THE STATE UNIVERSITY OF NEW JERSEY

RUTGERS

College of Engineering
Department of Electrical and Computer Engineering

332:322 **Principles of Communications Systems** **Spring 2005**

Quiz II

There are THREE questions. You have the class period to answer them. Show all work. Answers given without work will receive no credit. GOOD LUCK!

1. *(50 points)* **Pulse Amplitude Modulation:** Consider the signal

$$r(t) = m(t) \sum_{k=-\infty}^{\infty} p(t-kT)$$

where $m(t)$ is program material bandlimited to $\pm W$ Hz and $p(t)$ is an arbitrary waveform such that the sum $\sum_{k=-\infty}^{\infty} p(t-kT)$ exists.

   (a) *(20 points)* Suppose $p(t) = \delta(t)$. What condition on T insures that $m(t)$ can always be recovered from $r(t)$?

   (b) *(20 points)* Now suppose $W = 10$, $T = 10^{-3}$, and $p(t) = \frac{sin\pi t}{t}$. Since $p(t)$ exists for all time, you'll notice that the pulses $p(t)$ which comprise $\sum_{k=-\infty}^{\infty} p(t-kT)$ OVERLAP. If we apply an ideal band pass filter $H(f) = u(f+1010) - u(f+990) + u(f-990) - u(f-1010)$ (where $u()$ is the unit step function) to $r(t)$, show how can $m(t)$ be recovered from $r(t)$ (or not).

   (c) *(20 points)* Now suppose $W = 10$, $T = 10^{-3}$, and $p(t) = \frac{sin10^5\pi t}{t}$. Since $p(t)$ exists for all time, you'll notice that the pulses $p(t)$ which comprise $\sum_{k=-\infty}^{\infty} p(t-kT)$ OVERLAP. If we apply an ideal band pass filter $H(f) = u(f+1010) - u(f+990) + u(f-990) - u(f-1010)$ (where $u()$ is the unit step function) to $r(t)$, show how can $m(t)$ be recovered from $r(t)$ (or not).

2. *(50 points)* **Quantization:**

   (a) *(20 points)* What is the purpose of a quantizer? State your answer in words (no more than a short paragraph). NOTE: this is not an *optimality* question, just a simple question about what a quantizer is used for.

   (b) *(30 points)* We have seen in class that an optimal quantizer function $Q(x)$ seems to always be non-decreasing. Please PROVE that this observation is true (or not). You may assume that the PDF of X, the random variable to be quantized, exists and is non-zero for all $X \in \Re$. HINT: start from the Loyd-Max conditions and remember how these conditions were derived.

3. *(50 points)* **Cora and the Quantizer/Coder From Hell:**

Cora the communications engineer has been hired by Mephisto Incorporated to design a communications hot line (tee hee) for the Prince of Darkness himself. The Prince wishes to use the hot line to remotely measure the temperature, $x(t)$ in various parts of his domain. As one might imagine, the sample sequence for the temperature is constantly increasing. In fact in one particular area the sampled temperature follows

$$x_n = n/2$$

Assume Cora needs to encode and transmit this sequence.

Cora has a choice of two systems. The block diagram for the first scheme is given in FIGURE 1. Basically, a direct difference is computed for the input signal x_n and input to a 1-bit quantizer. A coder then outputs a binary 1 or 0 depending upon whether the quantizer output is $+1$ or -1 respectively. At the receiver, the 1's and 0's are converted into ±1s and cumulatively summed to obtain $\hat{x}_n$.

The block diagram for the second system is shown in FIGURE 2.

In this problem we will evaluate the effectiveness of both systems. For all parts assume that $\hat{y}_0 = 1$ and $q_n = 0$ for $n < 0$.

(a) *(10 points)* For system A in FIGURE 1, sketch the discrete sequence $\hat{y}_n$ for $n = 0, 1 \ldots 10$. What is the corresponding binary code sequence?

(b) *(10 points)* For system B in FIGURE 2, write down expressions for y_n and q_n by analyzing the block diagram and then sketch the discrete sequence $\hat{y}_n$ for $n = 0, 1 \ldots 10$. What is the corresponding binary code sequence?

HINT: It might help to put everything in a table.

(c) *(10 points)* For system A, carefully sketch the resulting $\hat{x}_n$. You may assume that $\hat{x}_0 = 0$.

(d) *(10 points)* Repeat the previous part for system B. Comment on any differences you find between the outputs generated by the two methods. Which, if either, does a better job? Why?

