THE STATE UNIVERSITY OF NEW JERSEY

RUTGERS

College of Engineering
Department of Electrical and Computer Engineering

332:322 **Principles of Communications Systems** **Spring 2006**

Quiz II

There are THREE questions. You have the class period to answer them. Show all work. Answers given without work will receive no credit. GOOD LUCK!

1. *(50 points)* **The Pulsing Passion of PAM:** Ok, now that I've got your attention, consider a pulse amplitude modulated system which uses a pulse shape $p(t) = \delta(t)$ so that the transmitted signal is

$$r(t) = \sum_k m(kT)\delta(t - kT)$$

where $m(t)$ is the program material.

   (a) *(15 points)* Assume $m(t)$ has non-zero spectral energy on the whole interval $\pm W$ Hz and zero spectral energy elsewhere. Furthermore, assume the spectrum of $m(t)$, $M(f)$ is continuous. Assume some spectral shape for $M(f)$, the Fourier transform of $m(t)$, and carefully sketch $R(f)$, Fourier Transform of $r(t)$.

   (b) *(15 points)* What is the maximum allowable value of sampling interval T which allows $m(t)$ to be recovered uncorrupted from $r(t)$?

   (c) *(20 points)* Now, suppose that

$$M(f) = [u(f + W) - u(f + W/2)] + [u(f - W/2) - u(f - W)]$$

   What is the maximum value of sampling interval T which will allow $m(t)$ to be recovered from $r(t)$? If we can recover $m(t)$ from $r(t)$, show exactly how it might be done.

2. *(50 points)* **Quantization:**

We have simply assumed that for an optimum quantizer, the $\{q_k\}$ are *ordered* – that is $q_k > q_{k-1}$. In this problem we will prove that the $\{q_k\}$ MUST be ordered from smallest to largest.

Please note that the bin boundaries $\{x_k\}$ are by definition ordered since we can always number successive boundaries that way. Thus, our derivation of the Loyd-Max rules is still valid since no ordering was assumed on the q_k in the derivation. The Loyd-Max rules are

$$q_k = E\left[X | X \in (x_{k-1}, x_k)\right]$$

and

$$x_k = \frac{q_{k+1} + q_k}{2}$$

Please prove that $q_{k+1} > q_k$ for all quantizer levels in an optimum quantizer.

3. *(50 points)* **Cora The Terminator:** Cora the communications engineer has been hired by SquirrelNOT, Inc. to predict the future positions of a particularly pesky squirrel named Martin P. Sciuridae. Given a sequence of his last known locations $\{s_n\}$, Cora is charged to develop a linear predictor for his position at the next time step. The company plans to use it's patented squirrel zapper at the predicted location. If Martin is there, he's history, but if not, such an attempt will just make him angry.

   And there is nothing worse than an angry squirrel.

   The form of Cora's linear predictor is

$$\hat{s}_n = \sum_{k=1}^{M} s_{n-k} w_k$$

   where the w_k are constants. You're going to help Cora find the appropriate $\{w_k\}$ to minimize a mean square error criterion

$$\text{MSE} = E\left[\left(s_n - \sum_{k=1}^{M} s_{n-k} w_k\right)^2\right]$$

   WARNING: This is NOT the same problem as in Quiz II 2004.

   (a) *(20 points)* Marty is a creature of habit and his position follows

$$s_{n+1} = \sum_{k=0}^{N-1} a_k s_{n-k}$$

   where the $\{a_k\}$ are constants. Find the weights $\{w_k\}$ which minimize the MSE. How many weights do you need? You must justify your result.

   (b) *(10 points)* Now suppose that Marty follows

$$s_{n+1} = -2s_n - s_{n-1} \tag{1}$$

   What are the $\{w_k\}$ which minimize the MSE? As before you must justify your result.

   (c) *(20 points)* Now suppose Cora does not know the a_k initially and only can observe Marty's position s_n. Cora decides to use an adaptive procedure – calculating the error gradient and adjusting the weights in the opposite direction:

$$w_j(n+1) = w_j(n) + \mu e_n s_j$$

   where

$$e_n = s_n - \sum_{k=1}^{M} s_{n-k} w_k(n)$$

   and $\mu > 0$.

   You may assume that Cora knows $M = 2$. Assume that $s_n = (-1)^n$ (verify that this satisfies the difference equation (1)). Please expand and then simplify the weight adaptation equation above to show that in this degenerate case, Cora will probably lose her job. Why do we say this case is degenerate? What if we assume $M = 1$?