

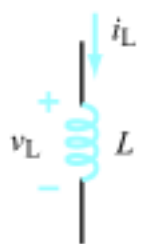


332:221 Principles of Electrical Engineering I

Passive circuit elements and their symbols

Element	Symbol	i - v Relationship
Resistor		$v_R = Ri_R$
Capacitor		$i_C = C \frac{dv_C}{dt}$
Inductor		$v_L = L \frac{di_L}{dt}$

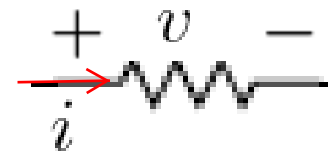
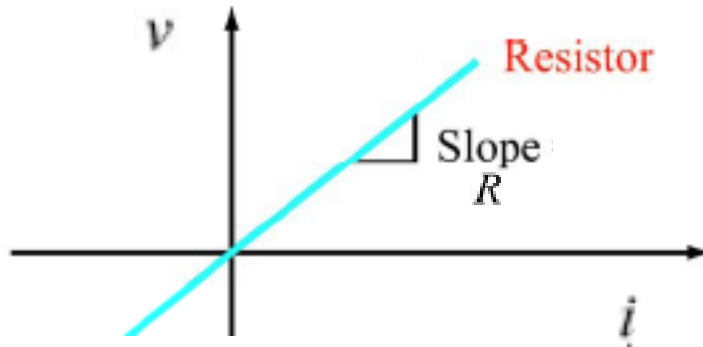
We concentrate first only on resistance. For a resistance with the direction of voltage v and current i as shown, $v= Ri$ (Ohm's law). Power consumed $=vi=i^2R=v^2/R$. **A resistance always consumes power.**

Conductance G of a resistance R is defined as the reciprocal of R , $G=1/R$.

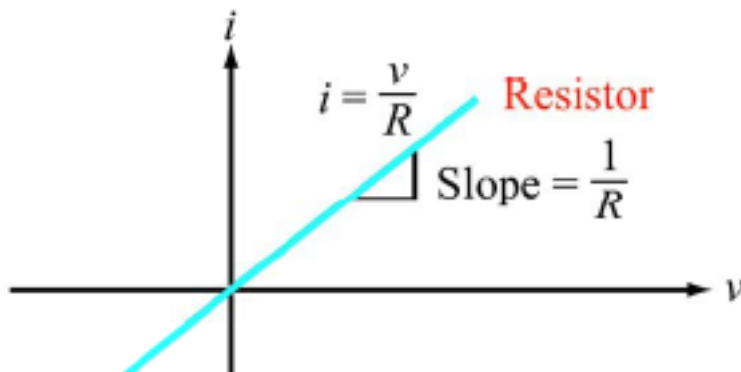
Unit of resistance is ohm, denoted as Ω
Capital Omega

Unit of conductance is mho, denoted by \mathcal{U}

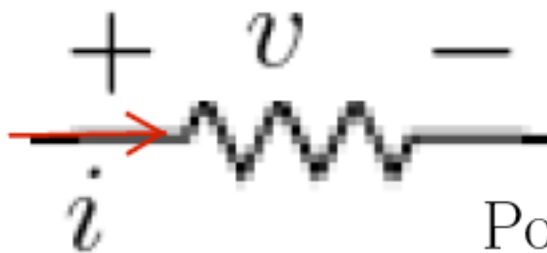
mho=siemen



$$v = iR$$



Resistor is a physical element. It has resistance. An ideal linear resistance has a linear v - i characteristic as shown. Practical v - i characteristic is non-linear.



$$v = iR$$

Power consumed by a resistance

$$p = vi = iRi = i^2R$$

$$p = v\frac{v}{R} = \frac{v^2}{R}$$

Resistance always consumes power.

Both rheostat and potentiometer are resistances, but cleverly used by utilizing a movable wiper.

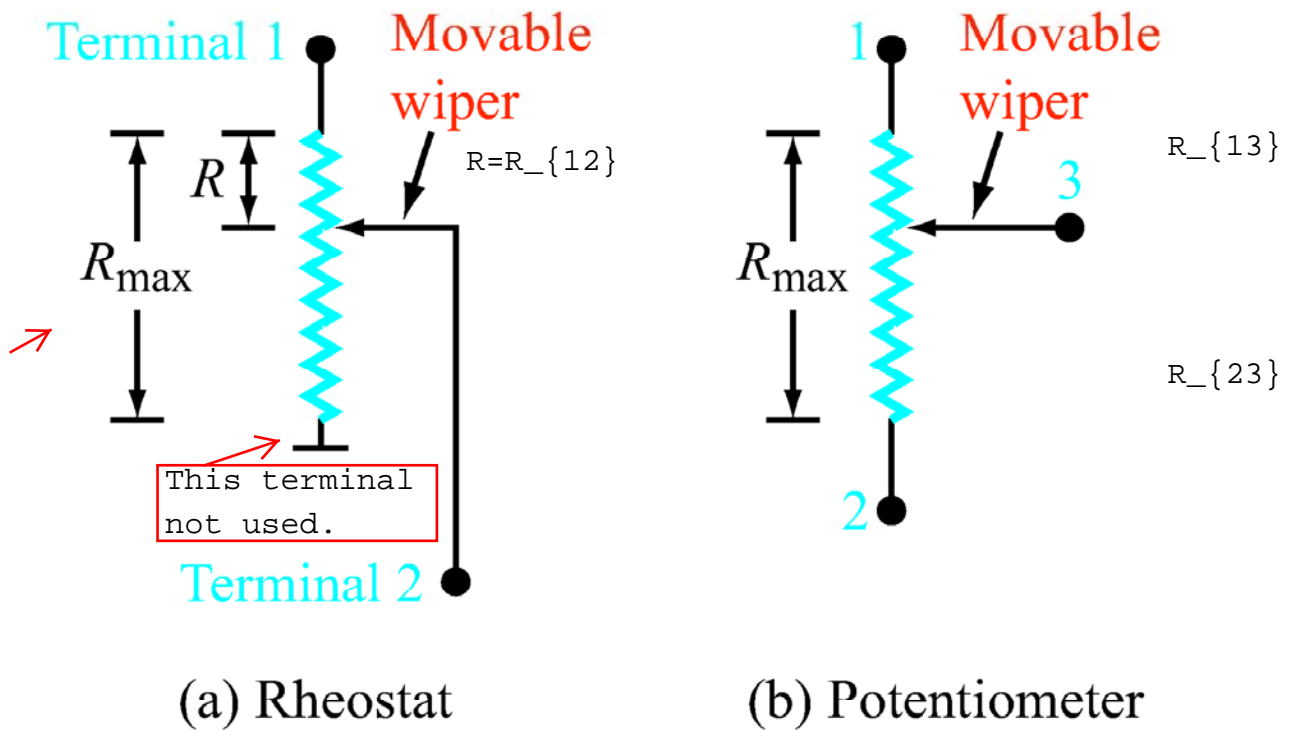
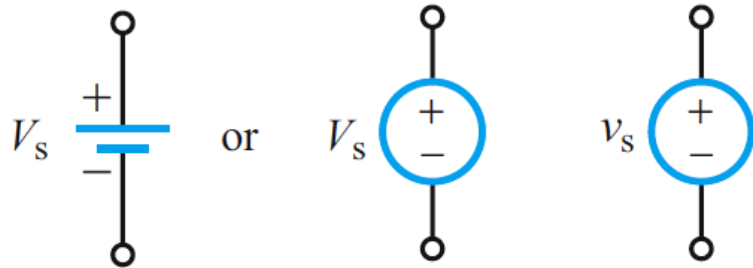


Figure 2-3: (a) Rheostat is used to set the resistance between terminals 1 and 2 at any value between zero and R_{\max} ; (b) location of wiper in potentiometer divides the resistance R_{\max} among R_{13} and R_{23} .

Circuit Elements: Independent Sources

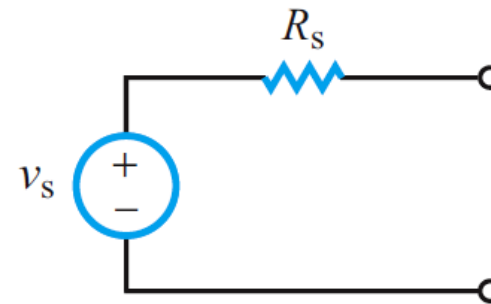
Independent Sources

Ideal Voltage Source



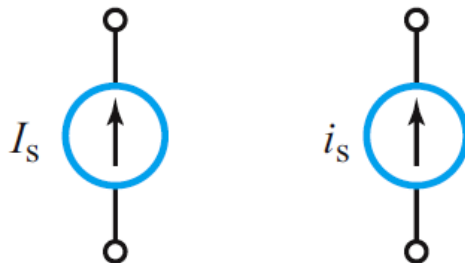
Battery

Realistic Voltage Source

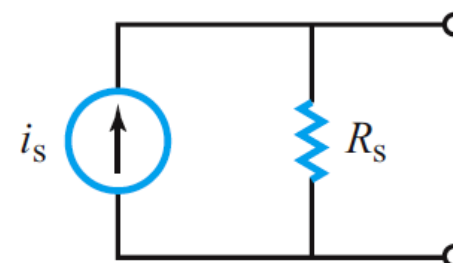


Any source

Ideal Current Source

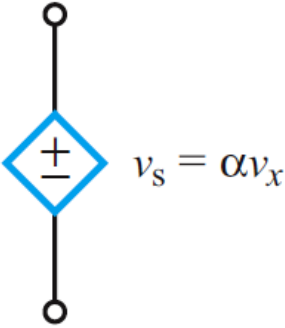
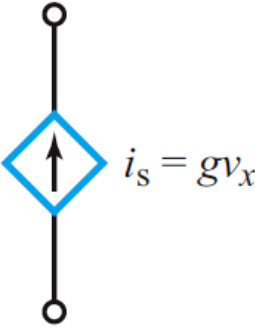
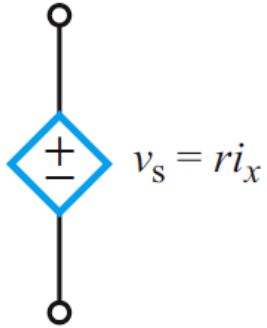
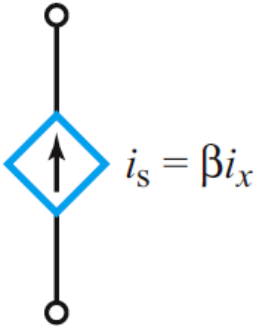


Realistic Current Source



Any source

Circuit Elements: Dependent Sources

Dependent Sources	
Voltage-Controlled Voltage Source (VCVS)  $v_s = \alpha v_x$	Voltage-Controlled Current Source (VCCS)  $i_s = g v_x$
Current-Controlled Voltage Source (CCVS)  $v_s = r i_x$	Current-Controlled Current Source (CCCS)  $i_s = \beta i_x$
<p><i>Note:</i> α, g, r, and β are constants; v_x and i_x are a specific voltage and a specific current elsewhere in the circuit. *Lowercase v and i represent voltage and current sources that may or may not be time varying,</p>	

Kirchoff's laws:

A **node** is a point where two or more circuit elements meet.

