

School of Engineering  
Department of Electrical and Computer Engineering

**332:223 Principles of Electrical Engineering | Laboratory**  
**Experiment # 5**  
**Title: Sinusoidal Steady State Analysis**

**1- INTRODUCTION:**

The objective of this experiment is to study series **R-C**, **R-L**, and **R-L-C** circuits and their sinusoidal steady state behavior. Phasor analysis is used to determine the input-output relationships.

**2- THEORETICAL CONSIDERATIONS:**

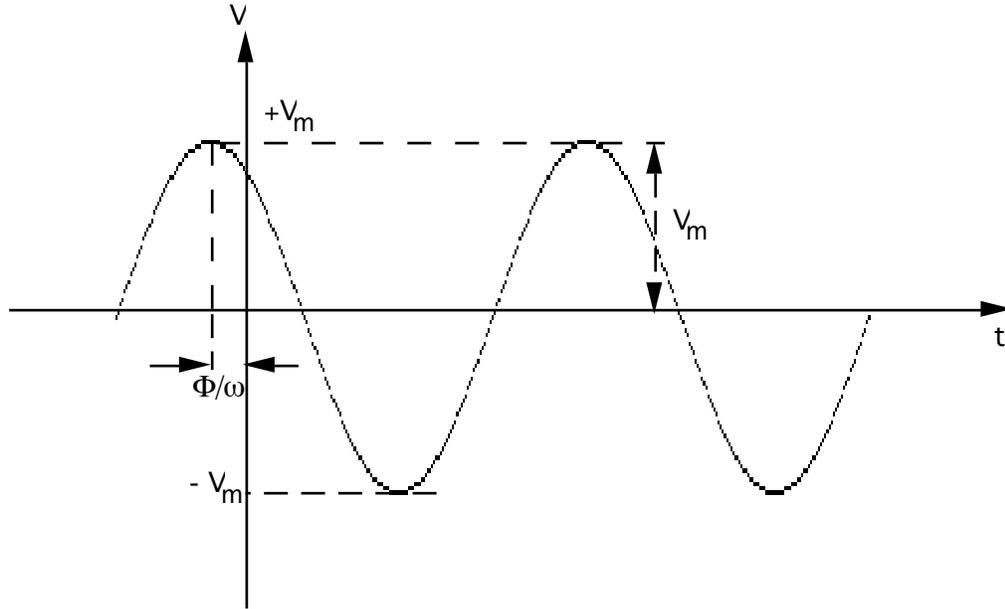
**2.1 The Sinusoidal Source:**

A sinusoidal voltage or current source (independent or dependent) produces a voltage or current that varies sinusoidally with time. Using the cosine function as a base, we can write a sinusoidally varying voltage as follows:

$$v(t) = V_m \cos(\omega t + \Phi) \quad (\text{Eq. 1})$$

where  $V_m$  is the maximum amplitude of the sinusoidal voltage,  
 $\omega = 2\pi f = 2\pi/T$  is the angular frequency of the sinusoidal signal in rad/sec.  
( $f$  is the frequency of the sinusoid in Hz,  
 $T$  is the period of the sinusoid in seconds)  
and  $\Phi$  is the phase angle of the sinusoidal voltage.

- Notes:**
- 1- Since the cosine function is bounded by  $\pm 1$ , the amplitude is bounded by  $\pm V_m$ . This is clearly shown in Fig. 1.
  - 2- Changing the phase angle  $\Phi$  shifts the sinusoidal function along the time axis, but has no effect on either the amplitude  $V_m$  or the angular frequency  $\omega$ .
  - 3- If  $\Phi$  is positive, the sinusoidal function shifts to the left.  
If  $\Phi$  is negative, the sinusoidal function shifts to the right.



