THE STATE UNIVERSITY OF NEW JERSEY

RUTGERS

College of Engineering
Department of Electrical and Computer Engineering

332:322 **Principles of Communications Systems** **Spring 2008**

Quiz I

There are 3 questions. You have the class period to answer them. Show all work. Answers given without work will receive no credit. GOOD LUCK!

1. *(50 points)* **Amplitude Modulation**

   (a) *(20 points)* Let $m(t) = e^{-t}u(t)$ where $u(t)$ is the unit step function. Let $r(t) = m(t)\cos 2\pi f_c t$ where $f_c \gg 1$. Provide an analytic expression for $R(f)$ the Fourier transform of $r(t)$ and carefully sketch $|R(f)|$.

   (b) *(10 points)* Carefully describe at least two different methods by which $m(t)$ can be recovered from $r(t)$. You must justify your answers.

   (c) *(10 points)* Let

   $$g(t) = \sqrt{\frac{1}{2\pi\sigma^2}}e^{-\frac{t^2}{2\sigma^2}}$$

   with Fourier transform

   $$G(f) = e^{-2f^2\sigma^2}$$

   We form a periodic carrier signal

   $$\phi(t) = \sum_{k=-\infty}^{\infty} g(t - \frac{k}{f_c})$$

   where f_c is the (fundamental) carrier frequency. Assume some general (i.e., different from the previous parts) program material $m(t)$ band limited to $B \ll f_c$. If $\sigma \ll \frac{1}{f_c}$, can $m(t)$ always be recovered from $r(t) = m(t)\phi(t)$ using an envelope detector? Carefully and analytically discuss why/why not.

   (d) *(10 points)* Suppose the diode in the envelope detector is faulty and functions as a simple resistor. Is the output of the envelope detector still $m(t)$? Always or never?

2. *(50 points)* **AM/FM/SchmeshM:**

   Hermaphrodites Inc. has just introduced their new modulation system. They take band limited non-negative program material $m(t)$ and form the signal

   $$r(t) = m(t)\sin\left(2\pi f_c t + \beta\int_0^t m(\tau)d\tau)\right)$$

   We assume that the bandwidth of $m(t)$ is $B \ll f_c$ and β is arbitrary so long as $\sin\left(2\pi f_c t + \beta\int_0^t m(\tau)d\tau\right)$ is always a high speed signal.

(a) *(25 points)* Is $r(t)$ an example of amplitude modulation? What is the output of an envelope detector with $r(t)$ as its input? Justify your answer.

(b) *(25 points)* Is $r(t)$ an example of frequency modulation? What is the approximate bandwidth of $r(t)$?

3. *(50 points)* **Cora, Marty and the FM Radio Station:**

Cora the communications engineer has decided to open her own FM radio station. Unfortunately, the transmission tower is home to one Martin T. Sciuridae, a squirrel and Cora's arch nemisis. Marty learns Cora is the new owner and sets out to sabotage her transmissions.

Assume Cora transmits

$$r(t) = \cos\left(2\pi f_c t - \beta \int_0^t m(\tau)d\tau\right)$$

and that the transmission is wideband FM. Cora's customers use the usual discriminators followed by envelope detectors as receivers.

Marty whips out a signal generator and corrupts $r(t)$ as

$$\tilde{r}(t) = \cos\left(2\pi f_c t - \beta \int_0^t m(\tau)d\tau\right) - \left(\beta \int_0^t m(\tau)d\tau\right) \sin\left(2\pi f_c t - \beta \int_0^t m(\tau)d\tau\right)$$

(a) *(10 points)* Calculate the output of the differentiator in Cora's listeners' receivers.

(b) *(15 points)* Does Marty's devlish plan work? Why/why not?

(c) *(15 points)* Cora learns of Marty's scheme and makes $\beta \gg 2\pi f_c$. She then tells her listeners to modify their receivers by placing a hard limiter after the output of the differentiator to produce

$$s(t) = \begin{cases} 1 & \frac{d\tilde{r}(t)}{dt} > 0 \\ 0 & \text{o.w.} \end{cases}$$

Rewrite $s(t)$ in terms of $\cos\left(2\pi f_c t - \beta \int_0^t m(\tau)d\tau\right)$?

(d) *(10 points)* Cora then asks her listeners to route $s(t)$ into another FM receiver. Is Marty's devilish plan foiled? Why/why not?

HINT: Where is the program material represented in an FM signal? Is that information preserved or corrupted by hard limiting?