Main topics: homework example on closed queueing network; introduction to multiaccess communication; examples of multiaccess channel; assumptions for an idealized multiaccess channel; introduction to Slotted Aloha.

1. Homework example on closed queueing network (Problem. 3.64):

In this problem, M customers are doomed forever to cycle between queue 1 and queue 2 in a closed queueing network. Service times at the queues are independent and exponentially distributed with mean $\mu_1$ and $\mu_2$. (assume $\mu_2 < \mu_1$) We are asked to calculate the steady state probabilities of the states and the true arrival rate at queue 2.

Method 1: The system can be represented by the following Markov chain:

\[
\begin{align*}
\mu_1 & \quad 0, M \\
\bar{\lambda}_2 & \quad \mu_2 \\
\lambda_1 & \quad \mu_1 \\
\mu_1 & \quad 1, M-1 \\
\bar{\lambda}_2 & \quad \mu_2 \\
\mu_1 & \quad 2, M-2 \\
& \quad \ldots \\
\mu_1 & \quad M, 0
\end{align*}
\]

Method 2: Using Jackson’s Theorem, let $\tilde{\lambda}_1 = \tilde{\lambda}_2 = \lambda$,

\[
\begin{align*}
P_{n_1, n_2} &= (1/G) \left( \frac{\bar{\lambda}_1}{\mu_1} \right)^{n_1} \left( \frac{\bar{\lambda}_2}{\mu_2} \right)^{n_2} \\
&= (1/G) (\alpha^{n_1+n_2} (\lambda/\mu_1)^{n_1}(\lambda/\mu_2)^{n_2})
\end{align*}
\]

where $G = \text{sum}(\alpha^{n_1+n_2} (\lambda/\mu_1)^{n_1}(\lambda/\mu_2)^{n_2})$ for all $n_1+n_2 = M$

* true arrival rate: let $A_2(T)$ denote the number of arrivals at queue 2 over $[0,T]$
\[
A_2(T)/T = \mu_1(1-P_{0,M}), \text{ as } T \text{ goes to infinity}
\]

2. Multiaccess Communication

The subnetworks considered thus far have consisted of nodes joined by point-to-point communication links. The implicit assumption about point-to-point links is that the received signal on each link depends only on the transmitted signal and noise on that link. There are many other widely used communication media, such as satellite systems, radio broadcast, multidrop telephone lines, and multitap bus systems, for which the received signal at one node depends on the transmitted signal at two or more other nodes. Shown below is a satellite system, in which many ground stations can transmit to a common satellite receiver, with the received messages being relayed to the ground stations.

The satellite system shown above, as well as other typical multiaccess systems all share the feature of a set of nodes sharing a communication channel. If two or more nodes transmit simultaneously, the reception is garbled, and if none transmit, the channel is unused. The problem is somehow to coordinate the use of the channel so that exactly one node is transmitting for an appreciable fraction of the time. We start by looking at a highly idealized mode. We shall see later that multiaccess channels can often be used in practice with much higher utilization than is possible in this idealized model, but we shall also see that these practical extensions can be understood more clearly in terms of our idealization.

**Assumptions for Idealized Multiaccess Channel:**

* **Slotted system** all transmitted packets have the same length and each packet requires on time unit (called a slot) for transmission. All transmitters are synchronized so that the reception of each packet starts at an integer time and ends before the next integer time.

```
            +---+---+---+---+
            |   |   |   |   |
            +---+---+---+---+
            |   |   |   |   |
            +---+---+---+---+

packets never overlap slots
```

This assumption has two effects. The first is to turn the system into a discrete-time system, this simplifying analysis. The second is to preclude, for the moment, the possibility of carrier sensing or early collision detection. Synchronization can be accomplished with relatively stable clocks, small amount of feedback from the receiver, and some guard time between the end of a packet transmission and the beginning of the next slot.
Assume packets arrive for transmission at each of the m transmitting nodes according to independent Poisson processes. Let \( \lambda \) be the overall arrival rate to the system, and let \( \lambda/m \) be the arrival rate at each transmitting node. This assumption is unrealistic for the case of multipacket messages.

Collisions or perfect reception: That is,

- 1 node sends → all receive perfectly
- \( \geq 2 \) nodes sends → collision

This assumption ignores the possibility of errors due to noise and also ignores the possibility of “capture” techniques, by which a receiver can sometimes capture one transmission in the presence of multiple transmissions.

0, 1, e immediate feedback: Assume that at the end of each slot, each node obtains feedback from the receiver specifying whether 0 packets, 1 packet, or more than one packet (e for error) were transmitted in that slot. This is also unrealistic, particular in the case of satellite channels. It is made to simplify the analysis, and we shall see later that delayed feedback complicates multiaccess algorithms but causes no fundamental problems.

Each packet in a collision must be retransmitted until it is successful. This is certainly reasonable in providing reliable communication.

a) No buffering \((m < \infty)\) This says if one packet is currently waiting for transmission or colliding with another packet during transmission, new arrivals at that node are discarded and never transmitted.

b) Infinite set of nodes \((m = \infty)\) In this case, every packet is at a node by itself.

Our interest in this section is in multiaccess channels with a large number of nodes, a relatively small arrival rate \( \lambda \), and small required delay. Under these conditions, the fraction of backlogged nodes is typically small, and new arrivals at backlogged nodes are almost negligible. Thus the delay for a system without buffering should be relatively close to that with buffering. Also, the delay for the unbuffered system provides a lower bound to the delay for a wide variety of systems with buffering and flow control. The infinite-node assumption alternatively provides us with an upper bound to the delay that can be achieved with a finite number of nodes.

Protocol 1 ---- Slotted Aloha:
The basic idea of this algorithm is:

- packet arrives at a node in slot I
- transmit in slot \( i+1 \)
  - if feedback = 1 (success) done
  - if feedback = e (collision) wait a random time and try again

This is illustrated in the following figure:
Here the total number of retransmissions and new transmissions in a given slot is approximated as a Poisson random variable with some parameter $G > \lambda$. With this approximation, the probability of a successful transmission in a slot is $Ge^{-G}$. Finally, in equilibrium, the arrival rate, $\lambda$, to the system should be the same as the departure rate, $Ge^{-G}$. This relationship is shown below:

The maximum possible departure rate occurs at $G=1$ and is $e^{-1} \approx 0.368$. 

Throughput = $P[\text{success}] = P[1 \text{ arrival / slot}] = Ge^{-G}$

![Graph showing throughput vs. G]

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