THE STATE UNIVERSITY OF NEW JERSEY

RUTGERS

College of Engineering
Department of Electrical and Computer Engineering

332:322 **Principles of Communications Systems** **Spring 2006**

Quiz I

There are 3 questions. You have the class period to answer them. Show all work. Answers given without work will receive no credit. GOOD LUCK!

1. *(50 points)* **Linear Systems Warmup:**

   (a) *(10 points)* Write down the forward and reverse Fourier Transform which relates $x(t)$ and its Fourier Transform $X(f)$.

   (b) *(20 points)* Show that if $x(t)$ has Fourier Transform $X(f)$, then the Fourier Transform of $\frac{dx}{dt}$ is $j2\pi f X(f)$

   (c) *(20 points)* The energy in a signal $x(t)$ is

   $$\mathcal{E}_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

   What is the energy in the signal $x(t) = \frac{2\cos 2\pi t}{t} - \frac{\sin 2\pi t}{\pi t^2}$?
   HINT: Use your knowledge of Fourier Transforms and their properties. Also, work smart, not hard.

2. *(50 points)* **Amplitude Modulation:** Suppose the program material is a periodic square wave

   $$m(t) = \sum_k p(t-2k)$$

   where $p(t) = u(t) - 2u(t-1) + u(t-2)$.

   (a) *(10 points)* Sketch the AM waveform $r_1(t) = m(t)\cos 2\pi f_c t$ on the interval $[0,4]$ where $f_c = 2$

   (b) *(10 points)* Sketch the large carrier AM waveform $r_2(t) = (1+m(t))\cos 2\pi f_c t$ on the interval $[0,4]$ where $f_c = 2$.

   (c) *(15 points)* Now assume $f_c = 10^6$ and sketch the output waveforms when $r_1(t)$ and $r_2(t)$ are applied as inputs to ideal envelope detectors. Sketch the associated outputs.

   (d) *(15 points)* Suppose $r_1(t)$ is demodulated synchronously using $\cos(2\pi f_c t + \phi)$ where ϕ is a phase offset. What is the output? You must show your work and justify your result.

3. *(50 points)* **Frequency/Phase Modulation:**

(a) *(15 points)* Frequency modulation of a signal $m(t)$ has the form

$$r(t) = \cos\left(2\pi f_c t + \beta \int_0^t m(\tau) d\tau\right)$$

where β is a constant.

Assume $f_c = 10^8$Hz, $\beta = 100$ and a single-sided bandwidth of $f_m = 10^4$Hz for the program material $m(t)$. If $r(t)$ is applied to an ideal envelop detector. What is the resulting output? How might you answer change if $f_m = 10^6$Hz?

(b) *(15 points)* We wish to design a superheterodyne receiver for radio stations whose carriers are between 1GHz and 2GHz and adjacent channels are separated by 200KHz. We can design tunable filters in the GHz range with Qs of only about 10. This means that our filters will have a bandwith about 10% as wide as the carrier. So for a 1GHz carrier, we can have a 100MHz wide tunable bandpass filter which cannot separate out a single 200KHz channel in the passband.

What is the smallest value intermediate frequency (IF) can we use for our heterodyne receiver?

(c) *(20 points)* Cora the communications engineer knows that single sideband AM can be used to reduce the bandwidth of the radiated signal $r(t)$ and thereby allow radio channels to be more densely packed in frequency domain.

A phase modulated (PM) signal has the form

$$r(t) = \cos(2\pi f_c t + \beta m(t))$$

Please help Cora devise, if possible, a comparable scheme for narrowband and wideband PM. Quantitatively argue your case. If it is possible, show how to do single sideband PM (modulation and demodulation). If not, show why not. You can assume any ideal component you'd like.