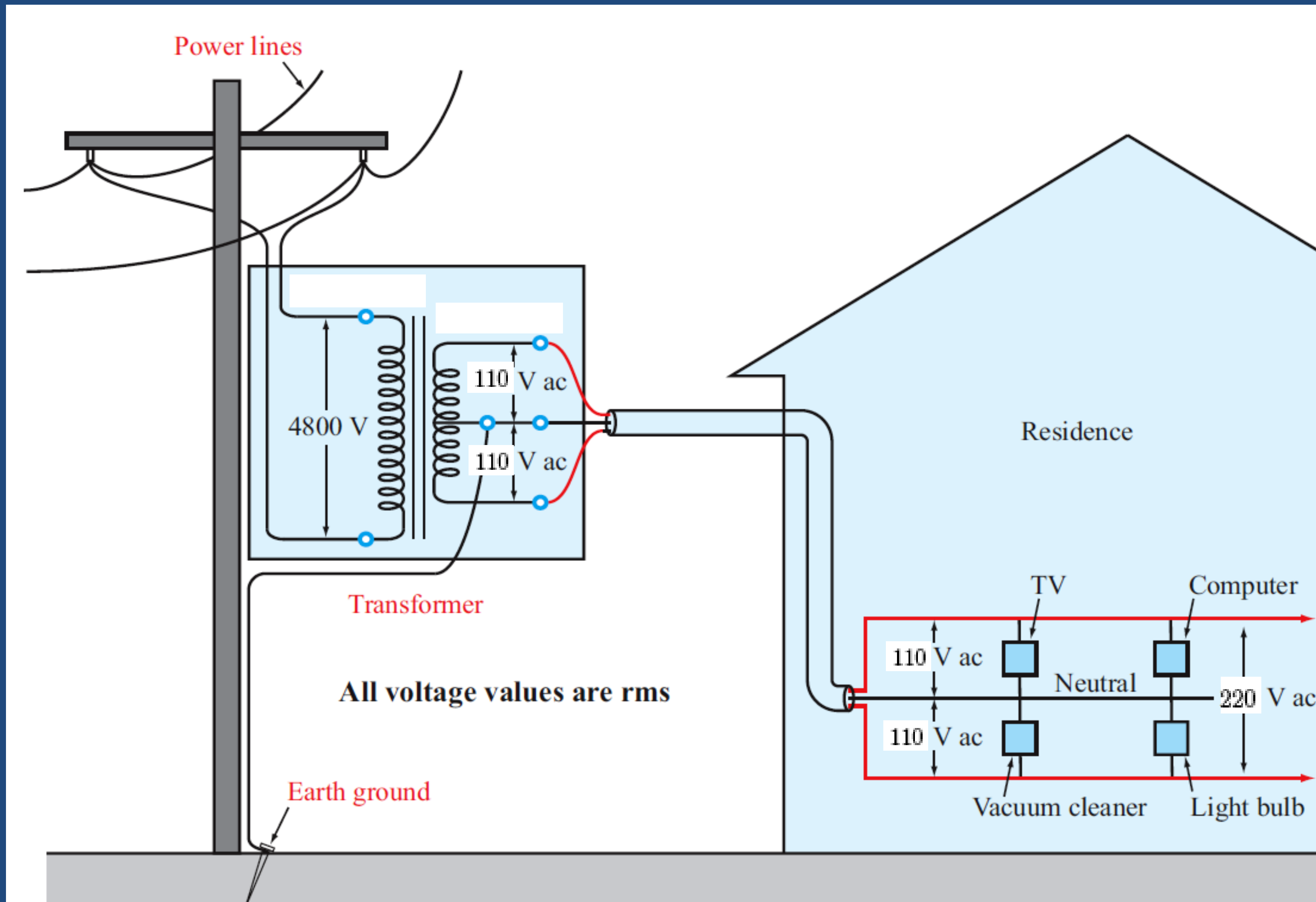
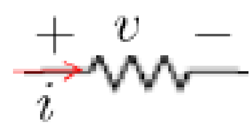

Power lines
4800 V
110 V ac
110 V ac
Transformer
All voltage values are rms
Earth ground
Residence
TV
Computer
110 V ac
Neutral
220 V ac
110 V ac
Vacuum cleaner
Light bulb

POWER

Effective Value of a time varying signal

A wall power outlet gives 110 V.
However, it is a time varying sinusoidal signal.
Then, what do we mean by saying it is 110 V?
We mean by it that it has the same **power delivering capability** of a constant (DC) signal of 110 V has.
How do we define the **power delivering capability** of a time varying signal?
It is defined on one ohm basis. To explain this, consider a resistance.
The voltage versus current relationship of a resistance is

$v(t) = i(t)R.$



Thus,

the power delivered to a resistance R is given by $= v(t)i(t) = i^2(t)R = \dfrac{v^2(t)}{R}$.

If $R = 1\Omega$, power delivered equals the **square of the signal**.

The **power delivering capability** of a time varying signal $x(t)$ on one ohm basis is normally called simply as the **Power of the signal**, and it simply equals $x^2(t)$. If the signal $x(t)$ varies with respect to time, so does $x^2(t)$.

The **Effective Value** of a time varying signal is that value of a DC signal that can deliver the same amount of power as the time varying signal does on the **average**.

Let us consider a periodic signal $x(t)$ of period T. Then, the average power of the signal $x(t)$ over the period T can be computed as

$$\text{Average of } p(t) \text{ over the period } T = \frac{1}{T}\int_0^T x^2(t)dt.$$

For a constant signal (DC signal) E, the average as well as the instantaneous power are the same as E^2.

The effective value E of a time varying periodic signal $x(t)$ with period T is given by

$$E^2 = \frac{1}{T}\int_0^T x^2(t)dt.$$

Hence

$$\text{Effective value of } x(t) = \sqrt{\frac{1}{T}\int_0^T x^2(t)dt}.$$

Clearly, the effective value of $x(t)$ is computed by performing **three operations** on $x(t)$.

These three operations are **Root, Mean, and Square** of $x(t)$ as illustrated below,

$$\text{Effective value of } x(t) = \underset{\text{Root} \nearrow}{\sqrt{\underbrace{\frac{1}{T}\int_0^T}_{\text{Mean}} \underbrace{x^2(t)}_{\text{Square}}\, dt}}.$$

Because of the significance of the three operations, Root, Mean, and Square, the *effective value is also normally called as the **R**oot **M**ean **S**quare value or for short **RMS** value.*

Determination of the RMS value of a sinusoidal signal:
Let us consider a sinusoidal signal

$$x(t) = A\cos(\omega t + \theta).$$

Then, the RMS value of $x(t)$ is given by

$$\sqrt{\frac{1}{T}\int_0^T x^2(t)dt} = \sqrt{\frac{1}{T}\int_0^T A^2\cos^2(\omega t+\theta)\, dt}$$

$$= \sqrt{\frac{A^2}{2T}\int_0^T [1+\cos(2\omega t + 2\theta)]\, dt}$$

$$= \sqrt{\frac{A^2}{2T}\int_0^T dt + \frac{A^2}{2T}\int_0^T \cos(2\omega t+2\theta)\, dt}$$

$$= \sqrt{\frac{A^2}{2T}\int_0^T dt} = \frac{A}{\sqrt{2}}.$$

In the above, the second equality follows by the trigonometric equality $2\cos^2(\alpha) = 1+\cos(2\alpha)$, and the rest follows by obvious integrations. It is worth noting that the average of a sinusoidal signal over a period $T = \frac{2\pi}{\omega}$ is indeed zero.