Kirchoff's Current Law (KCL):

The algebraic sum of all the currents at any node in a circuit equals zero.

$$\sum_{n=1}^N i_n = 0 \quad (\text{KCL})$$

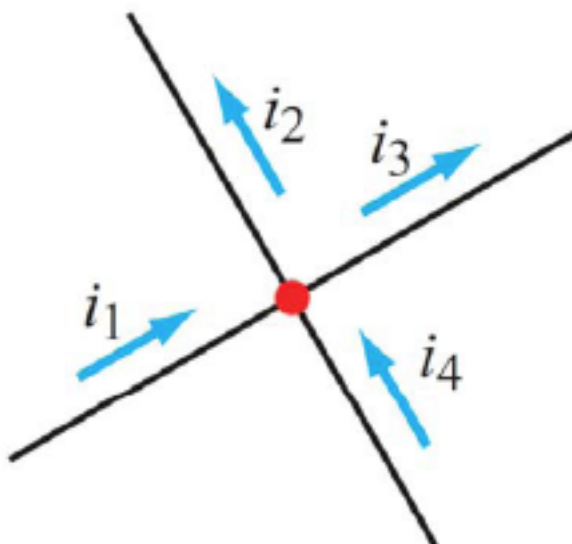
A closed-path or loop:

Starting at any arbitrary node, walk along an element connected to it to move to another node, repeat this as often as necessary, and come back to the original node. The path you traced is a closed path or loop.

$$\sum_{n=1}^N v_n = 0 \quad (\text{KVL})$$

Kirchoff's Voltage Law (KVL):

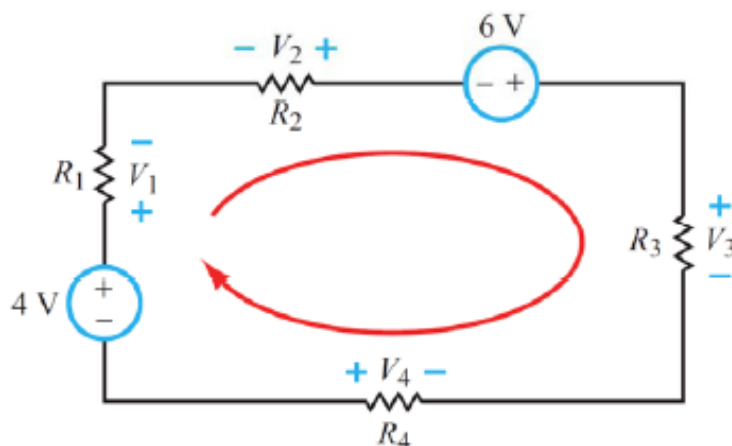
The algebraic sum of all the voltages around any closed-path in a circuit equals zero.



$$i_1 - i_2 - i_3 + i_4 = 0$$

$$i_1 + i_4 = i_2 + i_3$$

$$-i_1 + i_2 + i_3 - i_4 = 0$$

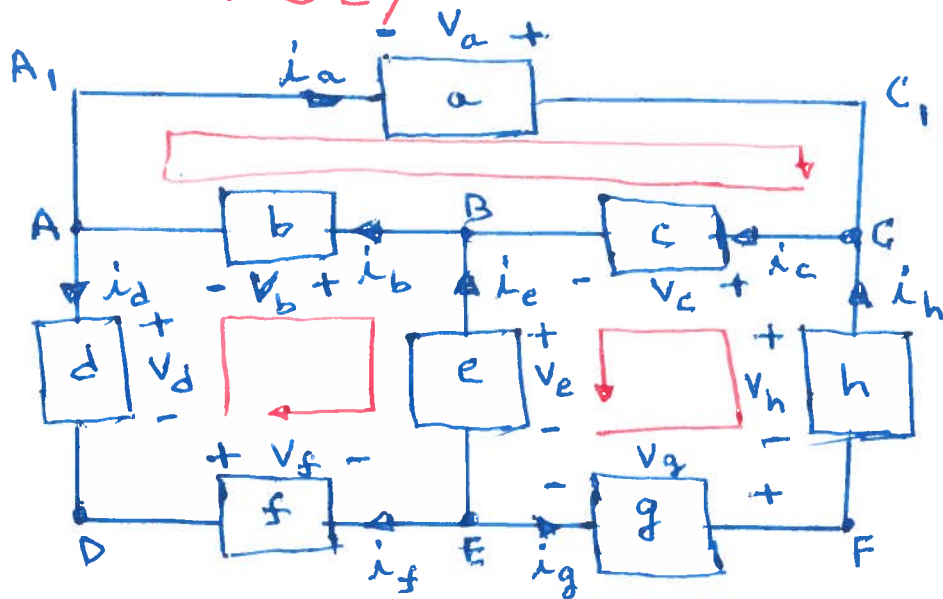


$$-4 + V_1 - V_2 - 6 + V_3 - V_4 = 0$$

$$4 - V_1 + V_2 + 6 - V_3 + V_4 = 0.$$

$$4 + V_2 + 6 + V_4 = V_1 + V_3.$$

Example: KCL/KVL



KCL

$$\begin{aligned} i_a + i_d - i_b &= 0 \\ i_b - i_c - i_e &= 0 \\ -i_a + i_c - i_h &= 0 \\ -i_d + i_f &= 0 \\ i_e + i_f + i_g &= 0 \\ -i_g + i_h &= 0 \end{aligned}$$

ONLY
FIVE
EQUATIONS
ARE
INDEPENDENT

Add all these equations,
You get $0=0$

KVL

Loop A-B-C-B-A

$$-V_a + V_c + V_b = 0$$

Loop A-B-E-D-A

$$-V_d - V_b + V_e - V_f = 0$$

Loop E-F-G-B-E

$$-V_g - V_h + V_c + V_e = 0$$

All are independent equations.

You can find other loops. However, KVL equations of those loops will not be independent from the above.

Two important types of interconnection of circuit components:

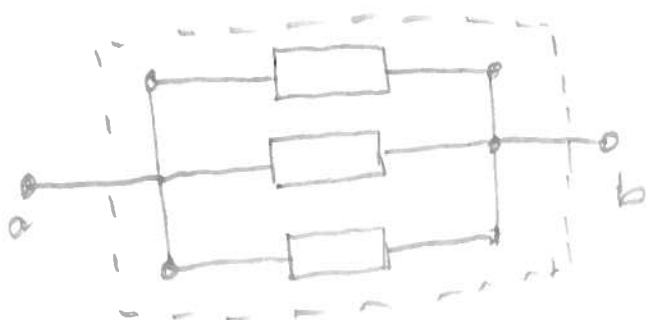
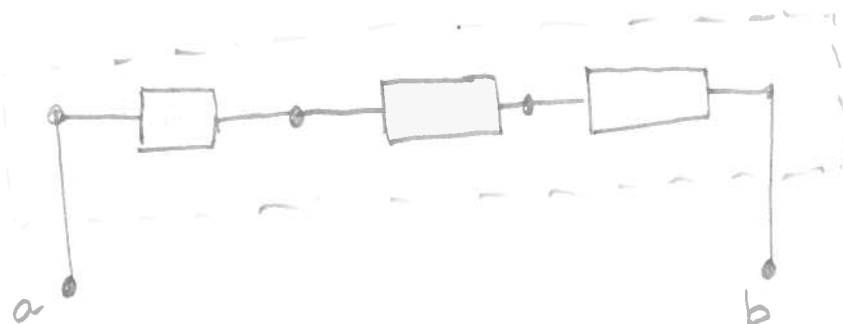
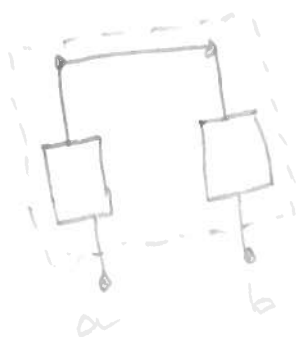
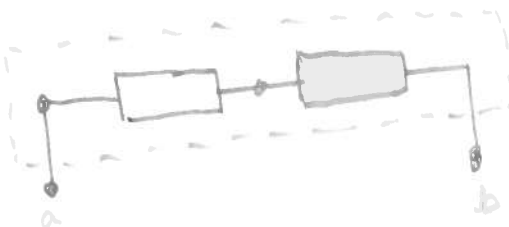
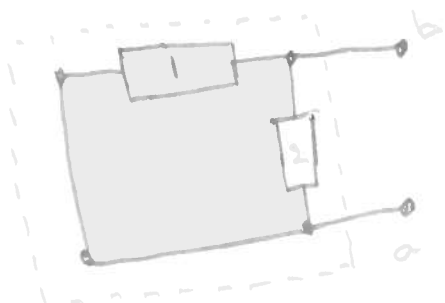
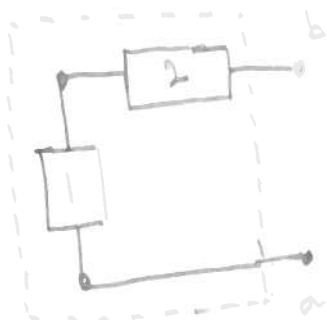
Series Connection:

