Problems from the Book:
The following problems from our textbook:
Chapter 7: 7.1.5, 7.2.2, 7.6.3, 7.7.1
Chapter 8: 8.3.1, 8.3.2, 8.4.1, 8.4.2
Chapter 9: 9.2.1, 9.3.1

Supplemental Problems:
1. We have examined the problem of detecting a signal in the presence of noise, which was merely a test to decide between two different Gaussian distributions with different means. In this problem, we look at the problem of deciding between two different Gaussian distributions with the same mean, but different variances.

Suppose, under $H_0$ our measured data $x(n) \sim N(0, \sigma_0^2)$ while under $H_1$ our measured data is $x(n) \sim N(0, \sigma_1^2)$, for $n = 0, 1, \cdots, N - 1$.

a) Calculate the likelihood ratio $L(x)$.

b) Using the Neyman-Pearson Theorem, show that region of $x$ such that $L(x) > \gamma$ is equivalent to the region of

$$T(x) = \frac{1}{N} \sum_{n=1}^{N-1} x^2(n) > \gamma_0$$

for

$$\gamma_0 = \frac{2}{N} \ln \gamma + \ln \frac{\sigma_1^2}{\sigma_0^2} \quad \frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}.$$ 

Observe that in this case, the test statistic is just an estimate of the variance.