The above development leads to a very commonly used expression,
the RMS value of a sinusoidal signal $= \frac{\text{Amplitude}}{\sqrt{2}}$.
Note that the RMS value of a sinusoidal signal depends only on its amplitude, but neither on its frequency nor on its phase angle.

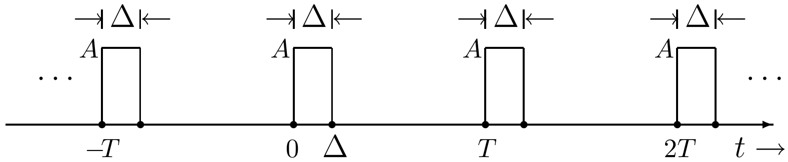
Phasors: We introduced and worked with Amplitude of a sinusoidal signal as the magnitude of its phasor. Instead, we can work with **RMS phasors** as well where we use the RMS value as the magnitude of the phasor.

Time varying Sinusoid		Amplitude Phasor	RMS Phasor
$A\cos(\omega t+\theta) = \sqrt{2}A_{RMS}\cos(\omega t+\theta)$	$\Rightarrow$	$A\angle\theta$	$A_{RMS}\angle\theta.$

One can work **consistently** with either Amplitude Phasors or RMS Phasors. It is common to work with RMS Phasors.

Example 1: Determine the RMS value of the periodic pulse train $p(t)$ shown below where the width of each pulse Δ equals $0.2T$. (The pulse train $p(t)$ is formed when a DC signal A is **ON** for a period Δ and **OFF** for a period $T-\Delta$ and so on. In this regard Δ is called the duty period.)

Periodic pulse train $p(t)$



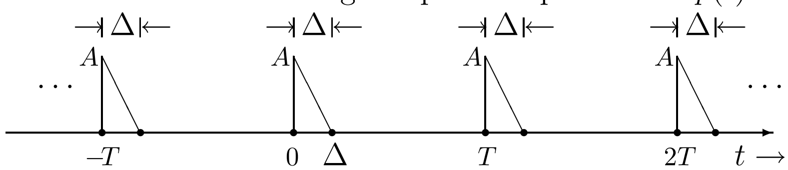
$$\text{The RMS value of } p(t) = \sqrt{\frac{1}{T}\int_0^T x^2(t)dt} = \sqrt{\frac{A^2}{T}\int_0^{0.2T} dt}$$
$$= \sqrt{\frac{A^2}{T}(0.2T)} = \sqrt{\frac{A^2}{5}} = \frac{A}{\sqrt{5}}$$

Home-Work: Based on the above example, construct a signal whose RMS value equals

$$\frac{\text{Peak value of the signal}}{2\sqrt{2}}.$$

Example 2: Determine the RMS value of the triangular periodic pulse train $p(t)$ shown below where the width of each pulse Δ equals $0.2T$.

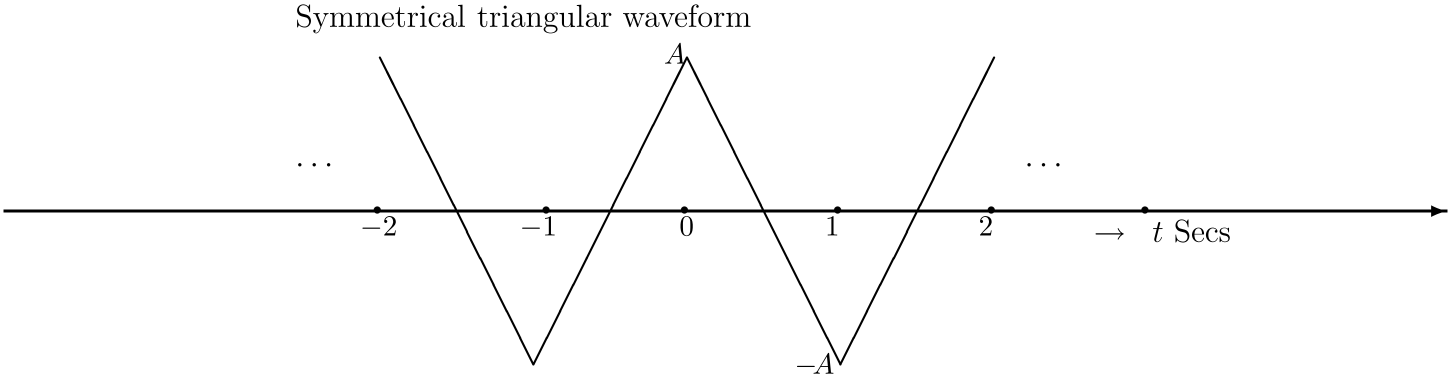
Triangular periodic pulse train $p(t)$



$$\text{The RMS value of } p(t) = \sqrt{\frac{1}{T}\int_0^T x^2(t)dt} = \sqrt{\frac{1}{T}\int_0^{0.2T} \left(\frac{A}{0.2T}(0.2T-t)\right)^2 dt}$$
$$= \sqrt{\frac{A^2}{3T}(0.2T)} = \sqrt{\frac{A^2}{15}} = \frac{A}{\sqrt{15}}$$

Home-Work: Show that the RMS value of the symmetrical periodic triangular signal given below equals $\frac{A}{\sqrt{3}}$.