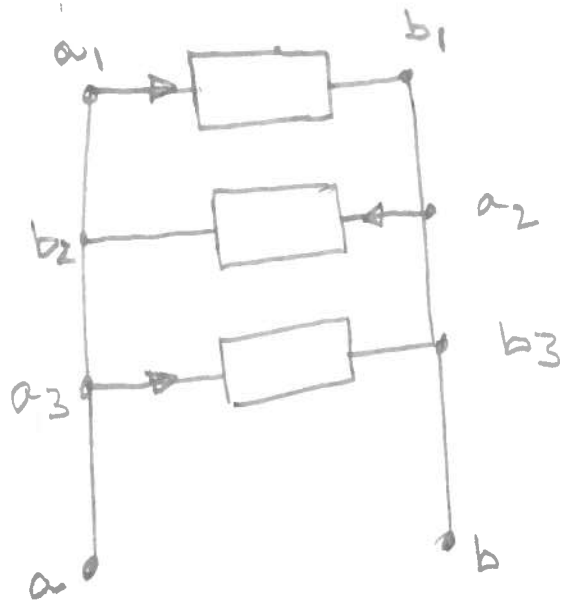
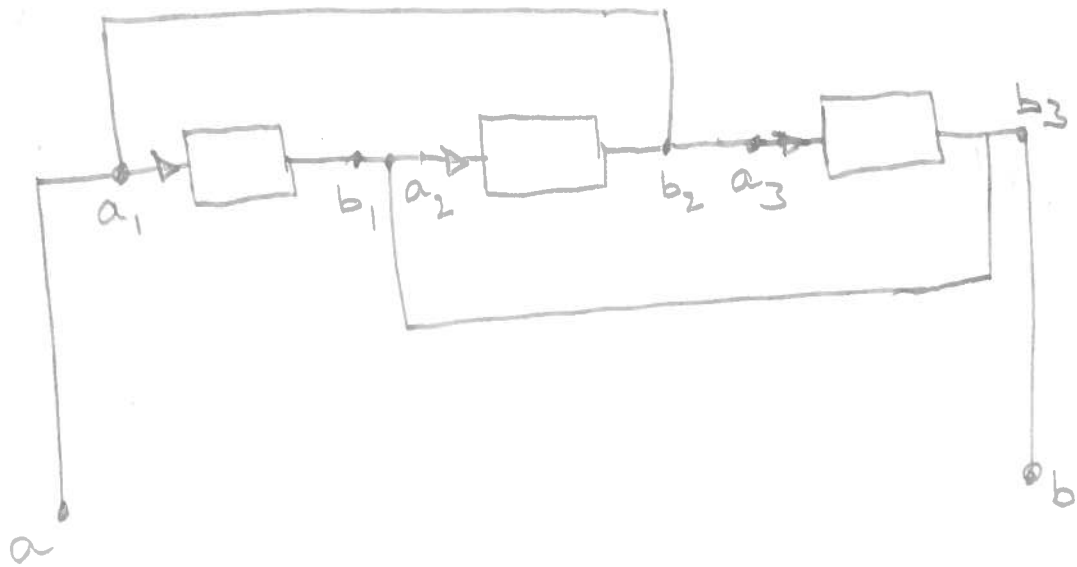
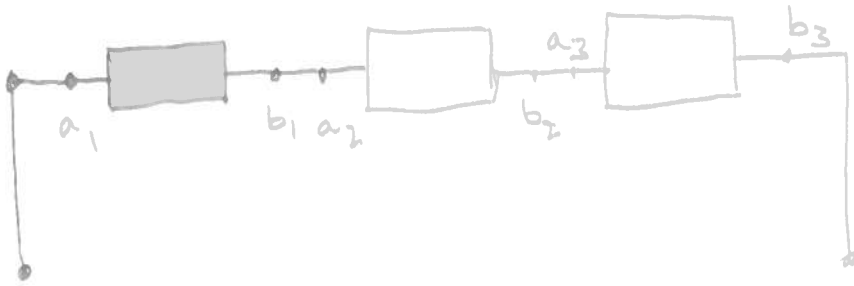
Two or more elements (branches, components)
are said to be interconnected in series
if the currents through each one of them are equal.

Parallel Connection:

Two or more elements (branches, components)
are said to be interconnected in parallel
if the voltages across each one of them are equal.

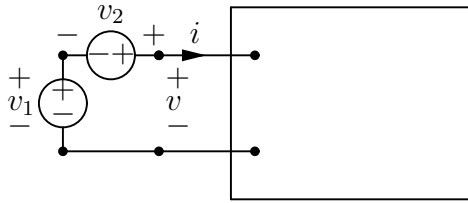
In the following diagrams, two elements are interconnected forming an equivalent one element with two terminals a and b. Decide as far as the terminals a and b are concerned, whether the two elements are interconnected in series or in parallel.





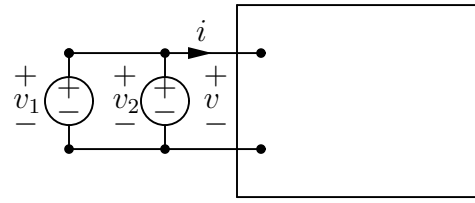
Principles of Electrical Engineering I – Interconnection of Sources

This is always feasible.



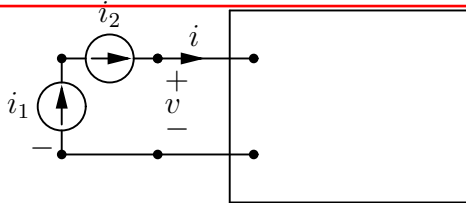
Voltage sources connected in series

This is feasible only when $v_1 = v_2$



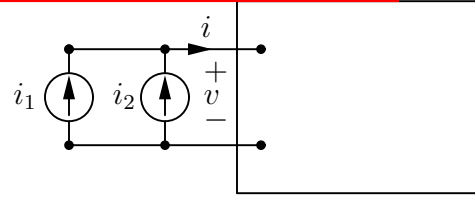
Voltage sources connected in parallel

This is feasible only when $i_1 = i_2$



Current sources connected in series

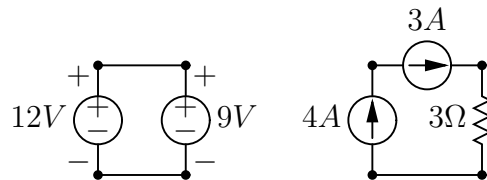
This is always feasible.



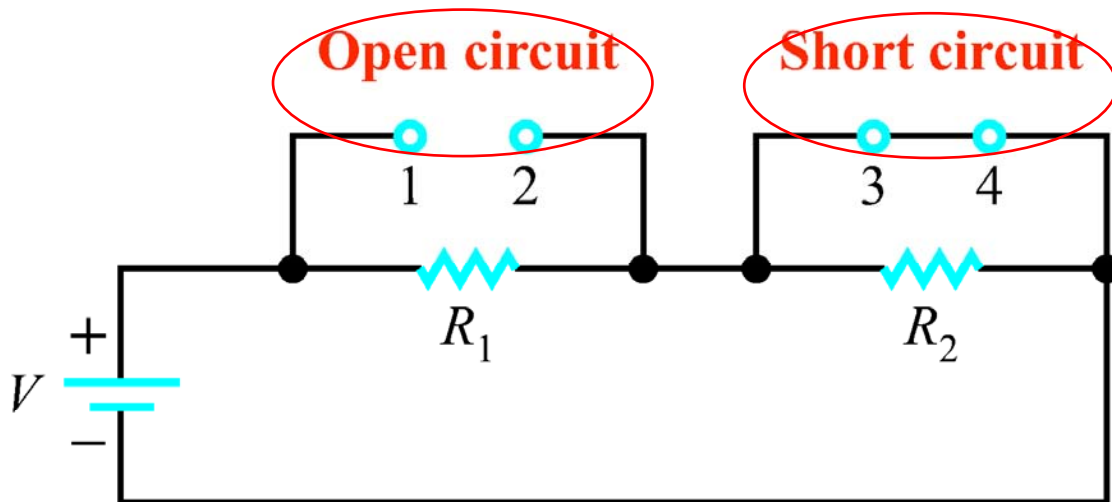
Current sources connected in parallel

All interconnections that do not satisfy Kirchoff's laws are illegal.

Illustration of illegal interconnections:



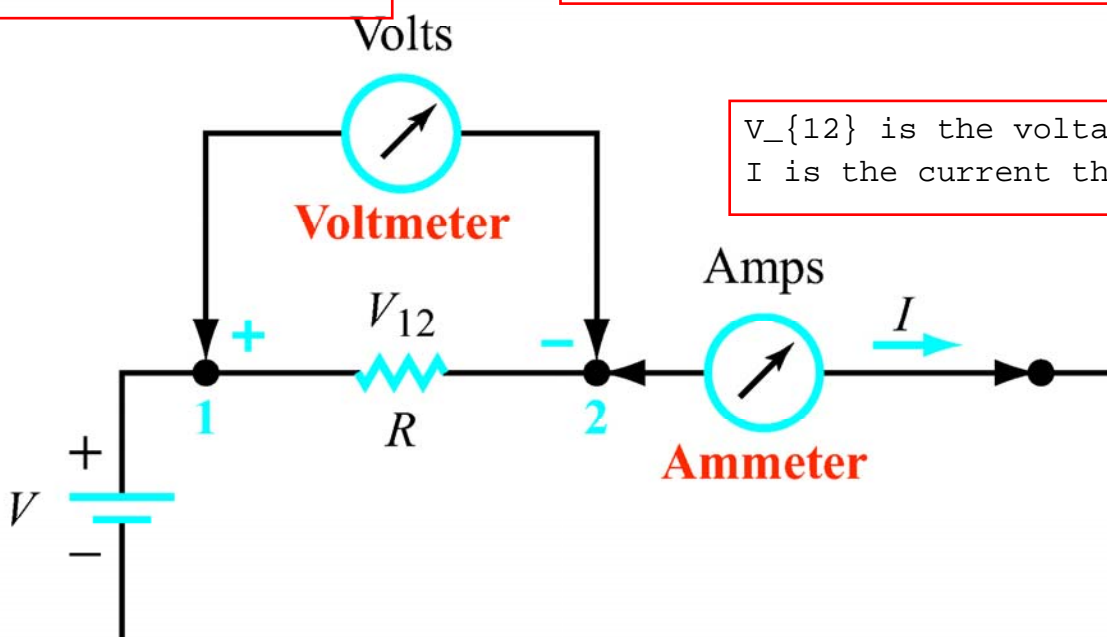
We often use two phrases 'open circuit' between two terminals or 'short circuit' between two terminals. In the following figure, terminals 1 and 2 are open circuited, while terminals 3 and 4 are short circuited. The terminology applies to only those specified terminals.



