THE STATE UNIVERSITY OF NEW JERSEY

RUTGERS

College of Engineering
Department of Electrical and Computer Engineering

332:322 **Principles of Communications Systems** **Spring**

Problem Set 3

1. **Discovered Angle Modulation** A signal $s(t)$ is measured and found to be described by

$$s(t) = A\cos(2\pi f_a t + \alpha \sin 2\pi f_b t)$$

   (a) We're later told that $s(t)$ is an angle modulated signal with sensitivity k_p. Using the standard angle modulation signal format found in your text, what is the information signal $m(t)$?

   (b) Now, imagine that you're told "WHOOPS! I meant FREQUENCY modulation. with frequency sensitivity k_f." Again using the standard signal format described in your text, please provide the information signal $m(t)$ and the instantaneous frequency $f_i(t)$.

2. **Two Tone Madness:** Consider a message signal with two tones at frequencies f_a and f_b respectively, defined as

$$m(t) = A_m \cos(2\pi f_a t)\cos(2\pi f_b t)$$

   (a) Find the corresponding phase modulated and frequency modulated signals.

   (b) Find the narrowband FM (i.e NBFM) signal using the FM modulated signal obtained in the previous part.

3. **Linear? Nonlinear?** Let $m_1(t)$ and $m_2(t)$ be two message signals, and let $s_1(t)$ and $s_2(t)$ be the corresponding modulated signals.

   (a) Carefully show that if the modulation is DSB-SC, SSB, or VSB, then

$$m(t) = m_1(t) + m_2(t)$$

   will produce a modulated signal

$$s(t) = s_1(t) + s_2(t)$$

   (b) Show that if the modulation is PM or FM, then

$$m(t) = m_1(t) + m_2(t)$$

   will not in general produce

$$s(t) = s_1(t) + s_2(t)$$

4. **Lecture Redux** Suppose we have a modulated signal

$$s(t) = A\cos(2\pi f_c t + \beta sin(2\pi f_m t))$$

with $\beta << 1$ (i.e., narrowband FM/PM).

(a) Find the spectrum of this narrowband FM/PM signal.

(b) Compare your previous result to the spectrum of an AM (suppressed carrier) signal

$$s(t) = A\sin(2\pi f_m t)\cos(2\pi f_c t)$$

Cite similarities and differences.

5. **Phase Locked Loops:** Consider a phase locked loop whose sole purpose is to lock on to the incoming sinusoid $\cos(2\pi f_c t)$ and produce $\sin(2\pi f_c t)$ at the output of the VCO (voltage controlled oscillator). The incoming sinusoid is multiplied by the output of the VCO and the result is sent through a low pass filter whose output is multiplied by $-K$ ($K > 0$ a constant). The output of the multiplier, call it $\dot{\phi}(t)$, is in turn sent to the input of the VCO which produces $\sin\phi(t)$.

(a) Sketch a system diagram of the PLL and show it will produce the desired result if the gain K is large enough. You may assume that $\phi(t) \approx 2\pi f_c t$ and then make appropriate approximations.

(b) Given the same input $\cos(2\pi f_c t)$ is it possible for the phase locked loop to "lock on" to frequencies other than f_c. If so, which ones? If not, why not?

HINT: Think about the bandwidth of the low pass filter – which we never actually specified in the previous part.