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330:501 Systems Analysis

Problem Set 9 SOLUTIONS

Once again, these were prepared quickly. In particular, I did not carefully check the book problem solutions to 6.1 and 6.2 and 6.21 since they are so similar to 5.1 and 5.2.

S & B: Ch.[6], prob. 1:
From the P3-1, we have the transition matrix

$$\phi(t,\tau)=\begin{bmatrix} t/\tau & 0 \\ 0 & t/\tau \end{bmatrix}$$

Thus, the observability Gramian

$$M(t_0,t_1)=(\frac{t_1^3}{3t_0^2}-\frac{1}{3t_0})\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Since the determinant equals zero, the system is not observable.

S & B: Ch.[6], prob. 2:
From P3-2, we have found the transition matrix $\phi(t,\tau)$. We can form the observability Gramian:

$$M(t_0,t_1)=(\frac{e^{6(t_1-t_0)}}{6}-\frac{1}{6})\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

The determinant of this matrix is zero. Therefore, the system is not observable.

S & B: Ch.[6], prob. 14:
If we define the $\tilde{y}=y-Du$, then the problem becomes, if the equation:

$$\begin{bmatrix} \tilde{y} \\ \dot{\tilde{y}} \\ \\ \overset{(n-1)}{\tilde{y}} \end{bmatrix}=\begin{bmatrix} C \\ CA \\ \\ CA^{n-1} \end{bmatrix}x$$

has unique solution. It is obvious that we can still form the observability matrix, which has the same form as of the (A,B,C) system. Therefore, the matrix D has no influence on the observability of the system.

S & B: Ch.[6], prob. 21:
Form the observability matrix for this linear time-invariant system:

$$L_1=\begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}=\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ \frac{i}{J} & -\frac{f}{J} & 0 \end{bmatrix}$$

The determinant of this matrix is $|L_1|=\frac{1}{J}\neq 0$, therefore, the system is observable.
For $C=[0,1,0]$, the observability matrix is:

$$L_2=\begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{J} & -\frac{f}{J} & 0 \\ a & b & 0 \end{bmatrix}$$

where a,b are some constants. Obviously, the rank of this matrix is less then 3. So the system is not observable.

S & B: Problem[6]:
Let's make this simple since all we need is a counterexample which shows that no matter what A is, we need not have complete controllability and observability for all possible B and C.

Suppose that the system is completely controllable. Well, if we can choose any C we like, then we can choose C as an EIGENVECTOR of A^T. In that case $CA = \lambda C$ and we're dead in the water right there (since λC is a scaled version of C. In fact, the whole L matrix is a set of scaled eigenvectors, $[C^T, \lambda C^T, \lambda^2 C^T ...]$.

So, can't put any condition on A which automatically makes the system both controllable and observable.