A **voltmeter** reads the voltage across an element, as such it is connected across (or parallel) to the element. An **ammeter** reads the current through an element, as such it is connected in series to the element.

What is an ideal meter ?

General parallel and series connections of elements are discussed in class.



V_{12} is the voltage across R
 I is the current through R .

Basics you should always remember

Electric Potential by itself is not power. Current has to be supplied or consumed to generate power. In other words, the interaction of Electric Potential and Electric current causes power to be generated or consumed. In the same way, **your inherent potential** by itself is not enough, it needs to be put to work by doing **Home-Work** in order to learn the material properly. Then only circuit analysis in its full detail can be absorbed by you.

Voltage is across a branch and current is through a branch:

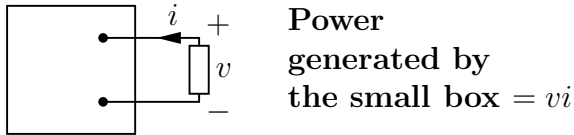


Figure 1

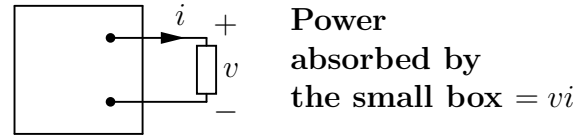


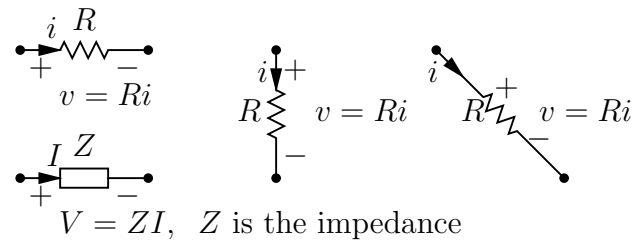
Figure 2

Resistance:

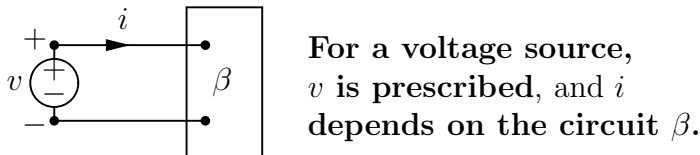
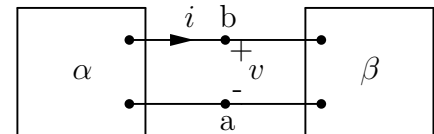
Water flows down the hill unless it is being pumped up.

Current flows down the potential hill through a resistance.

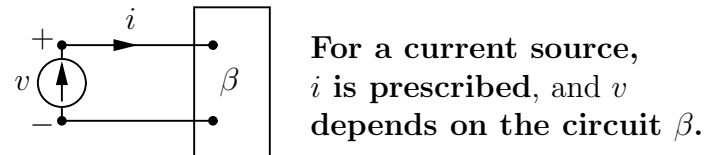
Resistance is a passive element. It always consumes power.



Source is an active element. A source could pump the current up the potential hill and thus generate power. A source could have a current flowing down the potential hill and thus consume power; in this case it is a sink. Think of a good battery charging a weak one. In the figure given on the right, two batteries a good one (denoted by α) and a weak one (denoted by β) are interconnected at terminals a and b. Which of the two batteries generates power and which one consumes? Is i positive if v is positive?



Voltage Source

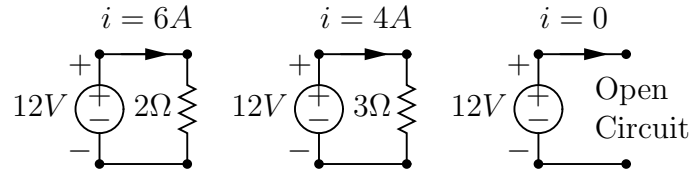


Current Source

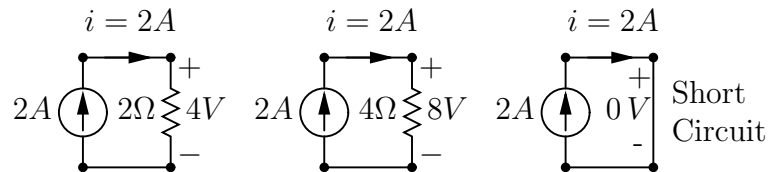
In circuit analysis, different situations arise.

- Both the voltage and the current of a branch might be known ahead. In this case the branch info is completely known.
- Either voltage or current of a branch but not both are known.
- Neither voltage nor current of a branch are known.

- In the case of a **voltage source**, voltage is known ahead but not current. That is, the voltage of a voltage source is prescribed ahead but the current through it is a variable that depends on the network to which it is connected.



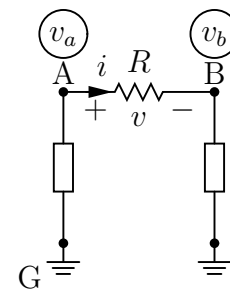
- In the case of a **current source**, current is known ahead but not voltage. That is, the current of a current source is prescribed ahead but the voltage across it is a variable that depends on the network to which it is connected.



- In the case of a **resistance**, the ratio between voltage and current (via Ohm's law) is known.

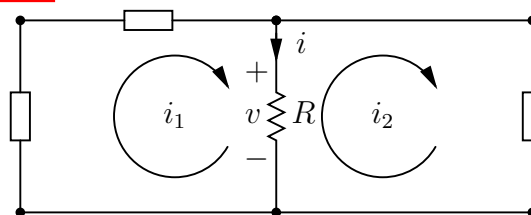
If a voltage or current is unknown, you can assign a variable to it. If we do so for each branch in the circuit, we end up with some unknown variables. **The aim of circuit analysis is to write as many equations (via KCL, KVL, Ohm's law) as there are unknown variables, and then solve those equations to determine the unknown variables.** The entire circuit analysis involves only this type of work. All we do in this course is to develop ways of simplifying the method(s) of writing equations, as small in number as needed.

Nodal Analysis: Consider a resistance R with two terminals marked A and B. Suppose the voltage of terminals A and B with respect to the ground G are respectively v_a and v_b as shown. Then the voltage across R is $v = v_a - v_b$, and the current through R is $i = \frac{v_a - v_b}{R}$.



We will discuss both Nodal Analysis and Mesh Analysis extensively later on.

Mesh Analysis: Consider a resistance R having two currents i_1 and i_2 pumped into it as shown. Then the net current through it is $i = i_1 - i_2$, and the voltage across R is $v = R(i_1 - i_2)$.



We will deal extensively with both **Nodal Analysis** and **Mesh Analysis** later on.

Principles of Electrical Engineering I

Modeling of Practical Sources

All physical devices need to be modeled appropriately in terms of circuit elements so that their characteristics can be taken into account in circuit analysis. In this notes, we emphasize modeling of practical voltage and current sources.

Modeling of a practical voltage source: An ideal voltage source keeps the voltage across its terminals the same whatever might be the current (and thus the power) it supplies to a load. Obviously, a practical voltage source can never do so because of its own internal behavior. The voltage across the terminals of a practical voltage source drops more and more as it supplies more and more current. As such, a practical voltage source can be modeled as an ideal voltage source in series with a resistance that can appropriately be called as its *internal resistance* (in general, impedance). Figure 1 shows a practical voltage source (denoted by α) whose terminals are a and b. A load β when connected to the source draws a current from it. A circuit model of a practical voltage source is shown in Figure 2 along with the load β .

Practical Source

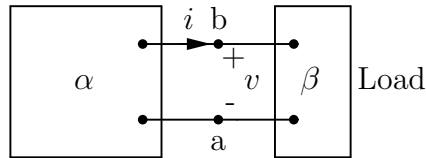


Figure 1

Model of a Practical Voltage Source

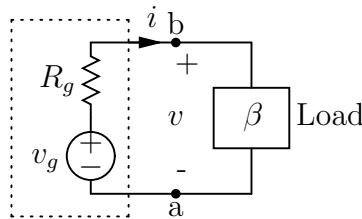


Figure 2

i (A)	v (V)
0	12
4	11
8	10
12	9
16	7.5
20	5
24	0

Table 1

In what follows, we illustrate by an example how to compute v_g and R_g for a given set of data, and also compute the range of current over which the circuit model of the given practical voltage source is valid.

Example 1: A set of experimental data is shown in Table 1 for a practical voltage source. The measurements are obtained experimentally showing how the voltage v across the terminals of a given practical voltage source changes as the current i it supplies to the load varies. Consider the model shown in Figure 2, and determine v_g and R_g , and the region in which the model is valid.

Modeling of a practical current source: An ideal current source keeps the current it supplies to a load the same whatever might be the voltage (and thus the power) a load demands. Obviously, a practical current source can never do so because of certain leakages inherent internal to it. The current it supplies to the load decreases as the voltage the load demands increases. As such, a practical current source can be modeled as an ideal current source in parallel with a resistance that can appropriately be called as its *internal resistance* or leakage resistance. Figure 1 shows a practical current source (denoted by α) whose terminals are a and b. A load β when connected to the source demands a voltage across it. A circuit model of a practical current source is shown in Figure 3 along with the load β .

