RUTGERS

School of Engineering
Department of Electrical and Computer Engineering

332:322 **Principles of Communications Systems** **Spring 2009**

Real Serious-As-A-Heart-Attack Quiz I

There are **THREE** problems. Each problem subpart is stated on a different sheet. Show all work on the stapled sheets provided (front and back). DO NOT DETACH THE SHEETS. You are allowed one side of an $8.5 \times 11\text{in}^2$ paper handwritten note sheet.

1. **Lightning Round:** This is a set of linear systems questions to keep your analytic muscles strong:

   (a) *(20 points)* The energy in a signal $s(t)$ is

$$E_s = \int_{-\infty}^{\infty} |s(t)|^2 dt$$

Assume the above integral exists (is finite) and show that

$$E_s = \int_{-\infty}^{\infty} |S(f)|^2 df$$

where $S(f)$ is the Fourier Transform of $s(t)$ HINT: Remember $|x|^2 = xx^*$.

SOLUTION:

$s(t)$ is the inverse Fourier Transform of $S(f)$

$$\begin{aligned} s(t) &= \int_{-\infty}^{\infty} S(f) e^{j2\pi ft} df \\ s^*(t) &= \int_{-\infty}^{\infty} S^*(f') e^{-j2\pi f' t} df' \end{aligned}$$

Since $|s(t)|^2 = s(t)s(t)^*$

$$\begin{aligned} E_s &= \int_{-\infty}^{\infty} s(t)s^*(t)dt \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(f)e^{j2\pi ft} S^*(f')e^{-j2\pi f' t} df df' dt \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(f)S^*(f') \int_{-\infty}^{\infty} e^{j2\pi(f-f')t} dt df df' \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(f)S^*(f')\delta(f-f') df df' \\ &= \int_{-\infty}^{\infty} S(f) \int_{-\infty}^{\infty} S^*(f')\delta(f-f') df' df \\ &= \int_{-\infty}^{\infty} S(f)S^*(f) df \\ &= \int_{-\infty}^{\infty} |S(f)|^2 df \end{aligned}$$

(b) *(20 points)* Suppose a periodic signal

$$s(t) = \sum_k s_k e^{j\frac{2\pi}{T}kt}$$

where T is the period and

$$s_k = \int_0^T s(t)e^{-j\frac{2\pi}{T}kt}dt$$

Show that

$$E_s = \int_0^T |s(t)|^2 dt = \sum_k |s_k|^2$$

SOLUTION:

$$\begin{aligned}
E_s &= \int_0^T \left(\sum_k s_k e^{j\frac{2\pi}{T}kt}\right)\left(\sum_k s_k^* e^{-j\frac{2\pi}{T}kt}\right) dt \\
&= \int_0^T \sum_k s_k s_k^* dt \\
&= T\sum_k |s_k|^2
\end{aligned}$$

Since we can prove that if k is different from $k^{'}$

$$\begin{aligned}
& \int_0^T s_k e^{j\frac{2\pi}{T}kt} s_{k'}^* e^{-j\frac{2\pi}{T}k^{'}t}dt \\
=& \int_0^T s_k s_{k'}^* e^{j\frac{2\pi}{T}(k-k^{'})t}dt \\
=& \frac{1}{j\frac{2\pi}{T}(k-k')t} s_k s_{k'}^* \left[e^{j\frac{2\pi}{T}(k-k^{'})t}\right]_0^T \\
=& \frac{1}{j\frac{2\pi}{T}(k-k')t} s_k s_{k'}^* (1-1) \\
=& 0
\end{aligned}$$

(c) *(20 points)* Let

$$q(t) = \int_{-\infty}^{\infty} s(t-\tau)h(\tau)d\tau$$

If $\mathcal{F}\{s(t)\} = S(f)$ and $\mathcal{F}\{h(t)\} = H(f)$, please provide an expression for the energy in $q(t)$ in terms of $S(f)$ and $H(f)$. You may use results stated as true from the previous parts.

SOLUTION:

Convolution in time domain results in multiplication in frequency domain

$$q(t) = h(t) * s(t)$$

$$Q(f) = H(f)S(f)$$

Use the result from part (a)

$$\begin{aligned} E_{q(t)} &= \int_{-\infty}^{\infty} |q(t)|^2 \, dt \\ &= \int_{-\infty}^{\infty} |Q(f)|^2 \, df \\ &= \int_{-\infty}^{\infty} |H(f)S(f)|^2 \, df \end{aligned}$$

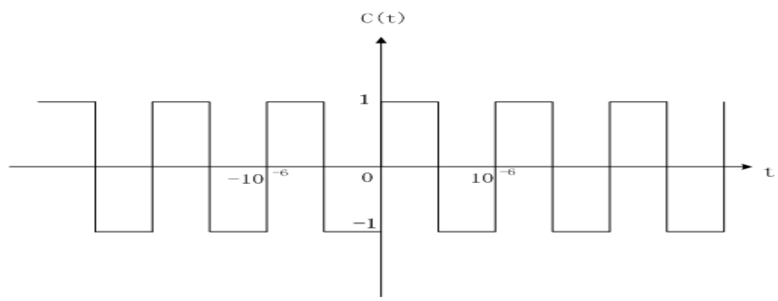
2. **AM Hodgepodge:**

   Let

$$c(t) = \sum_{k=-\infty}^{\infty} \left(u(t - \frac{k}{10^6}) - 2u(t - \frac{k + \frac{1}{2}}{10^6}) + u(t - \frac{k+1}{10^6}) \right)$$

   (a) *(20 points)* Carefully sketch $c(t)$.

