{264} fluke.

Consideration of Pr[the universe constructs TCA], under the initial condition constraint, is very intriguing. For example, it invites enthusiasts for the hypothesized successful TOE, faced with the following extremes of speculation, to anticipate where they think the truth resides.

(*a*) *View that advanced life-forms are very, very rare* (*TOE+IC indifferent*)

View. We go so far toward encouraging TCA as to posit a universe actualization, that is packed chock full of mass packets, so perfectly tailored to accommodate N, that only the bead mass values for the beads that are not separators, need to be chosen. Despite such manifest conduciveness, we can argue that an occurrence of TCA would be virtually impossible. In estimating Pr[the universe constructs TCA] we assume that TOE+IC is indifferent to N. The indifference assumption implies that TOE+IC does not favour N over the other 26 element sequences represented by the mass packets with different bead choices. For our realization of the universe, here on Earth of the order of 10^{27} nucleons would be convenient (e.g. if TCA were constructed using coins as tokens as mentioned at the close of §4*b*). We see that meeting a spacetime occupance of the order of $10^{45} n\tau_y$, instead of just the $n\tau_y$ implied by the distinct presence requirement, would not present any difficulty for us. However, we cannot be humankind chauvinistic, so we definitely must not use $10^{45} n\tau_y$. Instead, we overestimate Pr [the universe constructs TCA] by using only the paltry $n\tau_y$ nucleon-atomic years per mass packet needed to meet the distinct presence requirement. Note that from the inequality (1) in §3, the universe cannot have more than $10^{111} \approx 10^{114}/10^3$ mass packets. Since $n = 1245$ there are over $2^{1245} \approx 10^{375}$ mass packet sequence choices along with the one choice that gives N. Therefore, Pr [the universe constructs TCA] $< 10^{111}/10^{375} = 10^{-264}$.

In this paragraph we highlight some aspects of the bounding steps that we just employed. First, we stress that the estimation in the *View* was limited to only estimating an *upper bound* on Pr [the universe constructs TCA] subject to TOE + IC indifference. The number 1245 was large enough to afford great extravagance in coming up with an upper bound. We have proceeded allowing that we erred by many orders of magnitude, but we still make the case of an incredible fluke. While it is much too difficult to be accurate, we emphasize that it is rigorously correct to err in our bounding so long as we only err in the direction of coming up with a greater upper bound. In spite of this kind of erring we get an upper bound of 10^{-264}, so there is little motivation to be more accurate. There is just no need to enter into a very difficult line of argument striving for say, 10^{-10000}, when an upper bound of 10^{-264} already makes the case for an incredible fluke. See §6*c* below for more on this bound.

Barring a role of non-local connections in the emergence of advanced life-forms, one could make the same argument as in the *View*, for a galaxy, or star cluster, rather than a universe actualization, and thereby conclude that it is a very, very, safe bet that we are alone in our actualization of the universe.

(*b*) *Counterview: TOE + IC very strongly favours the construction of TCA*

Counterview. Perhaps a Pr [the universe constructs TCA] value near one is correct: In the above *View*, the ascribing of indifference must somehow be a colossal error (by over 264 orders of magnitude). Remarkably, as hard as it might be to show, there may be a great proclivity for the emergence of entities, call them advanced life-forms if you like, with the propensity to delve deep into the so-called Platonic realm and express TCA. In this sense, the equations on a tee-shirt, constrained by IC, are, beside everything else, an algorithm for the universe, with high probability, composing within itself TCA (a carrier of a complicated answer to a very deep question in mathematics), even multiple copies of TCA, from few materials.

The counterview is that the inevitability of TCA emerging resides in the universe's configuration at τ_a and how TOE prescribes it to evolve. To support it one would need evidence that is not extremely puny in the face of the 264 orders of magnitude. Is the counterview totally hopeless to shore up and defend, at least, for a start, even with a semblance of a quantitative explanation?

If correct, the counterview has a broader interpretation as we now explain. TCA, based on our choice of N, could be generalized to a family of canonical artefacts expressing large, highly distinguished, finite sequences from various fields of 'pure' mathematics. Just as with the aforementioned group orders, the expressions on the tee-shirt must be intimately connected to these sequences. One can say *a priori*: for TOE to be correct it must be intimately related to many areas of advanced 'pure' mathematics.

(*c*) *Remaining possibilities*

In the middle-ground between the view and the counterview one is left facing the implications of both: the speculation that we are very rare and TOE+IC has the feature of strongly favouring the occurrence of the event [the universe constructs TCA]. To conveniently express the continuum of speculations between the *View* and *Counterview*, it is useful to define a notion of the favouring of a certain element in the outcome of a quantum mechanics experiment. The 'experiment' will be the quantum cosmological one of Ψ actualization, and the element of interest to us will be the distinct presence of TCA.