**Fig. 1 Sinusoidal Signal**

## 2.2 Phasor Representation Of A Sinusoid:

Using the Euler identity,

$$e^{\pm j\theta} = \cos(\theta) \pm j \sin(\theta),$$

Eq. 1 can be written as follows:

$$\begin{aligned} v(t) &= V_m \operatorname{Re}\{e^{j(\omega t + \Phi)}\} = V_m \operatorname{Re}\{e^{j\omega t} e^{j\Phi}\} \\ v(t) &= \operatorname{Re}\{(V_m e^{j\Phi}) e^{j\omega t}\} \end{aligned}$$

The complex number  $V_m e^{j\Phi}$  is called the phasor representation or the phasor transform.

Eq. 1 can be represented in the phasor domain as follows:

$$V = V_m \angle \Phi$$

## 2.3 The Passive Circuit Elements In Phasor Domain:

If  $i(t) = I_m \cos(\omega t + \Phi)$  is flowing through a resistor, inductor, or a capacitor, then the voltage in the phasor domain is:

a- Across a resistor:

$$V = RI \quad (\text{Eq. 2})$$

b- Across an inductor:

$$V = (j\omega L)I \quad (\text{Eq. 3})$$

c- Across a capacitor:

$$V = (1/j\omega C)I = -(j/\omega C)I \quad (\text{Eq. 4})$$

where  $I = I_m \angle \Phi$  is the sinusoidal current  $i(t)$  in phasor domain.

## 2.4 Phasor Diagrams:

Consider a series RLC circuit with an input sinusoidal voltage  $V_s$  as shown in fig. 2.

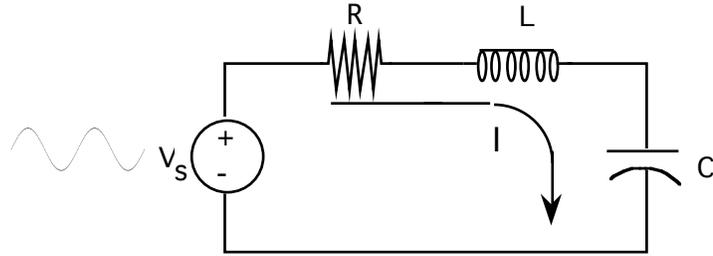


Fig. 2 A Series RLC Circuit

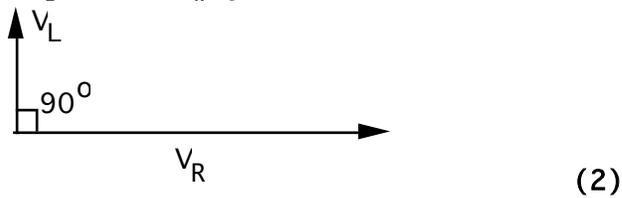
In the phasor domain,  $V_s = V_R + V_L + V_C = RI + (j\omega L)I + (-j/\omega C)I$ .

To draw the phasor diagram of Fig. 2, for ideal circuit elements, we do the following:

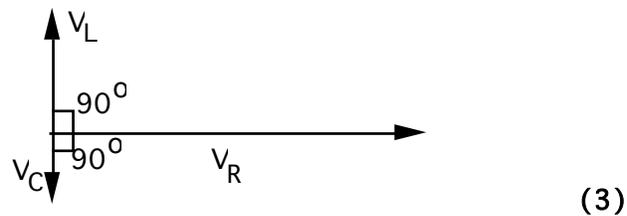
(1) Choose a reference voltage, say  $V_R$



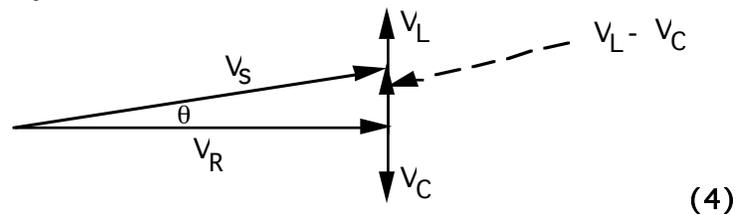
(2) From Eqs. 2 & 3 we see that  $V_L$  will lead  $V_R$  by  $90^\circ$