Symmetrical triangular waveform



Average Value

Calculus based, read yourself

The average value of a periodic function $x(t)$ with period T is given by

$$X_{av} = \frac{1}{T}\int_0^T x(t)\,dt. \tag{8.5}$$

Sine wave

$$V_{av} = \frac{1}{T}\int_0^T V_m \sin\frac{2\pi t}{T}\,dt$$

$$= \frac{V_m}{T}\left(-\frac{T}{2\pi}\right)\cos\frac{2\pi t}{T}\Big|_0^T = -\frac{V_m}{2\pi}[1-1] = 0.$$

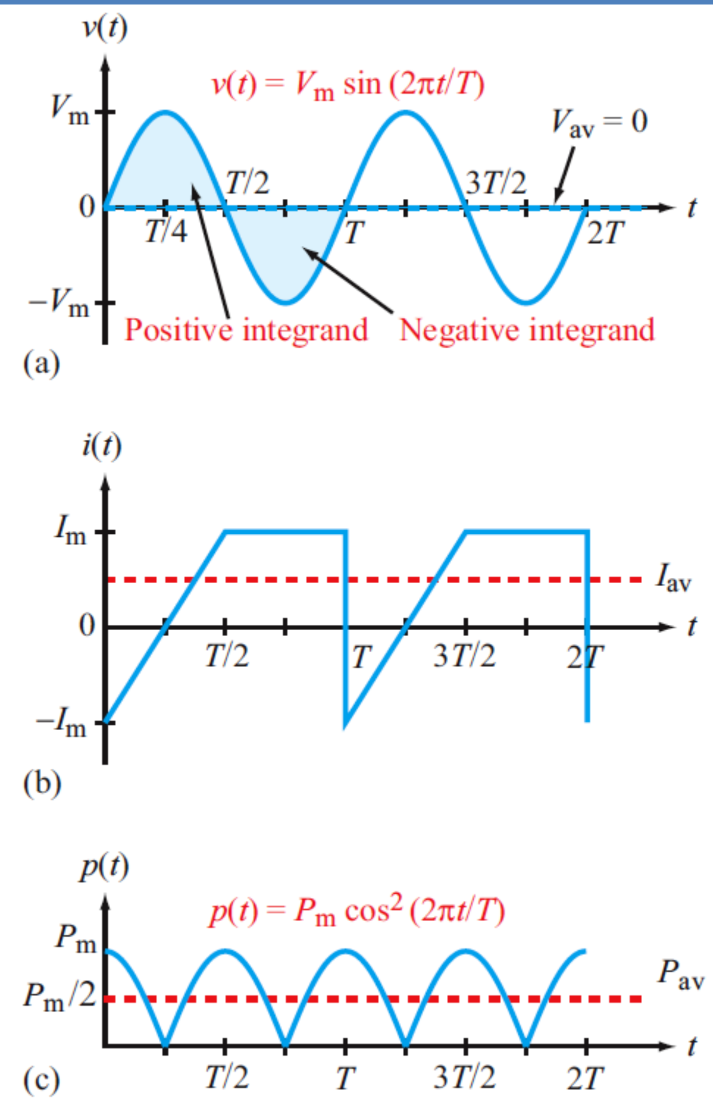
Truncated sawtooth

$$i(t) = \begin{cases} \left(\frac{4t}{T} - 1\right) I_m & \text{for } 0 \le t \le \frac{T}{2}, \\ I_m & \text{for } \frac{T}{2} \le t < T. \end{cases} \tag{8.8}$$

By Eq. (8.5), the average value of $i(t)$ is

$$I_{av} = \frac{1}{T}\int_0^T i(t)\,dt = \frac{1}{T}\left[\int_0^{T/2}\left(\frac{4t}{T} - 1\right) I_m\,dt + \int_{T/2}^T I_m\,dt\right]$$

$$= \frac{I_m}{2}. \tag{8.9}$$



We worked on this while discussing RMS Value

Average Value fo $p(t) = P_m \cos^2(2\pi t/T)$

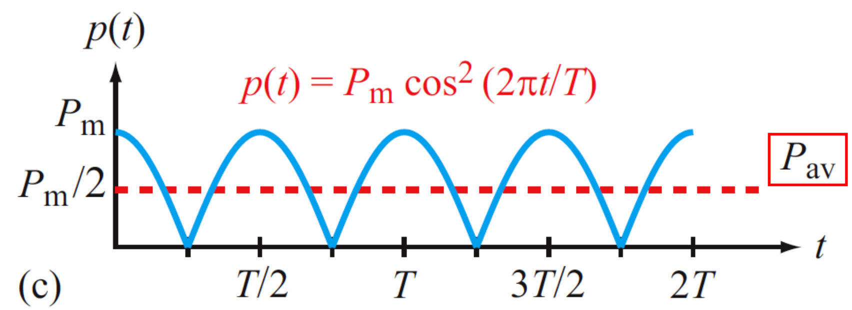
$$P_{av} = \frac{1}{T}\int_0^T P_m \cos^2\left(\frac{2\pi t}{T}\right) dt.$$

The integration is facilitated by applying the trigonometric relation

$$\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x,$$

which leads to the final result

$$P_{av} = \frac{P_m}{2}.$$



Average power &
Power fluctuations

$$\frac{1}{T}\int_0^T \cos^2\left(\frac{2\pi nt}{T} + \phi_1\right) dt = \frac{1}{2}$$

$$\frac{1}{T}\int_0^T \sin^2\left(\frac{2\pi nt}{T} + \phi_2\right) dt = \frac{1}{2}$$

These properties hold true for any values of φ_1 and φ_2

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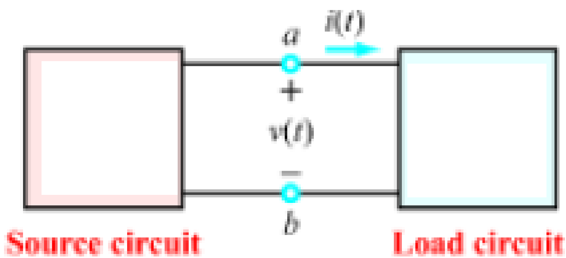
Power Calculations

Consider the circuit configuration shown where Source circuit delivers power to the Load circuit. Either Source circuit or Load circuit could contain independent or dependent sources or circuit elements such as resistances, capacitances, inductances, and mutual inductances or their combinations. Consider the terminals a and b. Let the voltage across the terminals a and b and the current delivered by the Source circuit to the Load circuit be respectively

$$v(t) = V_M \cos(\omega t + \theta_v) \quad \text{and} \quad i(t) = I_M \cos(\omega t + \theta_i).$$

For the directions of $v(t)$ and $i(t)$ as shown, Source circuit delivers instantaneous power $p(t) = v(t)i(t)$ to the Load circuit which consumes it.



For the directions of voltage and current as marked, Source Circuit delivers power to the Load Circuit.