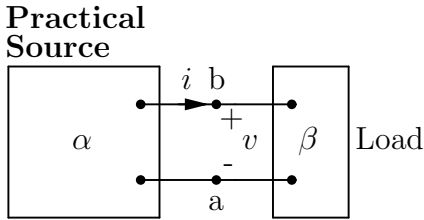


Figure 1

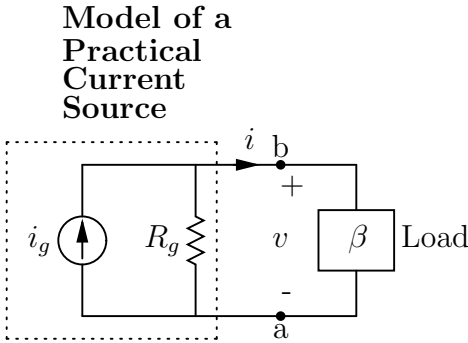


Figure 3

v (V)	i (mA)
0	10
25	8.75
50	7.5
75	6.25
100	5.0
125	3.0
150	0

Table 2

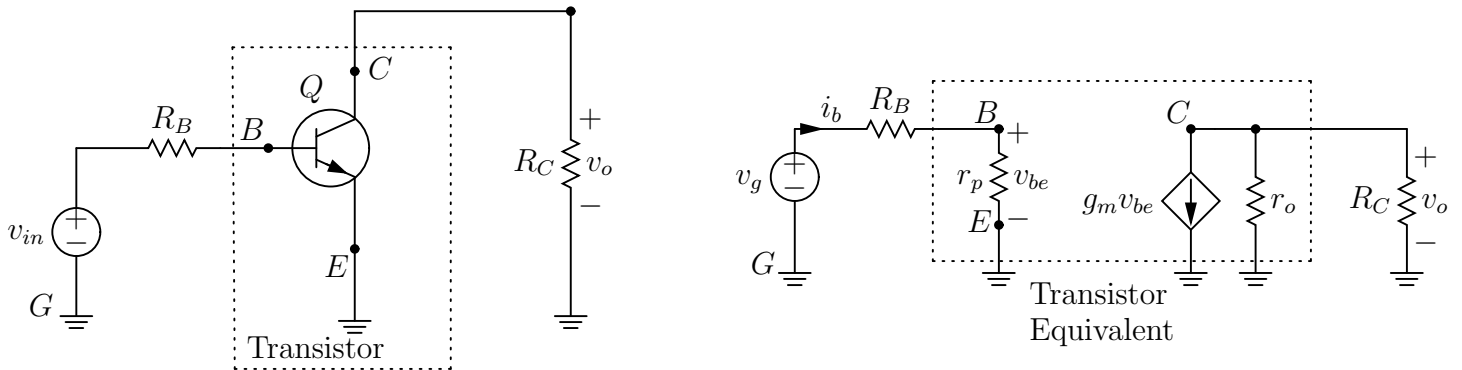
In what follows, we illustrate by an example how to compute i_g and R_g for a given set of data, and also compute the range of voltage over which the circuit model of the given practical current source is valid.

Example 2: A set of experimental data is shown in Table 2 for a practical current source. The measurements are obtained experimentally showing how the current i through the terminals of a given practical current source changes as the voltage v it supplies to the load varies. Consider the model shown in Figure 3, and determine i_g and R_g , and the region in which the model is valid.

Do this as Home Work (not collected).

Principles of Electrical Engineering I

A Transistor Amplifier Circuit



The circuit on the left is a simple amplifier circuit containing a **device** called a transistor (the circled component, denoted by Q) having three terminals B, E, and C.

Note: The transistor circuit given here is missing its bias voltages which make it work. You will learn the specifics of this circuit in a later course.

For some bias voltages, the so called small signal equivalent circuit of transistor circuit is shown on the right. Here r_p is the resistance of the so called plate between B and E, g_m is the transconductance of the transistor, and r_o is the output resistance associated with the transistor. The ratio of the output voltage v_o to the input voltage v_{in} is called the amplifier gain **A**. Compute **A**.

$$\mathbf{A} = \frac{v_o}{v_{in}} = -\frac{g_m r_p R_C}{R_B + r_p} = -\frac{g_m r_p}{R_B + r_p} \frac{r_o R_C}{r_o + R_C}.$$

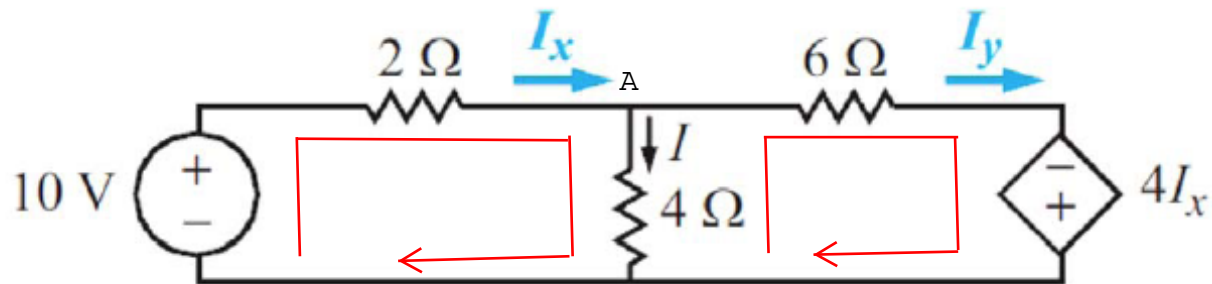
A design example: A pressure-gauge generates a voltage proportional to the pressure exerted on it. However, the magnitude of voltage generated is small and need a very sensitive meter to read it. As such, for proper reading, one needs to amplify the voltage generated by the pressure-gauge. The transistor amplifier circuit shown on the other side is to be used to amplify the voltage close to 200 fold, i.e., the amplifier gain \mathbf{A} needs to be as close as possible to 200. For the circuit shown (on the other side) v_{in} is the voltage generated by the pressure-gauge. A design engineer, based on other considerations, decides to use $R_B = 400\Omega$ and $R_C = 8K\Omega$, and then looks for an appropriate transistor. He/she finds in a catalog three different transistors whose parameters are tabulated below. Which of the three transistors is the most appropriate one to be used?

Transistor	r_p in $K\Omega$	g_m	r_o in $K\Omega$
1	6.4	0.02	64
2	1.6	0.03	60
3	0.8	0.05	40

Hint: Determine the gain \mathbf{A} for each transistor.

Example

Determine I_x and I_y in the circuit of Fig.



Solution: Application of KVL to the two loops gives

$$-10 + 2I_x + 4I = 0$$

$$-4I + 6I_y - 4I_x = 0.$$

Additionally,

KCL equation at A

$$I = I_x - I_y.$$

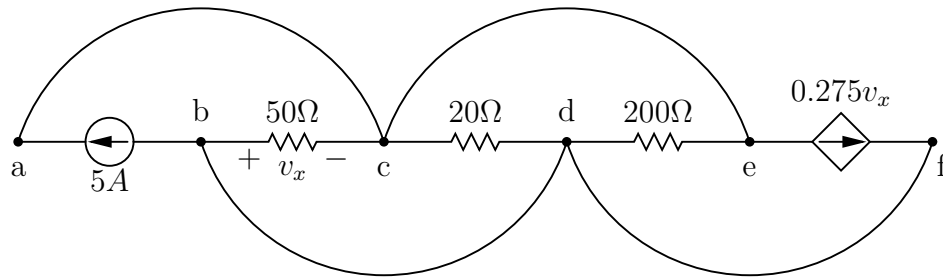
Solution of the three equations yields

$$I_x = 3.57 \text{ A}, \quad I_y = 2.86 \text{ A}.$$

332:221 Principles of Electrical Engineering I

Tracing a Circuit

Determine v_x in the circuit shown.



Often the practical layout of a circuit is completely different to the one required for easy analysis. It is hard to see what is going on in the circuit given above unless you are well experienced. The circuit given needs to be **retraced** so that all the interconnections are clear.

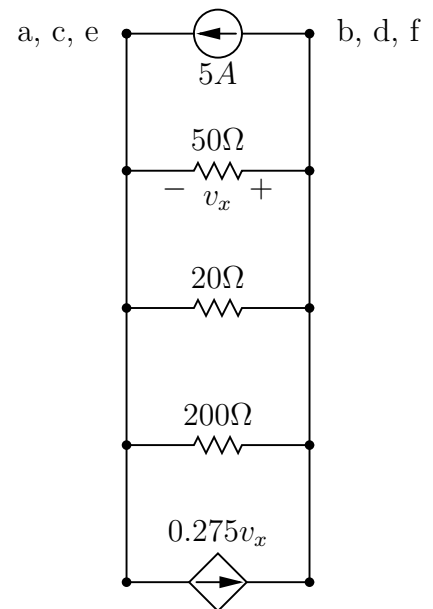
How do we retrace a circuit?

Hint: The circuit branches or components between any two nodes in the given circuit as well as the re-traced circuit must be the same, otherwise the given circuit and the re-traced circuit are not equivalent.

The retraced circuit is as shown on the right (justify it). We can now write the following equation easily,

$$v_x \left[0.275 - \frac{1}{50} - \frac{1}{20} - \frac{1}{200} \right] - 5 = 0.$$

This yields $v_x = 25$ V. Having determined v_x , we can find next the current in each resistance easily.