(3) From Eqs. 2 & 4 we see that  $V_C$  will lag behind  $V_R$  by  $90^\circ$



(4)  $V_s = V_R + V_L + V_C$



The magnitude of  $V_s$  is:  $|V_s| = [V_R^2 + (V_L - V_C)^2]^{1/2}$

The phase angle of  $V_s$  is:  $\theta = \tan^{-1}[(V_L - V_C)/V_R]$

### 3- PRE-LAB EXERCISES:

#### 3.1 R-C Branch:

Refer to Fig.3 and compute  $R$  and  $C$  for  $Z = 2556 \angle -38.5^\circ$  ohms at  $f = 1000\text{Hz}$ . Note that

$$Z = R - \frac{j}{\omega C} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}} \angle -\tan^{-1}\left(\frac{1}{\omega CR}\right).$$

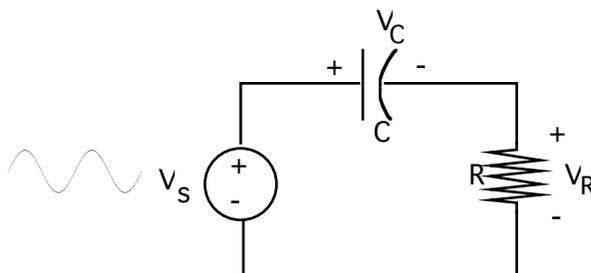
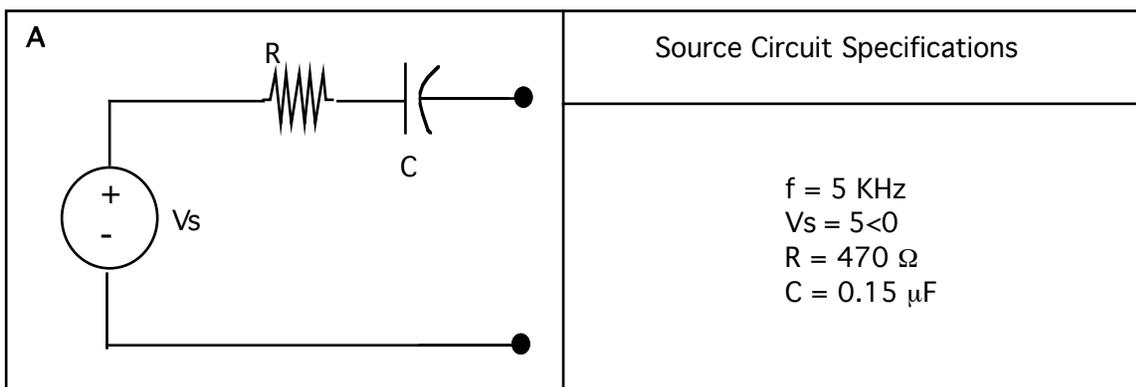


Fig. 3 R-C Branch

**3.2** For the following source circuit, design a load circuit that will extract the maximum power from the source circuit. Your design should minimize the number of parts and must be within 5% of transferring maximum power to the load. Also calculate the complex power that is delivered to the load.



#### 3.3 RL Branch:

Refer to Fig. 4 and compute  $R$  and  $L$  for  $Z = 1181 \angle 32.14^\circ$  ohms at  $f=1000\text{Hz}$ . Note that