Instantaneous power consumed or generated is given by

Two interpretations arise from this. Average power & Power fluctuations

$$p(t) = v(t)i(t) = V_M I_M \cos(\omega t + \theta_v)\cos(\omega t + \theta_i).$$

The above expression can be written in a number of ways by utilizing the following trigonometric equalities:

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}\,[\cos(\alpha+\beta) + \cos(\alpha-\beta)] \quad \text{and} \quad \cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta).$$

We rewrite $p(t)$ as follows:

$$\begin{aligned}
p(t) &= v(t)i(t) = V_M I_M \cos(\omega t + \theta_v)\cos(\omega t + \theta_i) \\
&= \frac{1}{2}V_M I_M \cos(\theta_v - \theta_i) + \frac{1}{2}V_M I_M \cos(2\omega t + \theta_v + \theta_i) \\
&= \frac{1}{2}V_M I_M \cos(\theta_v - \theta_i) + \frac{1}{2}V_M I_M \cos(2\omega t + 2\theta_i + \theta_v - \theta_i) \\
&= \underbrace{P}_{\text{Average real power}} + \frac{1}{2}V_M I_M \cos(\theta_v - \theta_i)\cos(2\omega t + 2\theta_i) - \frac{1}{2}V_M I_M \sin(\theta_v - \theta_i)\sin(2\omega t + 2\theta_i) \\
&= \underbrace{P}_{\text{Average real power}} + \underbrace{P\cos(2\omega t + 2\theta_i)}_{\text{Power fluctuations in a resistance}} - \underbrace{Q\sin(2\omega t + 2\theta_i)}_{\text{Power fluctuations in a reactance}}.
\end{aligned}$$

Constant

Periodic power fluctuations with zero average.

Terminology:

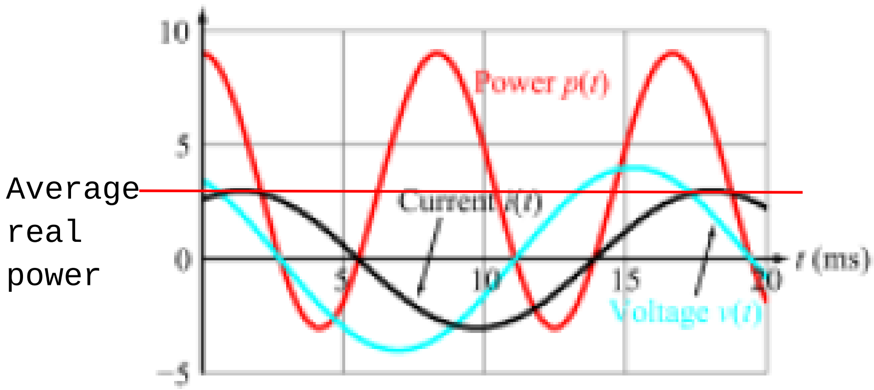
Real Power $\quad P = \frac{1}{2}V_M I_M \cos(\theta_v - \theta_i) = V_{RMS} I_{RMS}\cos(\theta_v - \theta_i) =$ Average real power

Reactive Pwer $\quad Q = \frac{1}{2}V_M I_M \sin(\theta_v - \theta_i) = V_{RMS} I_{RMS}\sin(\theta_v - \theta_i) =$ Peak of reactive power.

Power factor $= \cos(\theta_v - \theta_i)$, Reactive factor $= \sin(\theta_v - \theta_i)$.

The phase angle relationship $\theta_v - \theta_i$ plays an important role in power relationships.

Inductance & capacitance store energy



$$v(t) = 4\cos(377t + 30^\circ) \quad \text{(V)},$$
$$i(t) = 3\cos(377t - 30^\circ) \text{ (A)}$$
$$p(t) = v(t)\ i(t).$$

Next, we consider resistance, inductance, and capacitance each separately and simplify the expressions for P and Q. The basis of simplification is the knowledge of $\theta_v - \theta_i$ for each element.

Resistance:

$\theta_v - \theta_i = 0$, the voltage and current associated with a resistance are in phase.

$P = V_{RMS}I_{RMS} = \frac{V_{RMS}^2}{R} = I_{RMS}^2 R$, Note that $\cos(\theta_v - \theta_i) = \cos(0) = 1$.

$Q = 0$, Note that $\sin(\theta_v - \theta_i) = \sin(0) = 0$.

Resistance consumes only real average power. It does not consume any reactive power as there is no energy storage associated with a resistance.

Inductance:

$\theta_v - \theta_i = 90^0$, the current in an inductance lags its voltage by 90^0.

$P = 0$, Note that $\cos(\theta_v - \theta_i) = \cos(90^0) = 0$.

$Q = V_{RMS}I_{RMS} = \frac{V_{RMS}^2}{\omega L} = I_{RMS}^2\, \omega L$, Note that $\sin(\theta_v - \theta_i) = \sin(90^0) = 1$.

Inductance consumes only reactive power. On the average, it does not consume any real power as it merely stores in a magnetic field whatever power is consumed in some time interval and delivers it back in another interval (This illustration was done as Home-Work). By convention, reactive power of an inductive circuit is positive.

Capacitance:

$\theta_v - \theta_i = -90^0$, the current in a capacitance leads its voltage by 90^0.

$P = 0$, Note that $\cos(\theta_v - \theta_i) = \cos(-90^0) = 0$.

$Q = -V_{RMS}I_{RMS} = -V_{RMS}^2\, \omega C = -\frac{I_{RMS}^2}{\omega C}$, Note that $\sin(\theta_v - \theta_i) = \sin(-90^0) = -1$.

Capacitance consumes only reactive power. On the average, it does not consume any real power as it merely stores in an electric field whatever power is consumed in some time interval and delivers it back in another interval (This illustration was done as Home-Work). By convention, reactive power of a capacitive circuit is negative.

RMS Phasors

Complex power S:
Let $V = V_{RMS}\angle\theta_v$ be the RMS voltage phasor, and $I = I_{RMS}\angle\theta_i$ be the RMS current phasor, both associated with the same element. Then,

$$P = V_{RMS}I_{RMS}\cos(\theta_v - \theta_i)$$
$$Q = V_{RMS}I_{RMS}\sin(\theta_v - \theta_i).$$

$$VI^* = V_{RMS}\angle\theta_v I_{RMS}\angle -\theta_i = V_{RMS}I_{RMS}\angle\theta_v - \theta_i$$

$S = P + jQ = VI^*$

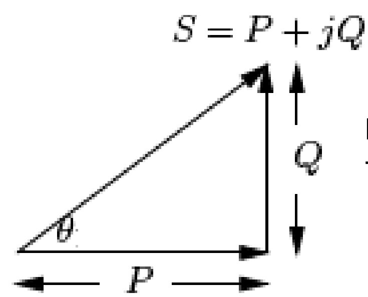
Magnitude of S is $V_{RMS}I_{RMS}$

Phase Angle of S is $\theta = \theta_v - \theta_i$

Define **Complex power** S as

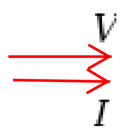
Complex number → $S = P + jQ$. ← Real numbers

A triangle called *Power Triangle* can be drawn with P as real part and Q as the imaginary part. It is easy to see that $S = P + jQ = VI^*$.