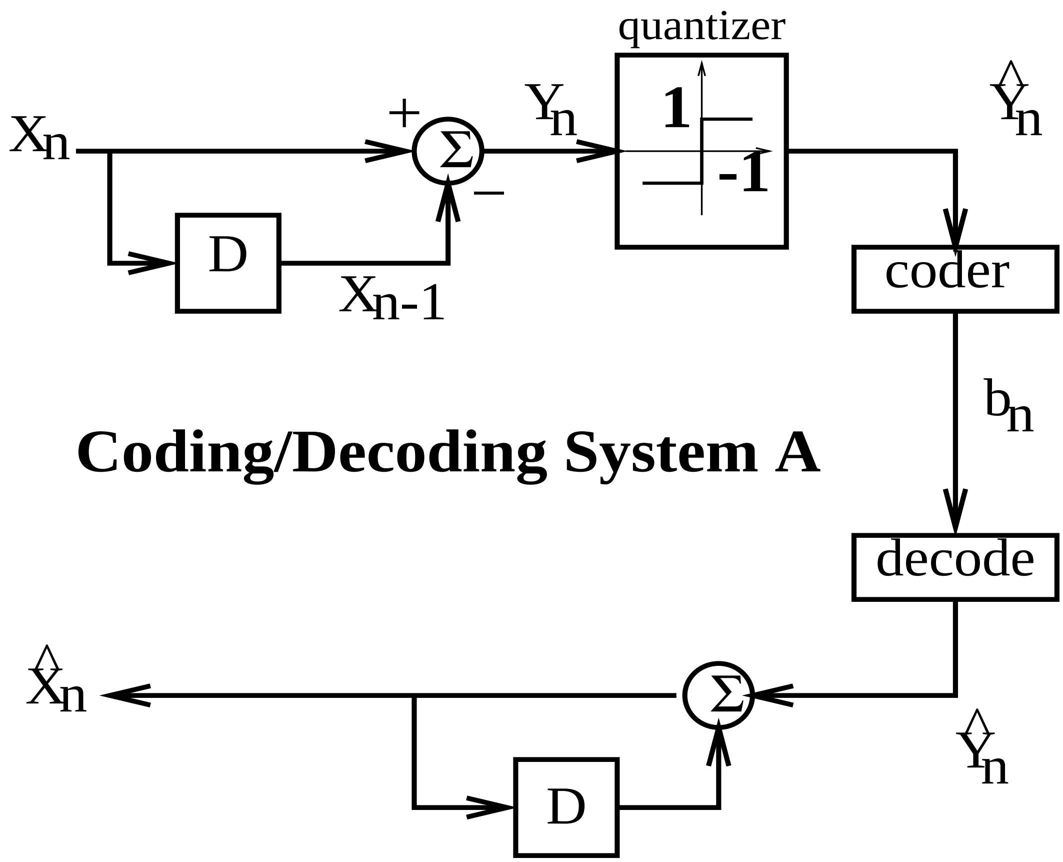

Figure 1: System A for problem 3

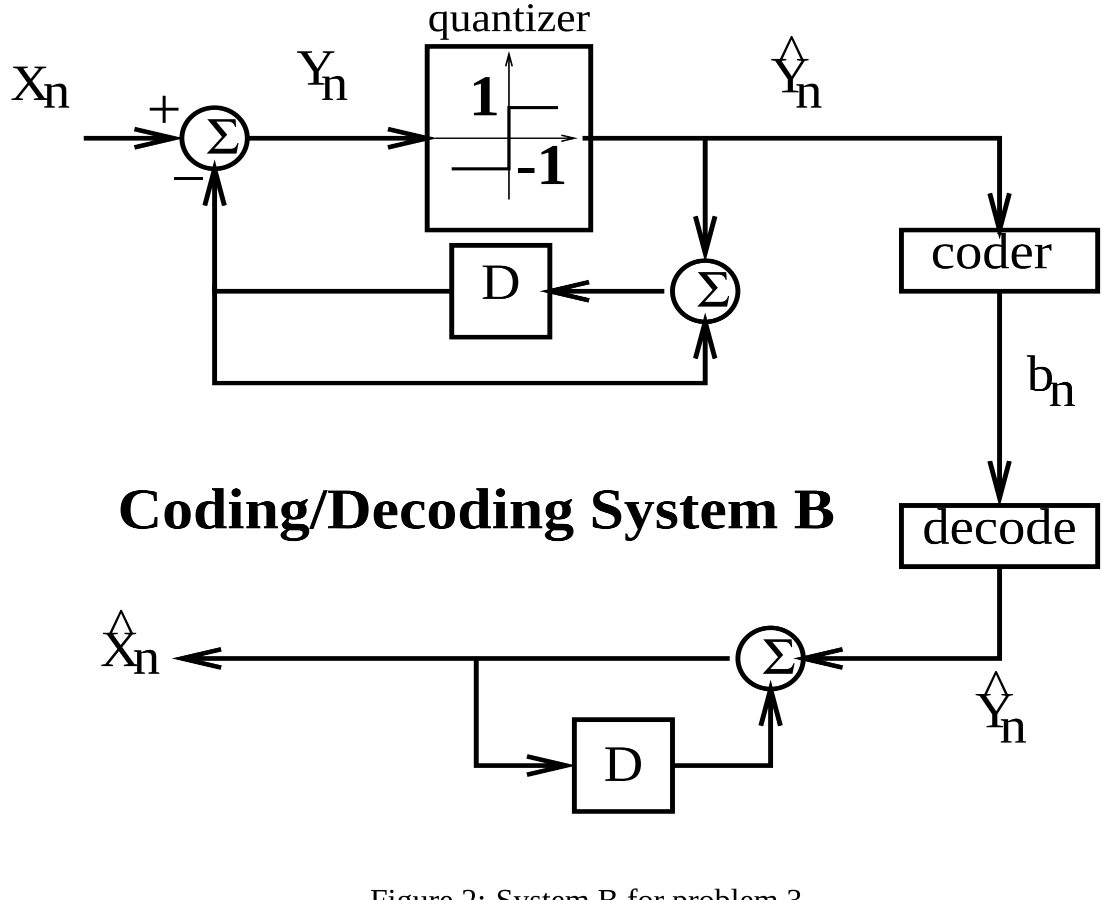

Figure 2: System B for problem 3