$$Z = R + j \omega L = \sqrt{R^2 + (\omega L)^2} \angle \tan^{-1}(\omega L/R).$$

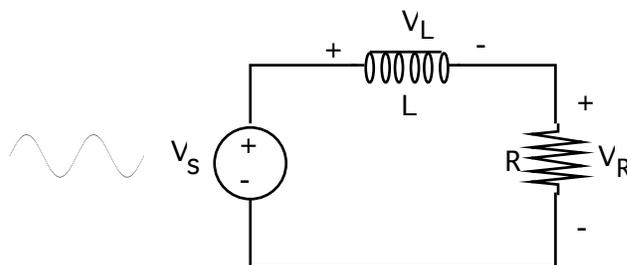
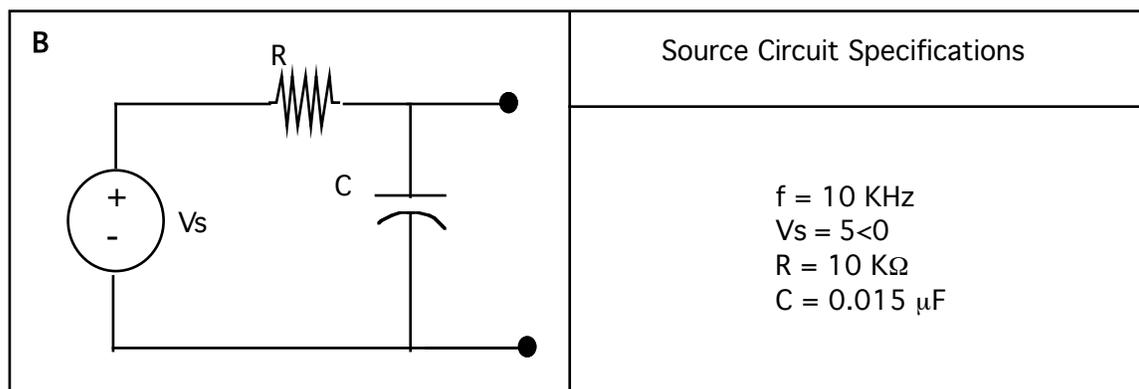


Fig. 4 R-L Branch

**3.4** - For the following source circuit, design a load circuit that will extract maximum power from the source circuit. Your design should minimize the number of parts and must be within 5% of transferring maximum power to the load. Also calculate the complex power that is delivered to the load.



**3.5** Refer to Fig. 5 and compute  $R$  and  $C$  for  $Z = 1388 \angle -43.9^\circ$  ohms at  $f = 1000\text{Hz}$ , given  $L = 100\text{mH}$ . Note that

$$Z = R + j(\omega L - 1/\omega C) = \sqrt{R^2 + (\omega L - 1/\omega C)^2} \angle \tan^{-1}[(\omega L - 1/\omega C)/R].$$

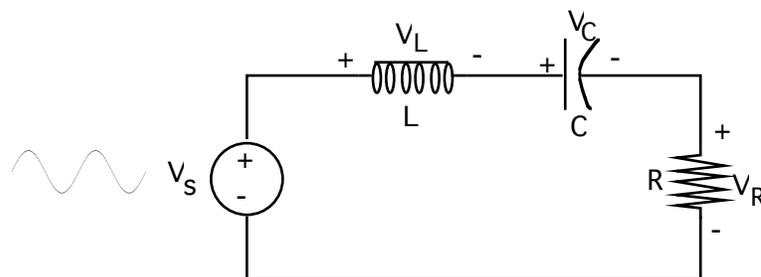


Fig. 5 R-L-C Branch

#### 4- EXPERIMENT:

Suggested Equipment:

- Tektronix FG 501A 2MHz Function Generator
- Tektronix DC 504A Counter-Timer
- Protek B-845 Digital Multimeter
- Agilent 54622A oscilloscope
- LS-400A Inductance Substitute Box
- 1  $\mu$ F Capacitor
- 100 K $\Omega$  resistor
- Various circuit elements to be determined by students

(**Important note:** Throughout this experiment, large resistance values will be used, and so, we will neglect the 50 $\Omega$  internal impedance of the function generator.)

**4.1 R-C Branch:** Note that, in all the following write up, notationally  $V_s$ ,  $V_R$ , and  $V_C$  are not phasors; they represent the magnitudes of the corresponding phasor voltages.

- With the values of  $R$  and  $C$  as computed in pre-lab exercise **3.1**, build the circuit of Fig.3.
- Connect the DVM (switched to AC) across the resistor and adjust the oscillator output until  $V_R$  has the proper value that corresponds to a current of 1mA flowing through the circuit.