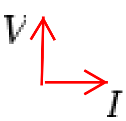


Power Triangle

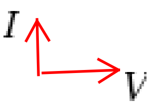
We consider some special cases:



Resistance: In this case, both V and I are in phase, hence $\theta_v - \theta_i = 0$.
Only $P = I_{RMS}^2 R = \frac{V_{RMS}^2}{R}$ exists and $Q = 0$. Triangle coalesces to a simple horizontal line.



Inductance: In this case, I lags V by 90^0, hence $\theta_v - \theta_i = 90^0$.
Only $Q = I_{RMS}^2 \omega L = \frac{V_{RMS}^2}{\omega L}$ exists and $P = 0$. Triangle coalesces to a simple vertical line.



Capacitance: In this case, I leads V by 90^0, hence $\theta_v - \theta_i = -90^0$.
Only $Q = -V_{RMS}^2 \omega C = -\frac{I_{RMS}^2}{\omega C}$ exists and $P = 0$. Triangle once again coalesces to a simple vertical line.

TABLE

Three Power Quantities and Their Units

QUANTITY	UNITS
Complex power	volt-amps
Average power	watts
Reactive power	vars

For sources, only way to calculate Complex power S is by VI^*. One cannot do any other way.

Conservation of Complex power:
Complex power in any circuit is conserved. That is, whatever is generated in some branches is consumed by the other branches of the circuit.

Maximum Power Transfer to a load: To be discussed later on.

Power triangle relates P Watts, Q VArs, $|S|$ VAs, and angle θ.
If we know any two of them, we can compute the others by utilizing triangle properties. For example, if we know P and θ, we get $|S| = \frac{P}{\cos\theta}$, and $Q = \frac{P\sin\theta}{\cos\theta} = P\tan\theta$.
Similarly, if we know $|S|$ and θ, we get $P = |S|\cos\theta$ and $Q = |S|\sin\theta$

Summary and Terminology

Read as HW

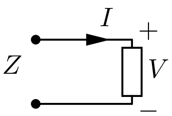
A **sinusoidal signal** is of the form $A\cos(\omega t+\theta)$ or $\sqrt{2}A_{\text{RMS}}\cos(\omega t+\theta)$,
A is the amplitude of the sinusoid and A_{RMS} is the RMS value of it,
θ is the phase angle of the sinusoid, ω is the radian frequency of the sinusoid; $\omega = 2\pi f$ where f is the frequency in Hertz; also $f=\frac{1}{T}$ where T is the period of the sinusoid.

For a given ω, a sinusoidal signal is **characterized** by its amplitude A (or by its RMS value A_{RMS}) and its phase angle θ. Thus, we can **represent** the sinusoid by a **phasor**. One can use either Amplitude phasor or RMS phasor,

$$\mathbf{A}\underline{/\theta} \;\Leftrightarrow\; \mathbf{A}\cos(\omega\mathbf{t}+\theta) \quad \text{or equivalently} \quad \mathbf{A}_{\text{RMS}}\underline{/\theta} \;\Leftrightarrow\; \sqrt{\mathbf{2}}\mathbf{A}_{\text{RMS}}\cos(\omega\mathbf{t}+\theta).$$

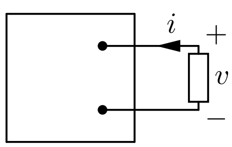
For sinusoidal inputs, in order to determine steady state values of circuit variables, one transforms the **time-domain** circuit to **phasor domain**, and then uses phasor analysis using the concept of **impedances**. Once a required phasor value is known, it can be transformed back to time-domain. Note that the ratio of a voltage phasor to the current phasor of an element (which is not a source) is called the impedance of the element, and is denoted by Z. The **generalized Ohm's law** $\frac{V}{I}=Z$ renders the phasor analysis conceptually exactly same as the analysis with resistances alone (chapters 1 to 5 of the text book).

$$\frac{V}{I}=Z$$

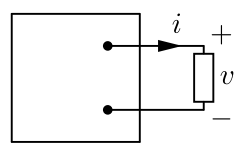


Generalized Ohm's law

Instantaneous Power $= p(t) = v(t)i(t)$



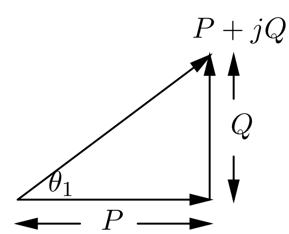
Power generated by the small box $= vi$



Power consumed by the small box $= vi$

Power computations are done easily in terms of phasors.
Complex Power Consumed or generated by an element $= P + jQ = VI^*$
where V and I are RMS phasors, $V = V_{\text{RMS}}\underline{/\theta_v}$ and $I = I_{\text{RMS}}\underline{/\theta_i}$.

Average real power $P = V_{\text{RMS}}I_{\text{RMS}}\cos(\theta_v-\theta_i)$ Watts
Reactive power $Q = V_{\text{RMS}}I_{\text{RMS}}\sin(\theta_v-\theta_i)$ Vars (Volt-Amp-Reactive)
Power triangle is shown on the right where $\theta_1=\theta_v-\theta_i$.
$\cos(\theta_v-\theta_i)$ is called **power factor**, and
$\sin(\theta_v-\theta_i)$ is called **reactive factor**.



Reactive power Q is positive for inductive reactive power, and negative for capacitive reactive power. Reactive power is the electrical power that oscillates between the magnetic field of an inductor and the electrical field of a capacitor; Reactive power is never converted to non-electrical power. It is absorbed in some interval and given away in some other interval. Q **is the maximum value of such power fluctuations**.

Volt-Amperes or Apparent Power:
It is the magnitude of complex power $= |S|$; VA (Volt-Ampere) is its unit.

Element	*Impedance* *Ohms*	**Average Power** **Watts**	**Reactive Power** **Vars**
Resistance R	R	$P = I_{\text{RMS}}^2 R = \frac{V_{\text{RMS}}^2}{R}$	$Q=0$
Inductance L	$j\omega L$	$P=0$	$Q = I_{\text{RMS}}^2\omega L = \frac{V_{\text{RMS}}^2}{\omega L}$
Capacitance C	$\frac{1}{j\omega C}$	$P=0$	$Q = -\frac{I_{\text{RMS}}^2}{\omega C} = -V_{\text{RMS}}^2\omega C$

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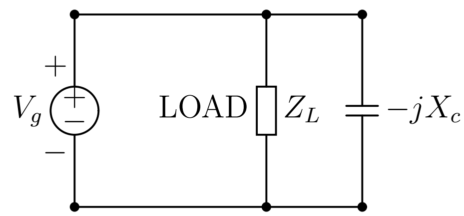
Power Factor Correction

Most of the industrial factories utilize induction motors to turn their wheels. An induction motor consumes both average real power as well as lagging (inductive) reactive power. Thus a typical industrial load has a power factor less than unity. Power companies charge for the consumption of real as well as reactive powers although at different rates. One can reduce the consumption of lagging reactive power from the power company by connecting an appropriate value of capacitance in parallel with the load. This is because a capacitance consumes a leading reactive power and thus the combined lagging and leading reactive powers can be as small as desired. This implies that the power factor of the load in parallel with a capacitance can be improved to a level as close to unity as desired. In other words, the cost of paying for the reactive power can be decreased to as small as desired. The following problem illustrates the concepts involved.

