THE STATE UNIVERSITY OF NEW JERSEY
RUTGERS

College of Engineering
Department of Electrical and Computer Engineering

332:322 **Principles of Communications Systems** **Spring 2006**

Quizlette I

FOR FUN, NOT FOR CREDIT

BUT: if you don't get a high score on this, you should worry

1. *(30 points)* **Linear Systems:**

   (a) *(10 points)* Once again write down the forward and reverse Fourier Transform which relates $x(t)$ and its Fourier Transform $X(f)$.

   SOLUTION:

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}df$$

   (b) *(20 points)* Show that if $x(t)$ has Fourier Transform $X(f)$, then the Fourier Transform of $x(t-t_0)$ is $e^{-j2\pi ft_0}X(f)$.

   SOLUTION:

$$\mathcal{F}\left\{x(t-t_0)\right\} = \int_{-\infty}^{\infty} x(t-t_0)e^{-j2\pi ft}dt = \int_{-\infty}^{\infty} x(s)e^{-j2\pi f(s+t_0)}ds = e^{-j2\pi ft_0}X(f)$$

2. *(30 points)* **Amplitude Modulation:**

   (a) *(10 points)* What is the Fourier Transform of $m(t)\cos 2\pi f_c t$ given the Fourier Transform of $m(t)$ is $M(f)$?

   SOLUTION:

$$\frac{1}{2}\left(M(f+f_c)+M(f-f_c)\right)$$

   (b) *(10 points)* What is the Fourier Transform of $m(t)\cos 2\pi f_c t + jm(t)\sin 2\pi f_c t$ given the Fourier Transform of $m(t)$ is $M(f)$?

   SOLUTION:

$$\frac{1}{2}\left(M(f+f_c)+M(f-f_c)\right)+j\frac{1}{2j}\left(-M(f+f_c)+M(f-f_c)\right) = M(f-f_c)$$

   (c) *(10 points)* The previous part is an (unrealizable) form of what sort of modultion?

   SOLUTION: *Single Sideband AM*

3. *(30 points)* **Quantization:**

(a) *(10 points)* What is the purpose of a quantizer? State your answer in words (no more than a short paragraph). NOTE: this is not an *optimality* question, just a simple question about what a quantizer is used for.

SOLUTION: *The purpose of a quantizer is to approximate samples (usually of a waveform) using a* finite *set of amplitude levels. Such quantization is a precursor for digital transmission of a signal since samples of a continuous real-valued waveform cannot otherwise be represented with a finite number of bits.*

(b) *(10 points)* The Loyd-Max conditions for optimal quantization are $q_k = E\left[X|X \in A_k\right]$ where A_k is the event that random variable $X \in (x_{k-1}, x_k)$ and $x_k = \frac{1}{2}(q_k + q_{k+1})$.

Suppose $f_X(x) = [u(x+1) - u(x-1)]/2$. Is a 1 bit quantizer with $q0 = -0.5$, $q1 = 0.5$ and $x0 = 0$ optimal? Why/Why not?

SOLUTION: *Yes.* $(q_1 + q_0)/2 = 0 = x_0$. $q_0 = E[X|X \in (-1,0)]$ *and* $q_1 = E[X|X \in (0,1)]$.

(c) *(10 points)* Sketch the output to this quantizer on the interval $t \in (0,6)$ when the input is the sawtooth waveform

$$m(t) = u_{-2}(t) + 2\sum_{k=0}^{\infty}(-1)^k u_{-2}(t - 2k + 1)$$

where $u_{-2}(t)$ is the unit ramp (the integral of the unit step). Then provide an analytic expression for $Q(m(t))$ in terms of the unit step function $u(t)$ (also known as $u_{-1}(t)$ in some circles).

SOLUTION:

$$Q(m(t)) = \frac{1}{2}\left(u(t) + 2\sum_{k=1}^{\infty}(-1)^k u(t - 2k + 1)\right)$$