(**Important note:** Notice that the oscillator output and the oscilloscope input have a common ground connection, consequently a component voltage cannot be displayed properly on the scope unless that component is also connected to the common ground. Make sure that you build your circuits exactly as shown in the figures.)

- Measure  $V_s$ ,  $V_R$ , and  $V_C$ .
- Display the time domain signals corresponding to  $V_s$  and  $V_R$  on the oscilloscope, and copy them from the oscilloscope screen.
- Measure the phase

The oscilloscope can automatically calculate the phase difference angle  $\theta$  between the phasors of  $V_s$  and  $V_R$  in the following way:

- press the **Quick Meas** button
- press the 2<sup>nd</sup> soft key to select phase
- press the 3<sup>rd</sup> soft key to measure the phase difference between the two signals

**4.2 R-L Branch:** Note that, in all the following write up, notationally  $V_s$ ,  $V_R$ , and  $V_{L+RL}$  are not phasors; they represent the magnitudes of the corresponding phasor voltages.

- Select an inductor which has the inductance value of  $L$  as computed in pre-lab exercise **3.3**.
- Measure the d.c. Resistance  $R_L$  of the inductor using the digital ohmmeter. You will need this value for your report.
- In pre-lab exercise **3.3**, a value for resistance  $R$  of Fig.4 was computed. Let  $R_1 = R - R_L$  where  $R_L$  is the d.c. Resistance of the inductor. Select a resistor whose resistance value equals

- $R_1$ . Note that the resistance value of the resistor is intentionally decreased to take into account the d.c. resistance of the inductor.
- With the values of  $R_1$  and  $L$  as described above, build the series circuit of Fig.4. Set up the circuit exactly as shown in Fig. 4 in order to avoid problems with the ground.
  - Adjust the oscillator until the output across the resistor  $V_R$  has the proper value that corresponds to a 1mA current flowing through the circuit. There is a notational ambiguity here. The notation  $V_R$  corresponds to the voltage across the physical resistor whose resistance value is  $R_1$ . As said earlier, we intentionally decreased the resistance value of the resistor to take into account the d.c. resistance of the inductor.
  - Measure the magnitudes of voltages corresponding to  $V_s$ ,  $V_R$ , and  $V_{L+RL}$  using the DVM (switched to AC). Note that the physical inductor is a series combination of  $R_L$  and  $L$ . We cannot physically measure the inductance voltage alone. We can only measure the voltage across the inductor which has both the resistance and inductance.
  - Display the time domain signals corresponding to  $V_s$  and  $V_R$  on the oscilloscope, and copy them from the oscilloscope screen.
  - Measure the phase angle  $\theta$  between  $V_s$  and  $V_R$  using the method explained in Section 4.1.

#### **4.3 R-L-C Branch:**

- Measure the d.c. Resistance of the inductor whose inductance value  $L = 100\text{mH}$ .
  - Build the series **R-L-C** series circuit of Fig. 5 by using the  $R$  and  $C$  values as computed in pre-lab exercise **3.5**. Make sure that the value of resistance  $R$  is decreased by the d.c. Resistance of the inductor as we did in the previous experiment with **R-L** branch.
- Follow the steps in Section **4.1** and measure  $V_s$ ,  $V_R$ ,  $V_{L+RL}$ ,  $V_C$ , and the phase angle  $\theta$  between  $V_s$  and  $V_R$ , and copy the waveforms corresponding to  $V_s$  and  $V_R$  from the oscilloscope.
- As we know, at resonance the reactance  $X$  is zero and the impedance is purely resistive. Since the imaginary portion of the impedance is zero the phase angle is equal to zero. Measure the resonant frequency for the circuit of Fig. 5. There are a number of ways of doing so. One way is to tune the frequency of the oscillator until  $V_s$  and  $V_R$  are in phase, i.e., the phase angle between them is zero.
- For all frequencies above the resonance, phase angles have the same sign and for frequencies below the resonant point have the opposite sign. Verify that this is true over a frequency range two orders of magnitude on both sides of the resonant frequency.
- Make sure that the voltages and the phase angles measured are correct because you will need them for your report.