Assume that an experimental outcome results in a random subset, K, of a finite set of possibilities L. Assume further that this set of all possibilities, L, has L elements. This outcome subset of distinct possibilities, K, could be empty. By $|\mathsf{K}|$ we mean the random number of elements in the outcome K. Let k be the least integer for which we can say $\Pr[|\mathsf{K}| \leqslant k] = 1$. Fix λ belonging to L. We now have the notation to form a definition that quantifies the tendency for λ to be favoured in the experimental outcome.

Our definition is a very crude one, but it will be adequate for our purposes. As a baseline for composing the definition, we take the hypothetical experiment in which there are k statistically independent, equally likely, choices from L, with replacement. For the baseline case, the probability that λ appears in the random set of at most k elements is easily seen to be $1-(1-(1/L))^k$, which, when L is large compared to k, is $\approx (k/L)$. Using this baseline, we quantify the favouring of λ, denoted $F(\lambda)$, in the outcome of the experiment of interest as

$$F(\lambda) = \Pr[\lambda \in \mathsf{K}]/[1-(1-(1/L))^k]. \tag{2}$$

For our application take K to be the set of mass packets generated in the 'experiment' of Ψ actualization, and take L to be the set of all possible mass packets. Let λ be TCA. First we write an inequality with N and n left open and then we specialize to the case we have been developing where N is the 26 orders and $n = 1245$. Consequently, $L = 2^n$, and from the comment following equation (1), along with the distinct presence requirement, $10^{114}/n$ is an upper bound on k. Since $[\lambda \in \mathsf{L}] =$ [the universe constructs TCA], using equation (2), so long as n is large, it follows that

$$\Pr[\text{the universe constructs TCA}] < F(\text{TCA}) \times 10^{114}/(n2^n). \tag{3}$$

For the special case of the 26 orders, where $n = 1245$, this becomes

$$\Pr[\text{the universe constructs TCA}] < F(\text{TCA}) \times 10^{-264}. \tag{4}$$

In the *View*, F(TCA) is taken to be one, conveying that TCA is not favoured at all beyond the filling of all the spacetime in the universe with the greatest number of mass packets. In the *Counterview* F(TCA) is taken to be 10^{264}, a totally unexplained, far greater favouring. In the context of TOE+IC, what F(TCA) actually is, is an open question.

A remaining possibility, distinct from those already mentioned here, is that the very notion of a TOE is untenable if one goes so far as to include the requirement of an explanation of certain fruits of thought, such as TCA. We see that consideration of Pr[the universe constructs TCA] has us facing a provocative range of viewpoints.

(d) *Demonstration of robustness*

If we had opened up TCAEC to TCAEC$^+$ as mentioned at the close of §4*c*, there would be much more freedom of choice than with TCAEC. We see such an opening as wrongly unrestrictive. However, to demonstrate the great robustness of the artefact idea in drawing the *View–Counterview* contrast, it is worthwhile to explore what happens if one makes the opening. As we now show, all that would happen is that 10^{264} would be replaced by a much smaller, but still enormous, number. We first lower the number by 10^8 and then another factor of 10^{34} for the first two reasons mentioned in §4*c*. This gives 10^{222} in place of 10^{264}. We do an ultra-conservative lowering of 10^{222} to 10^{210} to allow 12 orders of magnitude for the other effects mentioned in §4*c*. With all this erring on the side of reducing 10^{264}, we are still left with 10^{210}, and the key points made in this section are by no means undetermined if 10^{264} is replaced by 10^{210}.

Next we explore what would happen if we were to allow binary representation. From the references (Gorenstein 1985; Thompson 1983) we see that the number $2^{363} \approx 10^{109}$ is the largest power of 2 dividing $N_1 \times N_2 \times \ldots \times N_{26}$, so the choice $m = 2$ would imbue the mass packet with long dull segments of identical beads. If we omit the dull segments in our bounding argument we need to lower 210 to 101. This amounts to a conservative bounding of the chance of occurrence on the grounds that natural formations, not requiring the agency of an advanced life-form, would so strongly favour the dull segments. Even this allowance of base two, which, at first, might seem to go a long way toward undermining the ultra-rarity argument, still leaves us with an enormous number so that ultra-rarity is left intact.

An enormous allowance, for what are highly distinguished sequences, constitutes an oxymoron. Yet we do have room for an enormous allowance of other choices for N. However, this last allowance requires careful accounting: any allowable N for which the corresponding number of pseudo-randomly encoded bits is less than the number 1245 that we have been using in our example results in replacing the number $10^{375} \approx 2^{1245}$ used in §6*a* by a correspondingly smaller number.

7. Summary of main result on emergence of advanced life-forms

The concept of TCA, a transcendent flag of advanced life, was introduced. Consideration of an initial attempt at defining TCA, supported by some simple estimates, point to the truth of at least one of the following.

(i) Starting from IC, the chance emergence of a life-form with an intelligence as advanced as humanity's is less than one part in at least 10^{264}. See §6*a*.