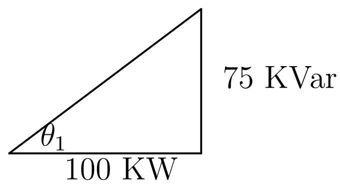
Example:

Consider the phasor domain circuit shown. Let the input voltage V_g be 220 V (RMS) at 60 Hz. The given load consumes 100 KW of real average power and has a lagging power factor of 0.8. Determine the value of capacitance C that needs to be connected in parallel with the load such that the combined power factor of the load and the capacitance in parallel with it is 0.95.

Repeat the above if combined power factor needed is unity.



The power triangle of the existing load is as shown by the triangle on the right where $\cos(\theta_1)$ is 0.8. We note that



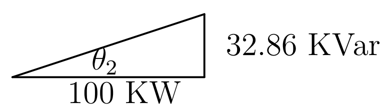
$$\cos(\theta_1) = 0.8 \quad \Rightarrow \quad \theta_1 = \angle 36.87^0, \ \sin(\theta_1) = 0.6, \text{ and } \tan(\theta_1) = 0.75.$$

Since $\tan(\theta_1) = 0.75$, the lagging reactive power consumed is 75 KVar.

We can construct a power triangle where $\cos(\theta_2)$ is 0.95. We note that

$$\cos(\theta_2) = 0.95 \quad \Rightarrow \quad \theta_2 = \angle 18.19^0, \ \sin(\theta_2) = 0.312, \text{ and } \tan(\theta_2) = 0.3286.$$

Since $\tan(\theta_2) = 0.3286$, the lagging reactive power consumed is 32.86 KVar. The triangle on the right illustrates this.



We need to have the resultant reactive power (lagging) as 32.86 KVar. This implies that the leading reactive power that needs to be supplied by the capacitance is $75 - 32.86 = 42.14$ KVar. We note that the leading reactive power that can be supplied by a capacitance of C Farads is $\omega C|V_g|^2$. Thus we have the required capacitance as

$$C = \frac{42140}{\omega|V_g|^2} = \frac{42140}{2\pi(60)|220|^2} = 2310\,\mu F.$$

If the required combined power factor is unity, the leading reactive power that can be supplied by the capacitance must annihilate the lagging reactive power of the original load. This implies that the capacitance must supply 75 KVar. In this case, we have the required capacitance as

$$C = \frac{75000}{\omega|V_g|^2} = \frac{75000}{2\pi(60)|220|^2} = 4110\,\mu F.$$

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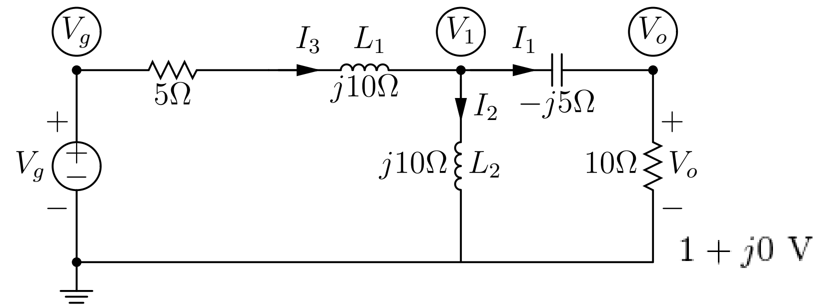
AC Power Calculation

Example 1:
Consider the phasor domain circuit shown. Let the output voltage V_o be $1 + j0$ V (RMS). Determine the power consumed or generated by each branch. Verify the principle of conservation of power in the given circuit.



We first need to solve the circuit before we compute power consumed or generated by each branch. One easy way of solving this circuit is to start at the output end on the right and proceed towards the source end while systematically calculating the currents I_1, I_2, and I_3 as well as voltages V_1 and V_g. We will compute these variables in the order of I_1, V_1, I_2, I_3, and V_g as shown below:

The current $I_1 = \frac{V_o}{10} = \frac{1}{10} = 0.1$ A.

Then, the voltage across the capacitance is $I_1(-j5) = -j0.5$ V.

This enables us to compute V_1 as $1 - j0.5$ V.

Now the current I_2 can be computed as $\frac{V_1}{j10} = \frac{1-j0.5}{j10} = -0.05 - j0.1 = -0.1118\underline{/63.43^0}$ A.

We note that $I_3 = I_1 + I_2 = 0.1 - 0.05 - j0.1 = 0.05 - j0.1 = 0.1118\underline{/-63.43^0}$ A.

Finally, we note that

$V_g = V_1 + I_3(5 + j10) = 1 - j0.5 + (0.05 - j0.1)(5 + j10) = 2.25 - j0.5 = 2.3\underline{/-12.53^0}$ V.

We note that the complex power generated by the source equals

$V_g I_3^* = (2.3\underline{/-12.53^0})(0.1118\underline{/63.43^0}) = 0.25714\underline{/50.9^0} = 0.1625 + j0.2$ VA.

This as well as other power calculations are shown in the following table.

Element	**Average Power generated watts**	**Average Power consumed watts**	**Reactive Power generated vars**	**Reactive Power consumed vars**
V_g *voltage Source*	0.1625		0.2	
5Ω *resistance*		$\lvert I_3\rvert^2 5 = 0.0625$		
$j10\Omega$ *inductance* L_1				$\lvert I_3\rvert^2 10 = 0.125$
$j10\Omega$ *inductance* L_2				$\lvert I_2\rvert^2 10 = 0.125$
$-j5\Omega$ *capacitance*				$-\lvert I_1\rvert^2 5 = -0.05$
10Ω *resistance*		$\lvert I_1\rvert^2 10 = 0.1$		
Total of each column	0.1625	0.1625	0.2	0.2

Yes, there is power balance for both real and reactive powers.

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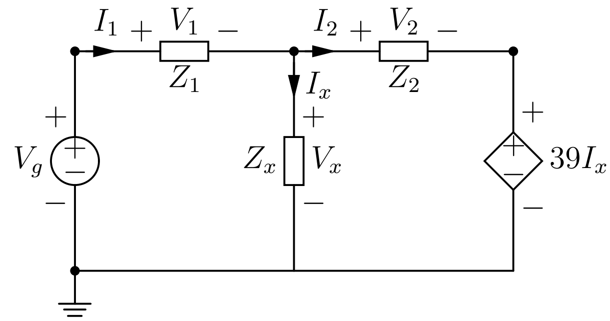
AC Power Calculation

Example 2:
Consider the phasor domain circuit shown where
$V_g = 150V$ (RMS), $Z_x = 12 - j16\ \Omega$,
$Z_1 = 1 + j2\ \Omega$, and $Z_2 = 1 + j3\ \Omega$.
Solve the given circuit, and determine the power consumed or generated by each branch. Verify the principle of conservation of power in the given circuit.