## 5- REPORT:

5.1 For the R-C branch shown in Fig.3, do the following:

- 1- Tabulate  $V_s$ ,  $V_R$ ,  $V_C$ , and  $\theta$  measured in the experiment.
- 2- Submit a copy of the waveforms of  $V_s$  and  $V_R$  on a graph paper with rectangular coordinates.
- 3- Prepare a phasor diagram to scale that represents in phasor domain  $V_s = V_R + V_C$ .
- 4- Calculate the phase angle  $\theta$  in three ways:
 
$$\theta = -\tan^{-1}(V_C/V_R) = -\sin^{-1}(V_C/V_s) = -\cos^{-1}(V_R/V_s)$$
 Compare  $\theta$  calculated with  $\theta$  measured.
- 5- Calculate the average power,  $P_{av}$ , in two ways:
 
$$P_{av} = V_R^2/R = V_s I \cos \theta$$
, Where  $\theta$  is the measured phase angle.
- 6- Does  $V_R$  lead or lag  $V_s$ ?
- 7- For the circuit of Fig. A in the pre-lab exercise 3.2, what is the load that consumes the maximum power and what is that value of maximum power.

5.2 For the R-L branch shown in Fig.4, do the following:

- 1- Tabulate  $V_s$ ,  $V_R$ ,  $V_{L+RL}$ , and  $\theta$  measured.
- 2- Submit a copy of the waveforms  $V_R$  and  $V_s$ .
- 3- Prepare a phasor diagram to scale. Take the effect of the d.c. resistance of the inductor into account.
- 4- From the phasor diagram, determine the phase angle  $\theta$  in three ways as in item 5.1 (step 4). Compare with the measured one.
- 5- Calculate the average power,  $P_{av}$ , in two ways as in 5.1.
- 6- Does  $V_R$  lead or lag  $V_s$ ?
- 7- For the circuit of Fig. B in the pre-lab exercise 3.4, what is the load that consumes the maximum power and what is that value of maximum power.

5.3 For the R-L-C branch in Fig. 5, repeat 5.2 (except part 7) for the impedance  $Z$  used in performing the experiment. Do not forget to take into account the d.c. resistance  $R_L$  of the inductor  $L$  in all subsequent calculations.

5.4 With  $V_s = 10 \sin(2\pi f t)$  where  $f=1\text{KHz}$ , use PSpice to plot  $V_s$  and  $V_R$  for the three circuits in figures 3, 4, and 5. Show 2 to 4 periods of the waveforms in your plot.  
(Hint: Use the **TRAN.** command.) Alternatively, you can use MULTISIM instead of Pspice.

5.5 With  $V_s = 10 \sin(2\pi f t)$  where  $f=1\text{KHz}$ , use Spice to determine the real part, imaginary part, magnitude, and phase of  $V_R$  for the circuits in figures 3, 4, and 5.  
(Hint: Use the **AC.** command.) Alternatively, you can use MULTISIM instead of Pspice.

