Supplemental Problem Solutions:

1. First, observe that \( E(X(t)) = \sum_j E(\xi_j)e^{i\omega_j t} = 0 \). Hence the process is first-order stationary. Now, to show WSS, we must show that \( R_X(t, t - k) = R_X(k) \). We will use \( X^*(t) \) to denote the complex conjugate of \( X(t) \).

\[
R_X(t, t - k) = E(X(t)X^*(t - k)) = \sum_j \sum_n e^{i(\omega_j t - \omega_n (t-k))} E[\xi_j \xi_n^*] = \sum_j \sigma_j^2 e^{i\omega_j k}
\]

Hence, since \( R_X(t, t - k) = R_X(k) \) is only a function of \( k \), we have that \( X \) is WSS. The power spectral density \( S_X(\omega) \) is calculated by taking the Fourier series of \( R_X(k) \), which is

\[
S_X(\omega) = \sum_{j=1}^M \sigma_j^2 \delta(\omega - \omega_j),
\]

which is simply a series of spikes in the frequency domain.

2. The key identity we use is

\[
Var(X(1) + \cdots + X(10)) = \sum_{k=1}^{10} Var(X(k)) + \sum_{j=1,j\neq k}^{10} \sum_{k=1}^{10} Cov(X(j), X(k)).
\]

Since \( X(t) \) is zero-mean, and stationary, \( Cov(X(j), X(k)) = R(|j - k|) \). Hence we have

\[
Var(Y) = 10R(0) + (R(1) + R(1) + R(2) + \cdots + R(8)) + (R(2) + R(1) + R(1) + \cdots + R(7)) + \cdots + (R(9) + R(8) + \cdots + R(1))
\]

Counting the terms, and using the fact \( R(k) = e^{-|k|} \) we get

\[
Var(Y) = 10 + 17e + 15e^2 + 13e^3 + 11e^4 + 9e^5 + 7e^6 + 5e^7 + 3e^8 + e^9.
\]