Instructions: The purpose of this exam is to anonymously assess the background of the students enrolled in the class. To keep the scores anonymous, please do NOT put your name on the exam. There are 4 problems, each of which requires a basic background expected of first year graduate students.

1. Suppose a random integer $x$ is chosen. What is the probability that $x^3$ ends in 11?

2. How many ways are there to uniquely rearrange the letters of the word:

   \[ \text{HELLO} \]

   (Hint: The L’s are considered identical.)
3. Find the determinant of the matrix

\[
\begin{pmatrix}
15 & 3 & 1 & 0 \\
3 & 16 & 6 & -2 \\
1 & 6 & 4 & 1 \\
0 & -2 & 1 & 3
\end{pmatrix}.
\]

(1)

4. Two points \(P_1\) and \(P_2\) are randomly, and independently selected from the \((X, Y)\) plane according to the distribution

\[
f_{XY}(x, y) = \begin{cases}
\frac{1}{4L^2} & (x, y) \in [-L, L] \times [-L, L] \\
0 & \text{elsewhere}
\end{cases}
\]

What is the probability that the line segment \(P_1P_2\) lies entirely within the region \(\Omega\) given by

\[
\Omega = \left\{(x, y) : x^2 + y^2 \leq L^2 \right\}.
\]