5.6 Explain why the following circuit is an Integrating Amplifier, that is explain in your own way why and when

$$V_o = -\frac{1}{RC} \int_0^t V_s(t) dt.$$

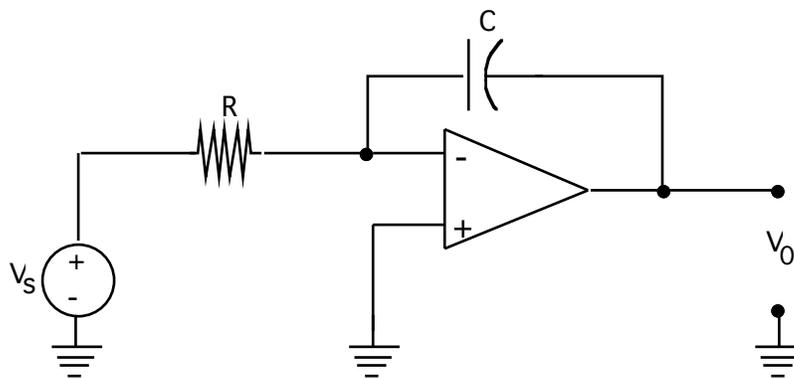


Fig. 6 Integrating Amplifier

- 5.7 Explain why the following circuit is a differentiating Amplifier, that is explain in your own way why and when

$$V_o(t) = -RC \frac{d(V_s(t))}{dt} .$$

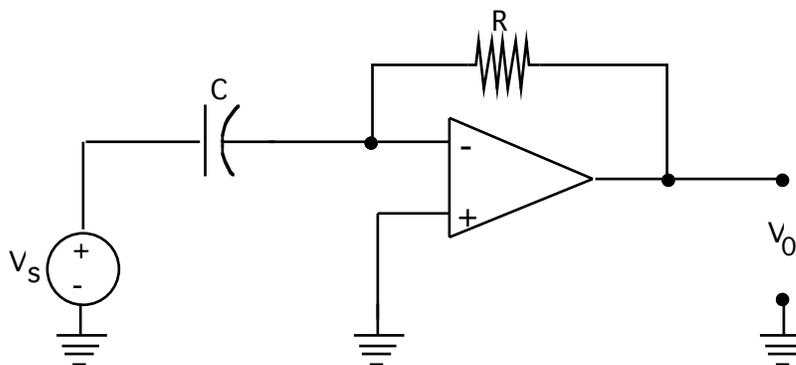


Fig. 7 Differentiating Amplifier

5.8 In the circuit of Fig. 8,  $R_1 = 10\text{K}\Omega$ ,  $R_2 = 1\text{K}\Omega$ ,  $R_3 = 10\text{K}\Omega$ ,  $C_1 = 1\text{nF}$ , and  $C_2 = 0.01\mu\text{F}$ . With  $V_{CC} = +15\text{ V}$  and the sinusoidal input  $V_g = 5 \cos(10^5 t)\text{ V}$ , simulate this circuit in Pspice or Multisim and display the waveforms of output voltage  $V_o$  and input voltage  $V_g$  in the same graph to compare them easily. What are the magnitude and phase angle of  $V_o$ ?

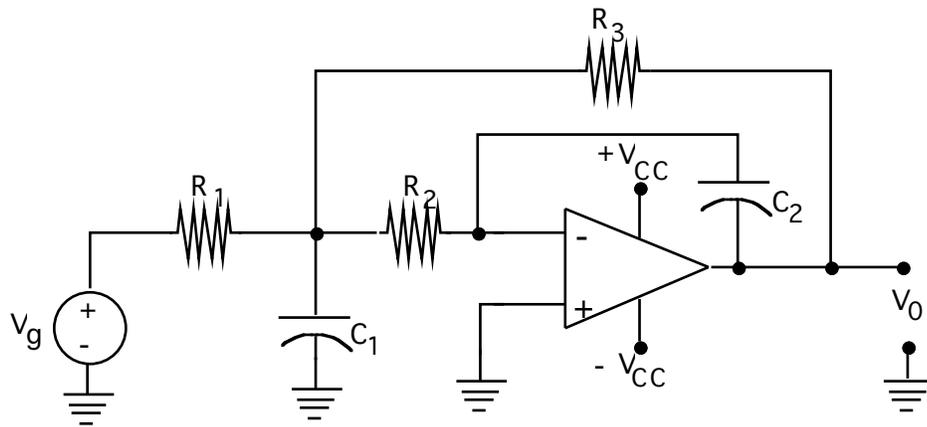


Fig. 8 Circuit for 5.8

5.9 Prepare a summary.