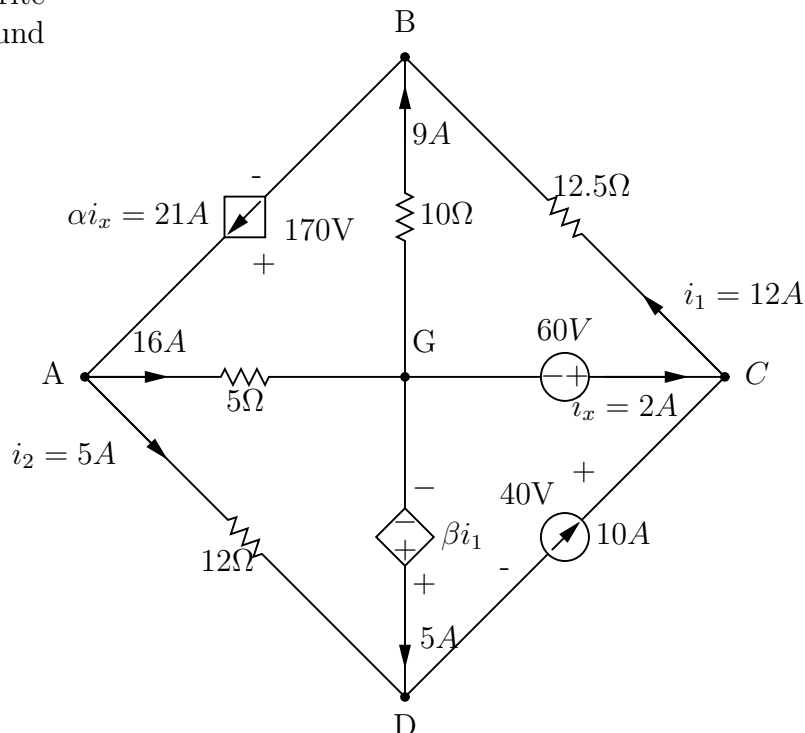
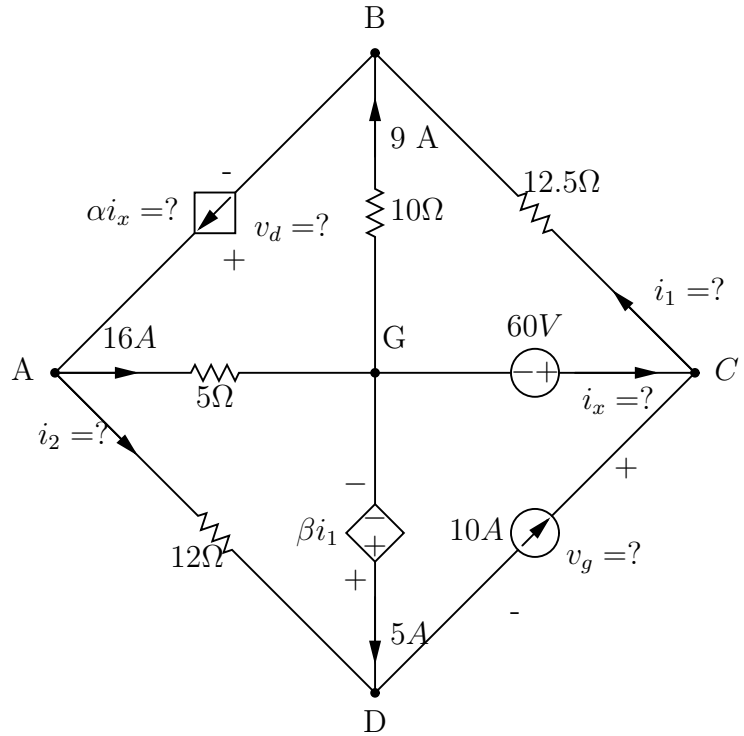
Principles of Electrical Engineering I

Example 1: KCL-KVL – Examples

Consider the partially solved circuit on the right with $\alpha = 10.5$ and $\beta = \frac{5}{3}$. Determine i_1 , i_x , αi_x , i_2 , v_d , and v_g in the order given.

Hints:

- To determine i_x , write a KVL equation around BGCB.
- To determine i_1 , write a KCL equation at C.
- αi_x is easy to determine once i_x is known.
- To determine i_2 , write a KCL equation at A.
- To determine v_d , write a KVL equation around AGBA.
- To determine v_g , write a KVL equation around DGCD.



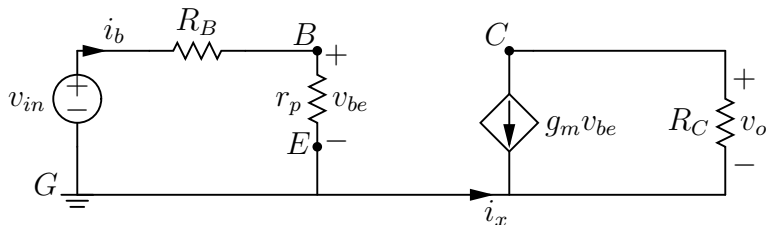
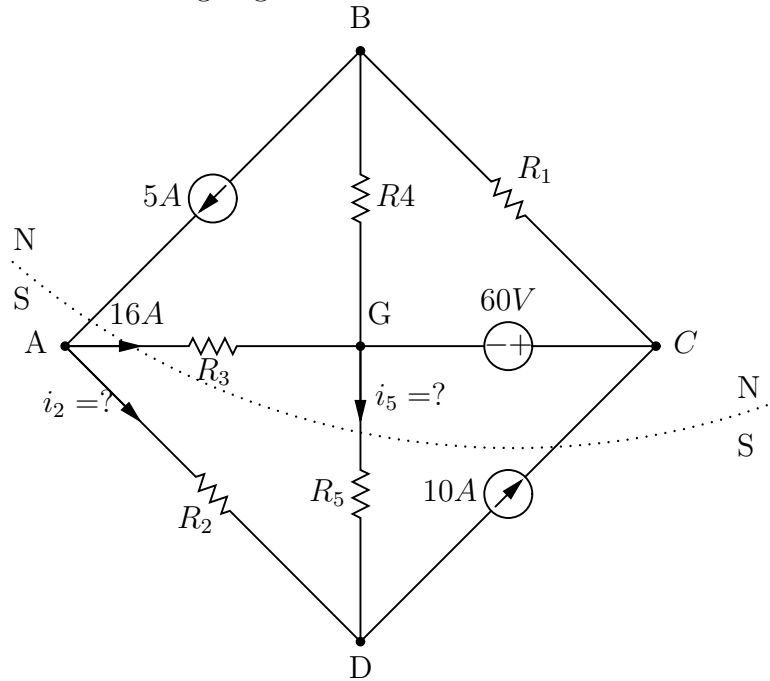
The complete solution which satisfies both the branch constraints and the intersection constraints is as shown on the right (justify it).

Cutset: Kirchhoff's Current Law (KCL) is normally written at a node where several branches meet. KCL can also be written for what are known as **cutsets**. A cutset of a circuit is a set of branches which when cut divides the circuit into two parts. Let one side of a cutset be denoted as side N and the other side as Side S. Then, KCL states that the sum of currents going from side N to side S equals the sum of currents going from side S to side N.

cutset partitions the circuit into two parts

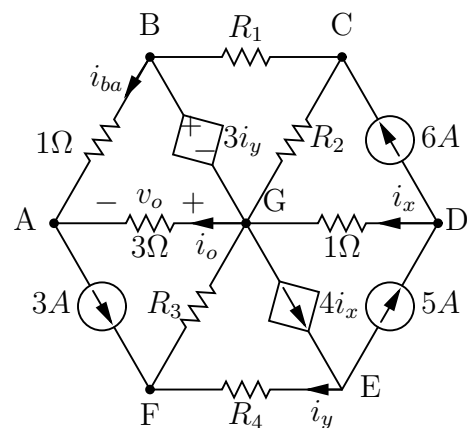


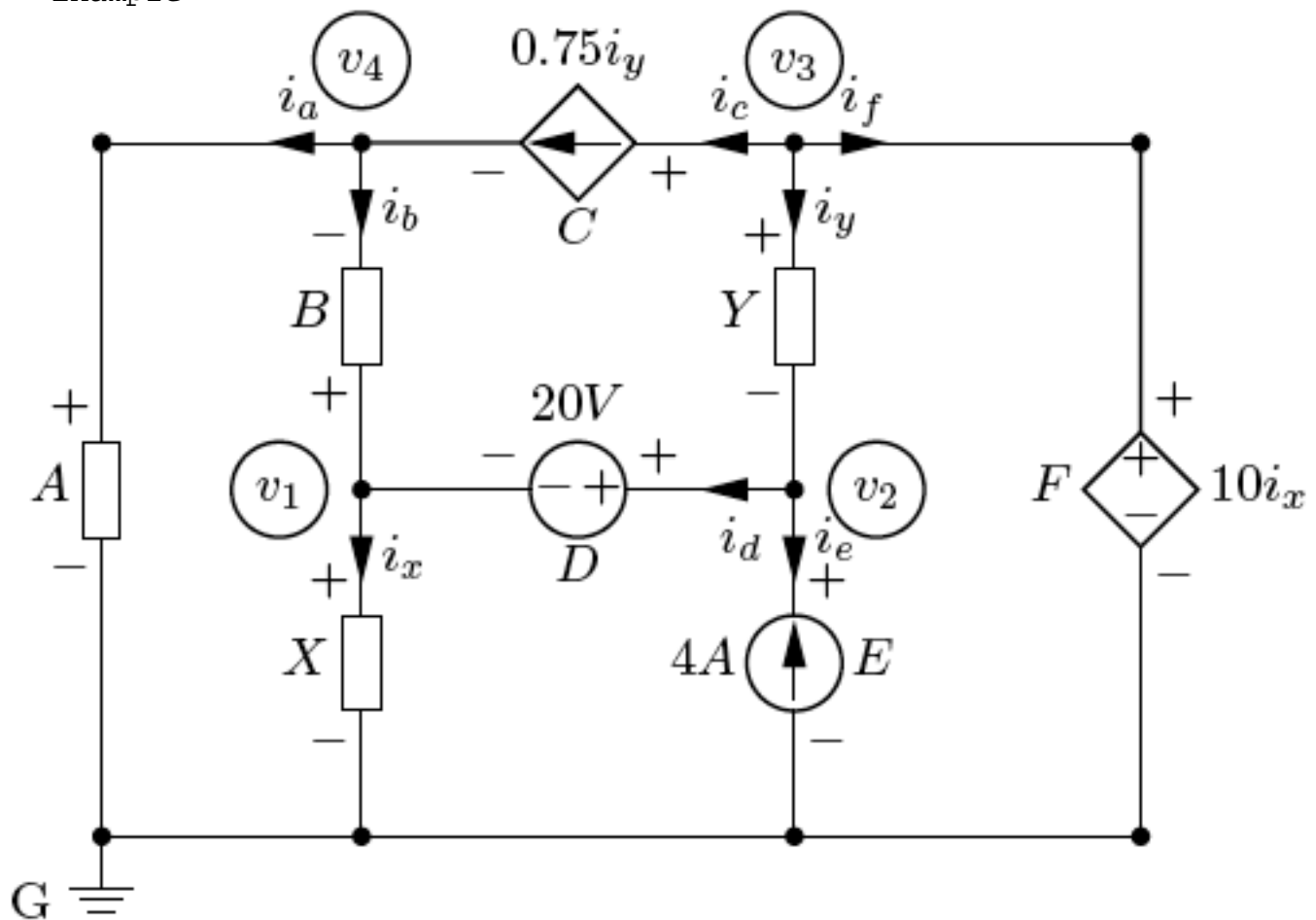
Example 2: In the circuit shown on the right, determine i_2 and i_5 by using node equations or cutset equations.



