Problems from the Book (Stark and Woods):
The following problems from our textbook:
Chapter 1: 1.30, 1.39
Chapter 2: 2.4

Supplemental Problems:
1. Another way to show that a sequence converges is to show that the sequence is a Cauchy sequence, and then use the fact that a sequence of real numbers converges if and only if it is a Cauchy sequences.
A sequence \( \{x_n\} \) of real numbers is said to be a Cauchy sequence if for each \( \epsilon > 0 \), there exists an \( N \) (depending on \( \epsilon \)) such that \( |x_n - x_m| < \epsilon \) for all \( n, m > N \).
Show that if a sequence \( \{x_n\} \) satisfies \( |x_{n+1} - x_n| \leq \alpha |x_n - x_{n-1}| \) for \( n = 2, 3, \ldots \), and for some fixed \( 0 < \alpha < 1 \), then \( \{x_n\} \) is a convergent sequence.
2. Suppose that \( \{A_j\} \) is an infinite collection of disjoint events in a sample space \( \Omega \). Let \( P \) be a probability measure defined on \( \Omega \). Show that
\[
\lim_{n \to \infty} P(A_j) = 0.
\]
3. Suppose \( A \) is a null event, that is, an event such that \( P(A) = 0 \). Show that
\[
P(B \cup A) = P(B - A) = P(B)
\]
for every event \( B \) in the sample space \( \Omega \). Here \( B - A = B \cap A^c \).
4. The following is an example that pairwise independence does not mean independent. Suppose you have a container with 4 tickets, numbered 1234, 2341, 3412, and 4123. Suppose a ticket is drawn. Define the events
A: First digit of the ticket drawn is 1 or 4
B: Second digit of the ticket drawn is 2 or 4
C: Third digit of the ticket drawn is 3 or 4
Show that \( A, B, \) and \( C \) are pairwise independent, but collectively are not independent.