   SOLUTION:

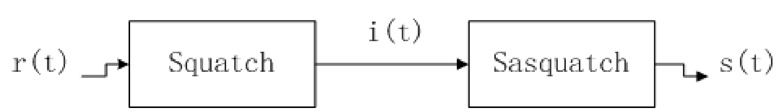


   (b) *(20 points)* For a signal $s(t)$ bandlimitted to $\pm$4kHz show how $s(t)$ can be recovered using a copy of $c(t)$ at the receiver (you may assume such a copy is available) if $r(t) = s(t)c(t)$. Is the low pass filter typical of most AM receivers strictly necessary here? Why? Why/not?

   SOLUTION: *At the receiver end, we just have to multiply $r(t)$ by $c(t)$ again. The output will be $r(t)c(t) = s(t)c^2(t) = s(t)$ because $c^2(t)$ becomes 1. No LPF is necessary here.*

   (c) *(20 points)* Assume $s(t) > 0 \ \forall t$ and that you want your message received on Planet Hodgepodge where diodes, multipliers and other such familiar EE devices do not exist. The only functions available to Hodgepodgians are Squatch$(x) = x^2$ and its inverse Sasquatch$(x) = +\sqrt{x}$. If possible, please design a receiver with which $s(t)$ can be derived from $r(t)$ on Planet Hodgepodge. You must show and justify your work.

   SOLUTION:



   $i(t) = r^2(t) = s^2(t)c^2(t) = s^2(t)$

   Since $s(t) > 0 \ \forall t$, we can get s(t) from the output of Sasquatch.

3. **Cora and the Chirping Squirrel:** Cora the communications engineer has a problem. Marty the squirrel, her arch-nemesis, has taken up residence in her attic. Marty has decided to drive Cora crazy with high frequency squirrel-chirps at night. These electromagnetic signals act

directly on Cora's brain, sending her the repeated message "you are crazy, you are crazy" in a subliminal (below the level of consciousness) way.

(a) *(10 points)* The device Marty uses to transmit signals has a fixed location. Cora tosses, turns and sleepwalks during the night. These movements rapidly vary the intensity of the signal her brain receives. Should Marty use AM or FM/PM to get his message across? Justify your answer.

SOLUTION:

If the amplitude is corrupted by motion, then FM/PM which does not depend on the envelope of the signal is better.

(b) *(20 points)* Suppose Marty uses FM and Cora's brain tissue first rectifies the incoming signal $r(t)$ and then passes it in sequence through a limiter, a discriminator and then an envelope detector. How does this sequence of operations differ from a normal FM receiver? Will Cora's brain still decode the message? Why/why not?

SOLUTION:

If you rectify, the discriminator gives you positive-going impulses at the zero crossings. Since it's the density of zerocrossings that matters for FM, rectifying the incoming signal has no effect.

(c) *(10 points)* Cora discovers Marty's plan and decides to set up a squirrel search and destroy mission. Which modulation method(s) will make it most likely that Cora can localize Marty's transmission, even if Marty's message $m(t)$ lasts only a few milliseconds?

i. $r(t) = m(t) \cos 2\pi f_c t$

ii. $r(t) = \cos(2\pi f_c t + \beta m(t))$

iii. $r(t) = (m(t) + |A|) \cos 2\pi f_c t$

where $A = \min_t m(t)$. You must justify your choice(s).

SOLUTION:

Marty wants to use the modulation method that does not emit a carrier when $m(t) = 0$. *Otherwise, it's always on and detectable. In contrast to first modulation method, second and third have higher chance of being detected since it's still on even if* $m(t) = 0$.

(d) *(20 points)* Marty decides to use PM and sneaks down to examine Cora's detection setup. He makes some measurements and sees that Cora's detector can sense signals at or above 2.4GHz. Marty is transmitting an information signal $m(t)$ with unit amplitude and bandwidth $B = 100$kHz (squirrels squeak) at a carrier frequency of $f_c = 2.3$GHz. If Marty's transmitted signal is

$$r(t) = \cos(2\pi f_c + \beta m(t))$$

how large can β be before Cora can detect the transmission?

SOLUTION:

$$\begin{aligned} 0.1 \times 10^9 &\geq B(\beta + 1) \\ 10^8 &\geq 10^5(\beta + 1) \\ 999 &\geq \beta \end{aligned}$$