Example 3: Determine i_x in the circuit shown on the left. The answer is zero. Justify it.

Example 4: Some times one does not need to solve the entire circuit to determine just what is needed. In the circuit on the right, determine the output voltage v_o .





Given data: Node voltages are with respect to G.

$v_1 = 30V$, $v_2 = 50V$, $v_3 = 100V$, and $v_4 = 120V$, while $i_x = 10A$ and $i_y = 4A$.

Determining currents:

$i_e = -4A$; $i_y = 4A$

KCL at v_2 gives $i_d = 8A$

KCL at v_1 gives $i_b = 2A$

Dependent current source, $i_c = 0.75i_y$

KCL at v_4 gives $i_a = 1A$

KCL at v_3 gives $i_f = -7A$

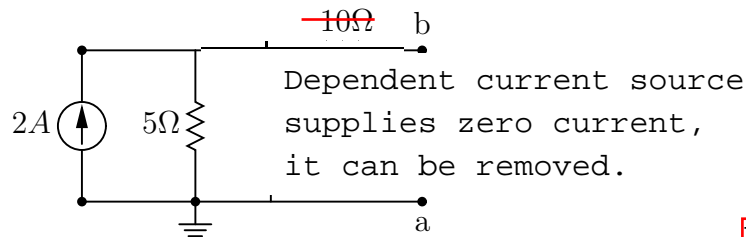
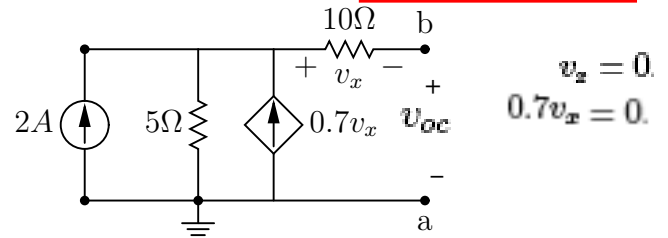
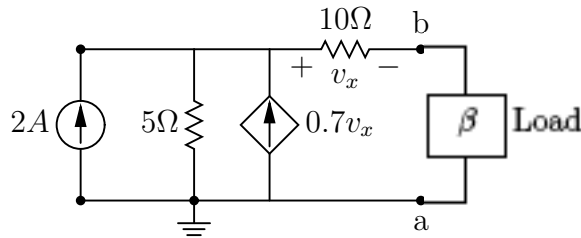
Branch voltages can be computed by taking the difference of voltages of nodes to which the branch is connected.

Element	Current A	Voltage V
<i>A</i>	1	120
<i>B</i>	2	-90
<i>C</i>	3	-20
<i>D</i>	8	20
<i>E</i>	-4	50
<i>F</i>	-7	100
<i>X</i>	10	30
<i>Y</i>	4	50

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Determination of open circuit voltage v_{oc} :

We note that there is no current through 10Ω resistance. Hence $v_x = 0$. Thus v_{oc} is same as v_{oc} . This implies that the dependent source current is zero. It is also easy to see that the 2 A current source simply supplies the 5Ω resistance establishing 10 V across it. The open circuit voltage at the terminals a and b is the same as the voltage across the 5Ω resistance, that is 10 V . Thus $v_{oc} = 10\text{ V}$.



Load is
short circuited

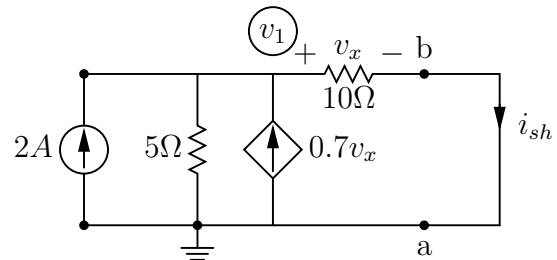
Determination of Short Circuit Current i_{sh} : Consider the circuit shown where the terminals a and b are short circuited. Mark the unknown node voltage as v_1 with respect to the ground. Note that $v_x = v_1$. We can then write the relevant KCL equation as

$$\frac{v_1}{10} + \frac{v_1}{5} = 0.7v_x + 2 = 0.7v_1 + 2.$$

By solving the above equation,

we get $v_1 = -5\text{ V}$.

Thus $i_{sh} = \frac{v_1}{10} = -0.5\text{ A}$.



Home-work: try to do the problem yourself before reading the solution given on the next page.

Solution: Write the KCL equations at the following nodes in that order to solve for each branch current:

Node Q, Node R,
Node L, Node M, Node P, Node N,
Node G, Node H, Node K, Node J,
Node C, Node F, Node E
Node B, Node A, and Node D.

One of the above equations just acts as a check or verification.

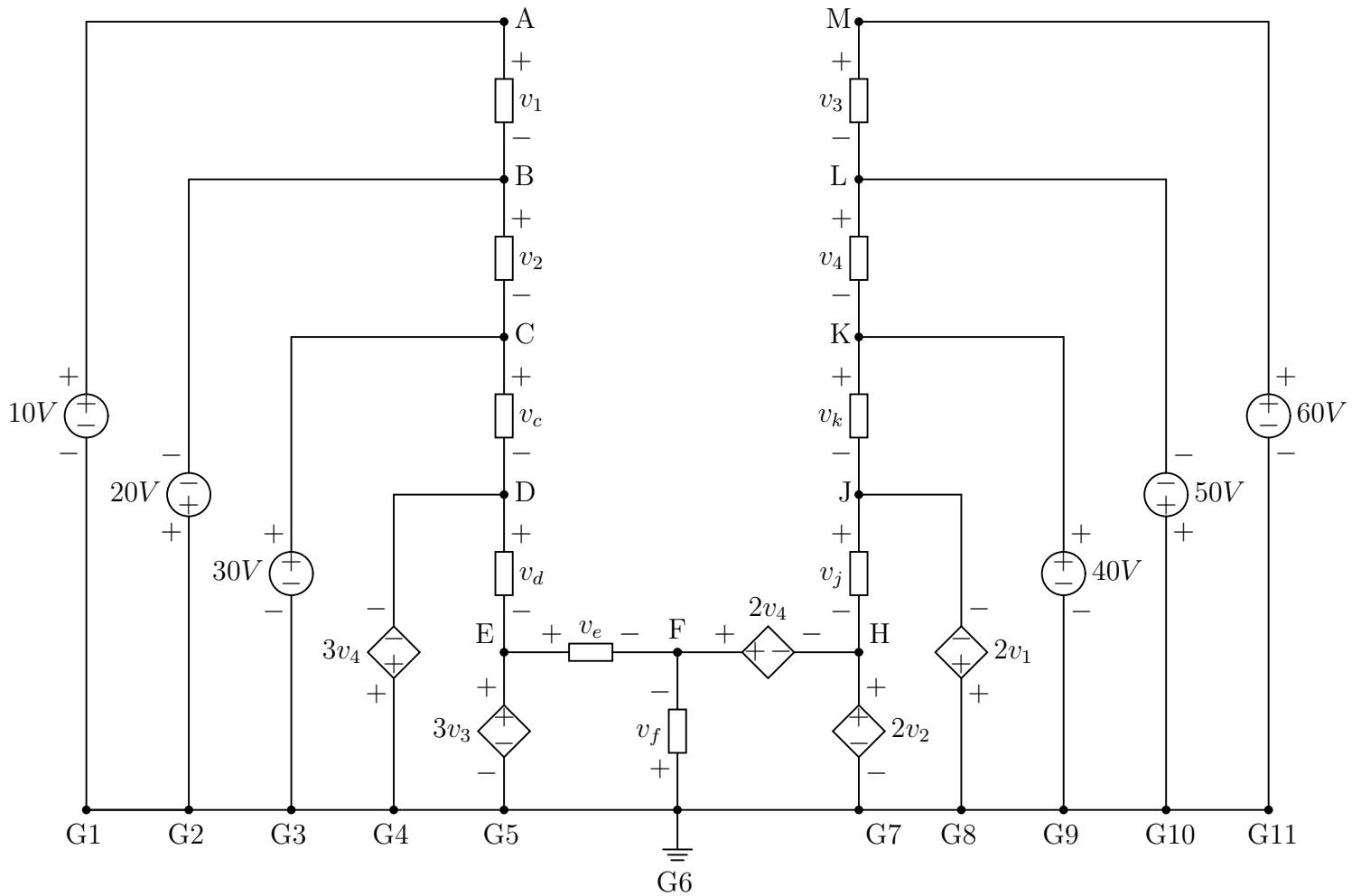
The answers are $i_1 = 1$ A, $i_2 = 1$ A, $i_3 = -3$ A, $i_4 = -2$ A, $i_a = 2$ A, $i_b = 2$ A, and $i_c = -4$ A.

In this problem, we just tried to illustrate Kirchoff's current law. We could solve here for the required currents by using only Kirchoff's current law. In a general circuit analysis we need both Kirchoff's current and voltage laws to determine the required variables. We will learn how to use these laws as we progress with the course.

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Home-work: try to do the problem yourself before reading the solution given on the next page.

Consider the circuit shown. We are interested in determining v_c , v_d , v_e , v_f , v_j , and v_k . Use Kirchhoff's voltage law systematically and repetitively to determine them.



