CHAPTER 7

Spread-spectrum Modulation

Problem 7.1

(a) The PN sequence length is

\[ N = 2^m - 1 = 2^4 - 1 = 15 \]

(b) The chip duration is

\[ T_c = \frac{1}{10^7} \text{s} = 0.1 \text{ } \mu\text{s} \]

(c) The period of the PN sequence is

\[ T = NT_c \]

\[ = 15 \times 0.1 = 1.5 \text{ } \mu\text{s} \]

Problem 7.2

<table>
<thead>
<tr>
<th>Shift number</th>
<th>Shift-register contents</th>
<th>Modulo-2 adder output</th>
<th>Shift-register output</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1000</td>
<td>0 + 0 = 0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0100</td>
<td>0 + 0 = 0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0010</td>
<td>0 + 0 = 0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1001</td>
<td>1 + 0 = 1</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>1100</td>
<td>0 + 1 = 1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>0110</td>
<td>0 + 0 = 0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>1011</td>
<td>1 + 0 = 1</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0101</td>
<td>1 + 1 = 0</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>1010</td>
<td>0 + 1 = 1</td>
<td>1</td>
</tr>
</tbody>
</table>
9       1101     1 + 0 = 1     0
10      1110     0 + 1 = 1     1
11      1111     1 + 0 = 1     0
12      0111     1 + 1 = 0     1
13      0011     1 + 1 = 0     1
14      0001     1 + 1 = 0     1
15      1000     0 + 1 = 1     1

The output sequence is therefore

\[ 11 \ 000100110101111 \ 0001 \]

\[ \text{one period} \]

**Problem 7.3**

(a) From both Table 7.2a and Table 7.2b we note the following:

**Balance property:**

Number of 1s in one period = 16  
Number of 0s in one period = 15

Hence, the number of 1s exceeds the number of 0s by one.

(b) **Run property:**

In both Tables 7.2a and 7.2b, we count a total of 8 runs of 1s and a total of 8 runs of 0s. Moreover, we note the following:

Runs of length 1 : 4  
Runs of length 2 : 2  
Runs of length 3 : 1

(c) **Autocorrelation function:**
\[ R_c(k) = \begin{cases} 1, & k = lN \\ -\frac{1}{N}, & k \neq lN \end{cases} \]

Hence, we have \((not\ to\ scale)\)
Problem 7.6

(a) The modulo-2 sum of \( b(t) \) and \( c(t) \), on a pulse-by-pulse basis, is as follows

\[ \begin{array}{c|cc}
  b(t) & 0 & 1 \\
  c(t) & 0 & 0 & 1 \\
       & 1 & 1 & 0 \\
\end{array} \]

(b) If symbol 0 is represented by a sinusoid of zero phase shift, and symbol 1 is represented by a sinusoid of 180° phase shift, the output of the modulo-2 adder takes on the same form as that described in Table 7.3 of the text.

Problem 7.7

\[
j(t) = \sqrt{2J} \cos(2\pi f_c t + \theta)
\]

The basis functions are

\[
\phi_k(t) = \begin{cases} 
\sqrt{\frac{2}{T_c}} \cos(2\pi f_c t), & kT_c \leq t \leq (k+1)T_c \\
0, & \text{otherwise}
\end{cases}
\]

\[
\tilde{\phi}_k(t) = \begin{cases} 
\sqrt{\frac{2}{T_c}} \sin(2\pi f_c t), & kT_c \leq t \leq (k+1)T_c \\
0, & \text{otherwise}
\end{cases}
\]

Hence, we may express the jamming signal \( j(t) \) as
\[ j(t) = \sqrt{J} T_c \cos \theta \sum_{k=0}^{N-1} \phi_k(t) \]

\[ - \sqrt{J} T_c \sin \theta \sum_{k=0}^{N-1} \phi_k(t) \]

Problem 7.8

The processing gain is

\[ \frac{T_b}{T_c} = \frac{1}{T_c} \]

The spread bandwidth of the transmitted signal is proportional to \(1/T_c\). The despread bandwidth of the received signal is proportional to \(1/T_b\). Hence,

\[ \text{Processing gain} = \frac{\text{spread bandwidth of transmitted signal}}{\text{despread bandwidth of received signal}} \]

Problem 7.9

\[ m = 19 \]
\[ N = 2^m - 1 = 2^{19} - 1 \approx 2^{19} \]

The processing gain is

\[ 10 \log_{10} N = 10 \log_{10} 2^{19} \]
\[ = 190 \times 0.3 \]
\[ = 57 \text{ dB} \]