THE STATE UNIVERSITY OF NEW JERSEY
RUTGERS

College of Engineering
Department of Electrical and Computer Engineering

332:322 **Principles of Communications Systems** **Spring**

Problem Set 8

Haykin: 3.7, 3.9-3.15

1. **Baby Quantization, Just in Case:** This is a simple problem, but it's worth doing just to make sure everyone is in synch. Suppose you have (for $t \geq 0$) a periodic signal

$$s(t) = u_{-2}(t) + 2\sum_{k=1}^{\infty}(-1)^k u_{-2}(t-2k+1)$$

for $t \geq 0$ where $u_{-2}(t)$ is the unit ramp function.

   (a) Sketch $s(t)$ over one cycle.

   (b) What might the PDF of $s(t)$ look like (use your instincts – you do NOT have the formal machinery to deal with this just yet).

   (c) $Q()$ is a one bit quantizer with $x_0 = 0$, $q_0 = -1$, $q_1 = 1$. Sketch he quantized signal $Q(s(t))$ and the error signal $s(t) - Q(s(t))$. Assume $s()$ is uniform on ± 1. Is $Q()$ an optimum one bit quantizer for $s(t)$? If not, what IS the optimum one bit quantizer for $s()$?

   (d) $Q()$ is a two bit quantizer with $x_k = -1/2 + k/2$, $k = 0, 1, 2$ and $q_k = -3/4 + k/2$, $k = 0, 1, 2, 3$. Sketch he quantized signal $Q(s(t))$ and the error signal $s(t) - Q(s(t))$. Is this quantizer optimal, assuming uniform $s()$ on ± 1?

2. **Quantization Recap:** For the two bit quantizer of the previous part, code q_0 as 00, q_1 as 01, q_2 as 10 and q_3 as 11. Determine the sequence of codes which would come out of the quantizer over one cycle. Assume that samples are taken every half second and that $Q(-0.5) = -1/4$, $Q(0) = 1/4$, $Q(0.5) = 3/4$.

   Discuss how one would reconstruct an approximation of the input signal using the sequence of codes at a receiver.

3. **Convexity:** Using the definition of convexity, determine for what values of α the function $f(x, y) = x^2 + \alpha xy + y^2$ is convex.

4. **Delta Modulation:**

   For the sawtooth waveform

$$s(t) = u_{-2}(t) + 2\sum_{k=1}^{\infty}(-1)^k u_{-2}(t-2k+1)$$

with $t \geq 0$, make sketches of the output of a delta modulator for sample rates $2Hz$ and $10Hz$ and step sizes $\Delta = 1/2$ and $\Delta = 1/10$ (four sketches total). Also sketch the outputs of the associated demodulators.

5. **Linear Prediction:** A one-step linear predictor operates on the sampled version of a sinusoidal signal. The sampling rate is equal to $10 f_o$ where f_o is the frequency of the sinusoid. The predictor has a single coefficient denoted by w_1.

   (a) Determine the optimum value of w_1 required to minimize the prediction error variance.

   (b) Determine the minimum value of prediction error variance.

   HINT: This is a direct application of linear prediction filter we studied in section 3.13. To be able to apply the standard equation, we need to find the autocorrelation funtion of the sinusiod of the form $m(t) = A \sin(2\pi f_o t)$. Note that the sinusiod being deterministic, the expectation is over one time period.