(ii) There is an at least 264 order of magnitude effect, that is unexplained by contemporary physics, that expresses the tendency of advanced life to emerge from the IC during the universe actualization process. (This type of rationalization suggests that physical laws must be related to areas of 'pure' mathematics in which highly distinguished large numbers have prominence. See §6*b* and the item about perfect numbers in the next section.)

(iii) Some combination of the two items above: if the number 10^{264} in (i) is reduced, the strength of the effect referred to in (ii) must therefore increase accordingly.

(iv) Even with our allowance of an exaggerated explanatory power of contemporary physics (see §2), the material expression of thought, TCA, cannot be explained, even in principle.

Apart from changing to a lower very large power of 10 to use in place of 264, this conclusion is not at all sensitive to the specific judgements made in defining TCAEC. We believe further work will reinforce that the initial judgements made here are not overly restrictive. However, even allowing a totally unwarranted opening of the defining architecture of TCAEC to TCAEC$^+$, and granting an ultra-conservative estimation, all that happens is that 10^{264} gets replaced by 10^{101}. We are still left with a strong conclusion.

8. Variations on the approach to defining TCA

Our best judgement for N, the 26 order choice, turned out to offer us an expositional advantage over more open-ended possibilities. In this section we discuss some considerations regarding other possibilities for N, and, in our last item, for the structure of TCA itself.

Look at the factor $10^{114}/(n2^n)$ on the right-hand side of inequality (3) in §6*c*. So long as this factor is exceedingly small and the artefact can be constructed, the TOE + IC indifference assumption enables us to obtain the ultra-rarity conclusion. Taking $\log_2$ of this factor we see that n is required to be considerably in excess of $\log_2 10^{114} \approx 379$. Note that while the n value of 1245 suited us very well, an N with an n of say 500 or 100000 would have been acceptable. These very different numbers would not alter the substance of the argument for the *View*. However, as n is lowered toward 379 our easy ultra-rarity argument is lost and as n gets too large the reasonableness of TCAS construction becomes questionable.

At the end of paragraph §6 we alluded to a generalization which opened up the definition of TCAEC in the direction of allowing for any of a number of highly distinguished sequences instead of only the 26 orders. Another variation on the theme is to discriminate advanced intelligences in terms of various specific highly distinguished N. We could say that a Ψ actualization contains an N-intelligence if and only if a canonical artefact expressing N is distinctly present.

The so-called perfect numbers offer an interesting range of choices for N. The perfect numbers are those positive integers equalling the sum of their divisors, where the number itself is not included in the sum. The numbers 6 and 28 are the first two perfect numbers and there are some extraordinarily large numbers that are perfect. Schroeder (1990) has a discussion about the known perfect numbers as of about 1985. There were only 30 known. At this writing 32 are known (and they are all variants of Mersenne primes). It is not known whether there are infinitely many perfect numbers. No odd perfect numbers are known and it is not known whether any exist. It is possible to achieve a great strengthening of the quantitative aspect of the conclusion in §7 by selecting from a *family* of artefacts based on the known perfect numbers (not that strengthening is needed). For example, an artefact based on the first 27 even perfect numbers results in replacing 10^{264} by a number of the order of 10^{97000}. Such an artefact turns out to require much more effort to construct than the artefact that we have described, but material expression is still very practical.

In devising TCA, each potential actualization with an advanced life-form contemplates the other such potential actualizations and the implication of there being others. As mentioned earlier, TCA is akin to a SETI but it is not a SETI signal. Unlike SETI, TCA's cost is trivial, and building it, once its design is settled, would be a very straightforward, but tedious project for a few graduate students. Since contemplation, not communication, is involved, a matter (frozen energy) form was advised here for TCA instead of a waveform. None the less, at this early point, the possibility that there is a way to generalize TCAEC to accommodate an outward propagating wave, cannot be dismissed. A spherical spatial structure for such a wave appeals to minimalist sensibilities. We cannot fail to point out the irony that would be involved, that a wave that, according to §6*a* professes our ultra-rarity, also constitutes a cry into the cosmic wilderness to say it isn't so.

9. TCA and a computer science question

In closing we mention an insight into a topical computer science question that our perspective offers. This graphic, some, like myself, would say vulgar, question is: Is the human brain just a wet computer? (Penrose (1989) has much of interest to say on this sort of question. Paraphrasing, one of his messages is: Don't let anyone bamboozle you into accepting that scientific evidence points to an answer of yes. His broad perspective of the possibilities of the physics that might be involved in thought processes is especially informative. Kurzwiel's book (1990) offers a wide range of views on such questions, from himself and numerous other contributing authors.)

Consider instead of the aforementioned question, the more fundamental question of this genre: Does contemporary physics go so far as to, in principle, explain all that a life-form as advanced as ours does? If contemporary physics is unable to explain how TCA evolves out of IC, the answer is no. Of course, after TCA has been defined, it is straightforward to devise a machine to produce it, but that aspect of the reach of contemporary physics is not at issue here.

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