The circuit can be solved by a number of methods. We use here mesh analysis and write two mesh equations as

$$150 = I_1 Z_1 + I_x Z_x \quad \Rightarrow \quad 150 = I_1(1 + j2) + (I_1 - I_2)(12 - j16)$$
$$0 = I_x Z_x - I_2 Z_2 - 39 I_x \quad \Rightarrow \quad 0 = (I_1 - I_2)(12 - j16) - I_2(1 + j3) - 39(I_1 - I_2).$$

We can solve the above equations to get $I_1 = -26 - j52$ A and $I_2 = -24 - j58$ A.
Then, we have $I_x = I_1 - I_2 = -2 + j6$ A,
$V_1 = I_1 Z_1 = 78 - j104$ V, $V_2 = I_2 Z_2 = 150 - j130$ V, and $V_x = I_x Z_x = 72 + j104$ V.
The complex power generated or consumed can be computed as VI^*. The computations are shown in the following table.

Element	**Power generated or consumed (Volt – amps)**
V_g Voltage Source	$V_g I_1^* = (150)(-26 + j52) = -3900 + j7800$ generated
Dependent Voltage Source	$39 I_x I_2^* = (-78 + j234)(-24 + j58) = -11700 - j10140$ consumed
Z_1	$V_1 I_1^* = (78 - j104)(-26 + j52) = 3380 + j6760$ consumed
Z_2	$V_2 I_2^* = (150 - j130)(-24 + j58) = 3940 + j11820$ consumed
Z_x	$V_x I_x^* = (72 + j104)(-2 - j6) = 480 - j640$ consumed

Yes, there is power balance for both real and reactive powers since the sum of consumed complex powers equals to the generated complex powers.

Principles of Electrical Engineering I
Maximum Power Transfer

Consider the interconnection of two circuits α and β as shown in Figure 1. The circuit α is an active circuit with sources (both independent and dependent) while the circuit β is a passive load on the circuit α. Let the equivalent impedance of circuit β be $R_L + jX_L$. Determine $R_L + jX_L$ such that power transferred by the circuit α to the load $R_L + jX_L$ is maximum

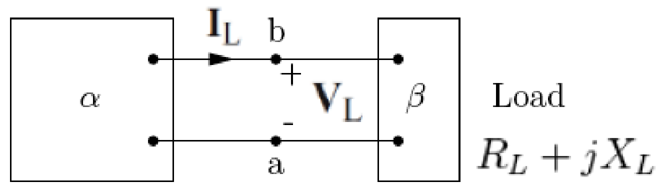


Figure 1

By replacing the circuit α by its Thevenin equivalent, we can redraw the circuit of Figure 1 as that in Figure 2.

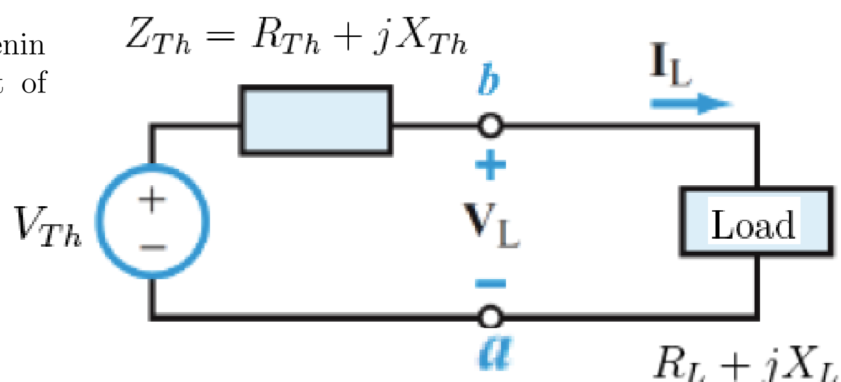


Figure 2

Let the resistance of load be R_L.
Current I is given by

$$\mathbf{I}_L = I = \frac{V_{Th}}{R_{Th} + R_L + j(X_{Th} + X_L)}$$

Power consumed by R_L (using RMS phasors),

$$P = |I|^2 R_L = \frac{|V_{Th}|^2 R_L}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}$$

We need to determine R_L and X_L such that P is maximum.
Let us first fix X_L and determine R_L such that P is maximized. To do so, set

$$\frac{\partial P}{\partial R_L} = 0.$$

Differentiation formula,

$$d\left[V^{-1}U\right] = V^{-1}\,dU - V^{-2}dV\,U = \frac{V\,dU - dV\,U}{V^2}.$$

Applying the above rule, we get

$$\frac{\partial P}{\partial R_L} = \frac{|V_{Th}|^2\left[(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2 - 2R_L(R_{Th} + R_L)\right]}{\left[(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2\right]^2} = 0.$$

This yields

$$R_L = \sqrt{(R_{Th})^2 + (X_{Th} + X_L)^2}.$$

One can verify that this maximizes the power P. Note that R_L equals the magnitude of the impedance corresponding to the rest of the elements in the circuit.
If we have the freedom to select X_L as well in order to maximize the power P, we need to set

$$\frac{\partial P}{\partial X_L} = 0.$$

This yields

$$\frac{\partial P}{\partial X_L} = \frac{-|V_{Th}|^2 \left[2R_L(X_{Th} + X_L)\right]}{\left[(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2\right]^2} = 0.$$

Thus,

$$X_L = -X_{Th}.$$

Now, when both R_L and X_L are free to be selected to maximize the power P, we have

$$X_L = -X_{Th} \text{ and } R_L = \sqrt{(R_{Th})^2 + (X_{Th} - X_{Th})^2} = R_{Th}.$$

In other words, the load impedance is the complex conjugate of the Thevenin impedance ($Z_L = Z_{Th}^*$) in order to maximize the power P consumed by the load.

Second derivative of P with respect to R_L as well as X_L confirms that P is maximum when $Z_L = Z_{Th}^*$. Verify yourself.

$$\text{Maximum Power} = \frac{R_{Th}}{(2R_{Th})^2}|V_{Th}|^2 = \frac{|V_{Th}|^2}{4R_{Th}} \text{ where } V_{Th} \text{ is the RMS voltage.}$$

Example Maximum Power

Determine the maximum amount of power that can be consumed by the load $\mathbf{Z}_L$ in the circuit of Fig.