Solution: Write the KVL equations for the following loops in that order to solve for each branch voltage:

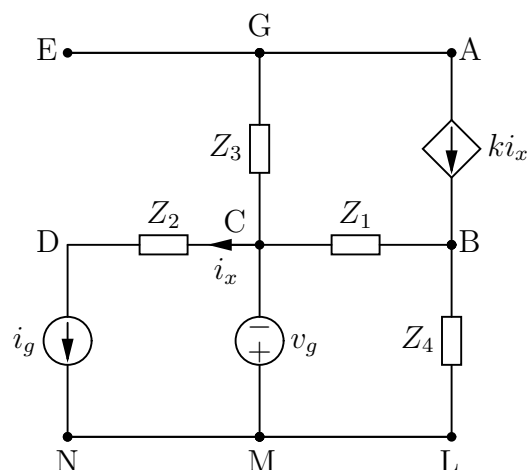
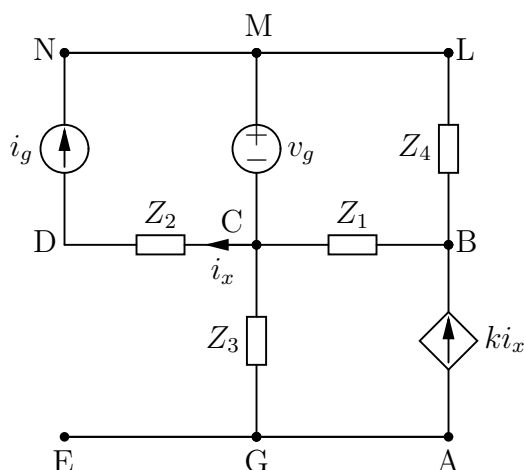
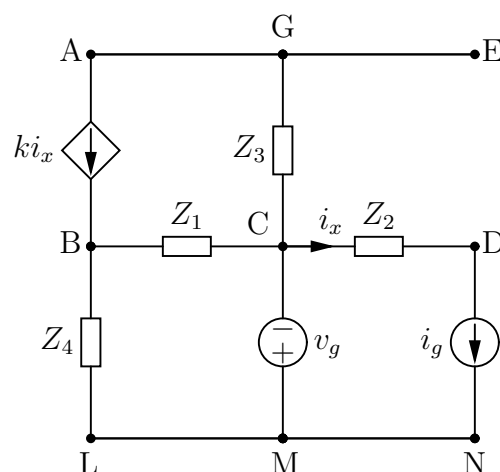
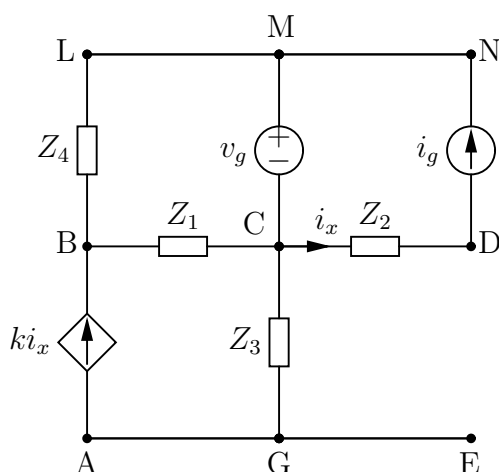
loop G1ABG2G1, loop G2BCG3G2, loop G11MLG10G11, loop G10LKG9G10,
 loop G3CDG4G3, loop G4DEG5G4, loop G9KJG8G9, loop G8JHG7G8,
 loop G7HFG6G7, and loop G5EFG6G5.

The answers are $v_1 = 30$ V, $v_2 = -50$ V, $v_3 = 110$ V, $v_4 = -90$ V, $v_c = -240$ V, $v_d = -60$ V, $v_e = 610$ V, $v_f = 280$ V, $v_j = 40$ V, and $v_k = 100$ V.

In this problem, we just tried to illustrate Kirchoff's voltage law. We could solve here for the required voltages by using only Kirchoff's voltage law. In a general circuit analysis we need both Kirchoff's current and voltage laws to determine the required variables. We will learn how to use these laws as we progress with the course.

Tracing Circuits – Example-1

Show or argue that the following four circuits are equivalent. Do it yourself first before reading the solution given below.



Note: Note that how the circuit is drawn is not important although visually one could feel better with one way of drawing the circuit than the other. What is important is what is connected between any two designated nodes.

Solution: Although the physical configuration or layout of each circuit is different, all the above circuits have exactly the same nodes, and moreover in between any two given nodes exactly the same branches exist. As such, all the circuits are electrically equivalent.

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This design problem is a home-work which will be collected and graded.

Consider the circuit shown. By writing a KCL at an appropriate node, determine i_s .

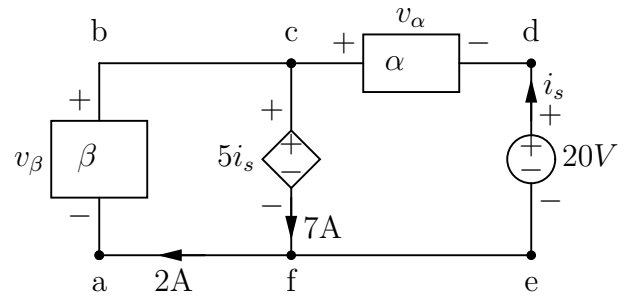


Figure 1

Determine the voltage across the dependent source in the direction shown.

By writing a KVL around an appropriate loop, determine the voltage v_α across the element α in the direction shown.

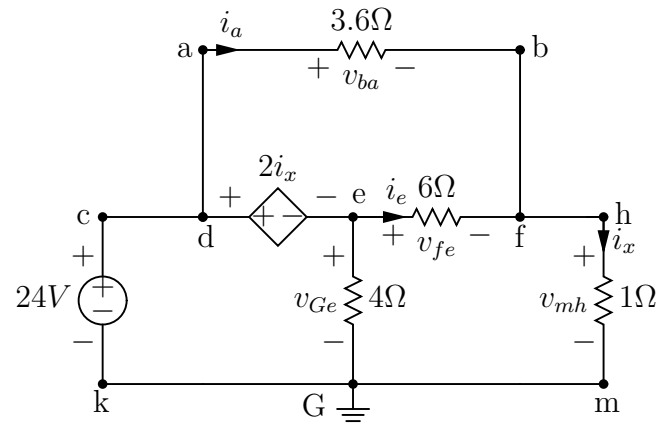
By writing a KVL around an appropriate loop, determine the voltage v_β across the element β in the direction shown.

Fill the following table regarding power consumption or generation. A particular element either consumes or generates power. You must put the appropriate value for a particular element in **only one** of the unfilled columns whichever is appropriate.

Element	Power Consumed	Power Generated
Element α		
Element β		
Independent Source		
Dependent Source		

Bridged Tee— Example

Consider the so called bridged T circuit that occurs commonly in applications. Some one partially solved the circuit shown on the right and determined v_{Ge} as 12 V. Determine the following by using appropriately, Ohm's law, KVL, and KCL. At each step, state the law you used.



Determine the voltage of the controlled source $2i_x$ and then determine the controlling current i_x . You may write the KVL for the loop kcdeGk.

After knowing i_x , determine v_{mh} .

Determine v_{fe} by using an appropriate KVL (look for a closed path which by now has only v_{fe} as unknown).

Determine i_e (Use Ohm's law, note that v_{fe} is known by now).

Determine i_a by using the KCL at the node h.

Proceeding in the same way solve for the current in each branch (mark the current values on the circuit) and verify whatever you have done above is correct. That is KCL equation at each node is satisfied and all three meshes, kcdeGk, efhmGe, and abfed, satisfy KVL equations.

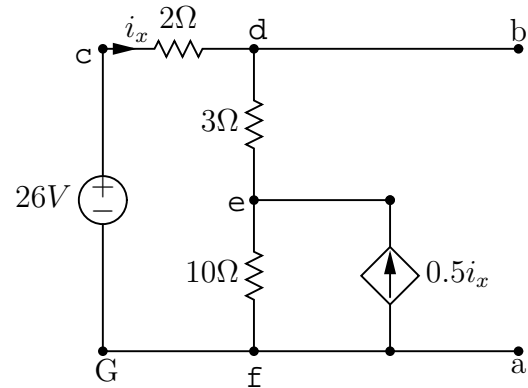
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This Home-Work problem is collected and graded

Determination of voltage of node b with respect to node a:

This can be done by a number of ways. At this time, we can use KCL and KVL judiciously in a straight forward way.

Step 1: Write a KCL at node e to determine the current in 10Ω resistance in terms of the variable i_x (Note that i_x is yet unknown).



Step 2: Write a KVL around the closed-loop GcdefG. Solve it to determine i_x .

Step 3: Walk along the path afGcdb and algebraically add the voltages encountered to determine the voltage of node b with respect to node a.

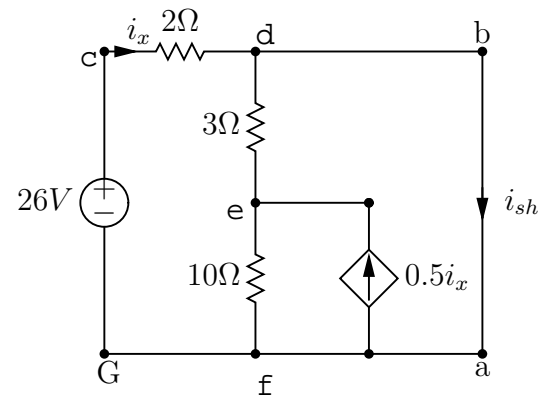
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HW 4 32

Determination of short circuit current i_{sh} :

This can be done by a number of ways. At this time, we can use KCL and KVL judiciously in a straight forward way.

Step 1: Write a KVL for the closed-loop afGcdba to determine i_x .



Step 2: Write a KCL at node d to determine the current in 3Ω resistance in terms of i_x and i_{sh} (Note that the variable i_{sh} is yet unknown).

Step 3: Write a KCL at node e to determine the current in 10Ω resistance in terms of i_x and i_{sh} (Note that the variable i_{sh} is yet unknown).

Step 4: Write a KVL for the closed-loop afedba and determine i_{sh} .