Solution: We start by determining the Thévenin equivalent of the circuit to the left of terminals (a, b). In Fig. the load has been removed so as to calculate the open-circuit voltage. Voltage division yields

$$\mathbf{V}_s = \mathbf{V}_{oc} = \frac{(4 + j6)}{4 + 4 + j6} \times 24 = 17.31\angle 19.44^\circ \text{ V},$$

RMS

where $\mathbf{V}_s$ is the Thévenin voltage of the source circuit to the left of terminals (a, b). The Thévenin impedance of the source circuit, $\mathbf{Z}_s$, is obtained by calculating the impedance at terminals (a, b), as shown in Fig. after deactivating the 24-V voltage source,

$$\mathbf{Z}_s = 4 \parallel (4 + j6) - j3 = \frac{4(4 + j6)}{4 + 4 + j6} - j3 = (2.72 - j2.04)\ \Omega.$$

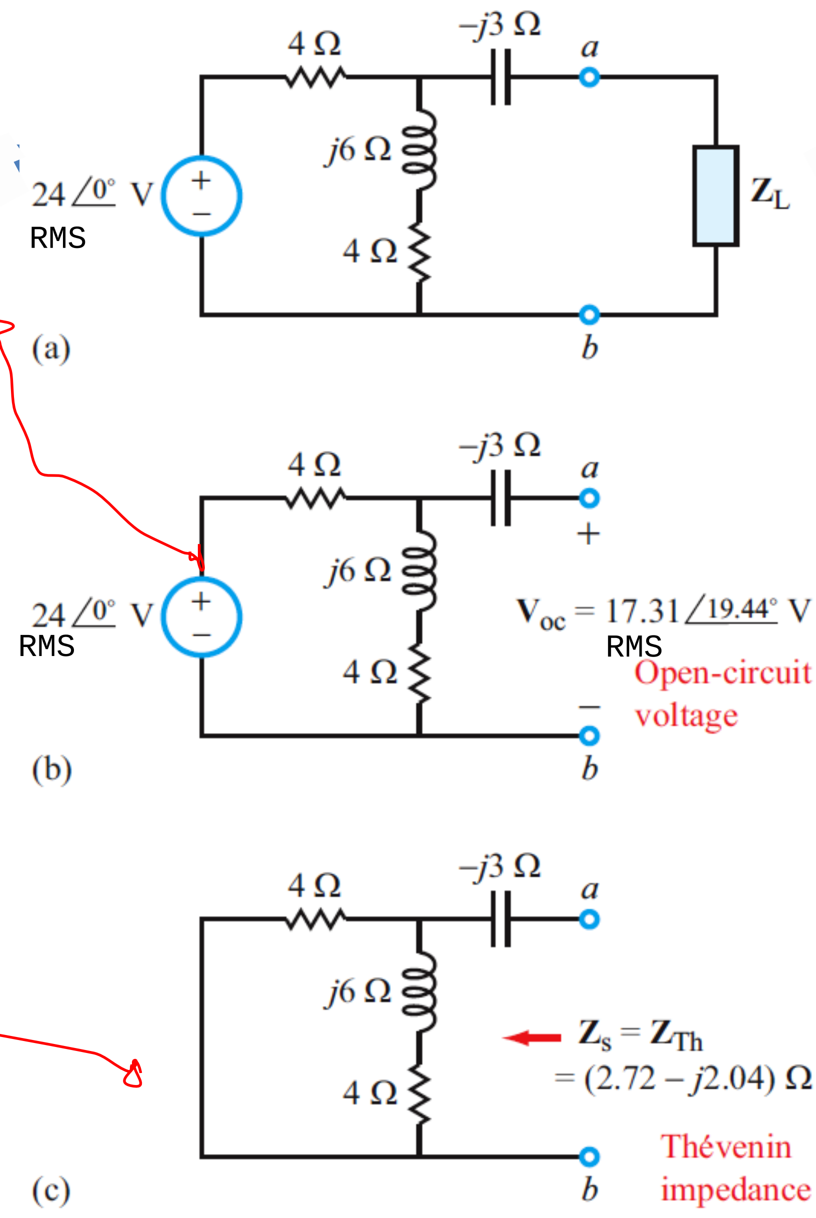


Figure 8-13: Circuit for Example 8-7.

Cont.

Example 8-7: Maximum Power

For maximum transfer of power to the load, the load impedance should be

$$\mathbf{Z}_L = \mathbf{Z}_s^* = (2.72 + j2.04)\ \Omega,$$

$$\text{Maximum Power} = \frac{R_{Th}}{(2R_{Th})^2}|V_{Th}|^2 = \frac{|V_{Th}|^2}{4R_{Th}} \quad \text{where } V_{Th} \text{ is the RMS voltage.}$$

$$P_{av}(\max) = \frac{|\mathbf{V}_s|^2}{4\,R_L} = \frac{(17.31)^2}{4 \times 2.72} = 27.54\,\text{W}.$$

All these problems are important although we do not collect the HW

HW from Nilsson and Riedel 8th and 9th editions

Nilsson and Riedel 8th edition:

10.2, 10.6, 10.17, 10.21, 10.29, 10.32

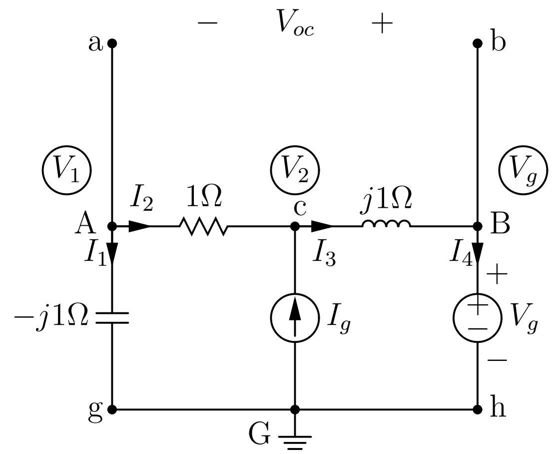
Nilsson and Riedel 9th edition:

10.1, 10.6, 10.18, 10.25, 10.30, 10.32

Principles of Electrical Engineering I

Student's name in capital letters:

Consider the circuit shown. Let $V_g = 2$ V and $I_g = 1$ A. It is known that $V_1 = (1-j2)\,V$ and $V_2 = (1+j)(1-j2) = 3-j1$ V. Also, $I_1 = 2+j$, $I_2 = -2-j$, $I_3 = -1-j$, $I_4 = -1-j$. Determine the average real power as well as the reactive power generated or absorbed by each element. Enter the values in the appropriate columns in the table below. Indicate the units. Is there a power balance within the allowable numerical accuracies? Both concepts and algebra are important. If your concepts are wrong, we cannot grade your algebra.



Element	Average Power generated	Average Power consumed	Reactive Power generated	Reactive Power consumed
V_g *Source*				
I_g *resistance*				
$-j1\Omega$ *capacitance*				
$j1\Omega$ *inductance*				
1Ω *resistance*				