Wireless Communications Technologies
Course No: 16:332:559 - (Spring 2002)

Homework 4

1. (a) The average delay is

\[
\frac{\int_0^\infty \tau \phi_e(\tau) d\tau}{\int_0^\infty \phi_e(\tau) d\tau} = \frac{A \mu_\tau}{A} = \mu_\tau
\]

The delay spread is \( \mu_\tau \)

\[
\sqrt{\frac{\int_0^\infty (\tau - \mu_\tau)^2 \phi_e(\tau) d\tau}{\int_0^\infty \phi_e(\tau) d\tau}} = \mu_\tau
\]

(b) DS spreading is used with two tap maximum ratio combining (MRC), where the taps are spaced at chip delay apart. We can say that the average SNR in each arm should follow an exponentially decaying power profile.

The average received bit energy due to MRC is given as

\[
\bar{\gamma}^{\text{mrc}} = \sum_{k=1}^2 \bar{\gamma}_k,
\]

where \( \bar{\gamma}_k = C e^{-k/\mu_\tau} \), \( k = 1, 2 \). Therefore,

\[
\bar{\gamma}^{\text{mrc}} = C(e^{-1/\mu_\tau} + e^{-2/\mu_\tau})
\]

Therefore, we can write

\[
C = \frac{e^{2/\mu_\tau}}{1 + e^{1/\mu_\tau}}
\]

and

\[
\bar{\gamma}_k = \frac{e^{(2-k)/\mu_\tau}}{1 + e^{1/\mu_\tau}} \bar{\gamma}^{\text{mrc}}
\]

Note that the SNR on each branch \( \gamma_k \) is exponentially distributed, i.e.,

\[
p(\gamma_k) = \frac{1}{\bar{\gamma}_k} e^{-\frac{\gamma_k}{\bar{\gamma}_k}}, \quad k = 1, 2
\]

The overall SNR due to MRC \( \gamma^{\text{mrc}} \) is the sum of two exponentials and the overall probability of error (for BPSK) is obtained for 2-branch diversity,

\[
P_b = \int_0^\infty Q(\sqrt{2\gamma}) \tilde{p}_{\gamma^{\text{mrc}}} (\gamma) d\gamma
\]
\[ P_b = \frac{1}{2} \left[ \frac{\tilde{\gamma}_1}{\tilde{\gamma}_1 - \tilde{\gamma}_2} \left\{ 1 - \sqrt{\frac{\tilde{\gamma}_1}{1 + \tilde{\gamma}_1}} \right\} + \frac{\tilde{\gamma}_2}{\tilde{\gamma}_2 - \tilde{\gamma}_1} \left\{ 1 - \sqrt{\frac{\tilde{\gamma}_2}{1 + \tilde{\gamma}_2}} \right\} \right] \]

Note the above expression can now be written entirely in terms of \( \tilde{\gamma}_1^\text{mr} \) and \( \mu_r \) by replacing \( \tilde{\gamma}_1 \) and \( \tilde{\gamma}_2 \) in terms of \( C \) (which depends on \( \tilde{\gamma}_1^\text{mr} \) and \( \mu_r \) and \( \mu_r \) as

\[ P_b = \frac{1}{2} \left[ 1 - \left\{ \frac{1}{1 - e^{-1/\mu_r}} \sqrt{\frac{\tilde{\gamma}_1^\text{mr}}{2 + e^{-1/\mu_r}}} + \frac{1}{1 - e^{1/\mu_r}} \sqrt{\frac{\tilde{\gamma}_2^\text{mr}}{2 + e^{1/\mu_r}}} \right\} \right] \]

(c) When channel is nondispersive, i.e., \( \mu_r = 0 \), then \( P_b = 10^{-3} \). Using the above equation, we see that \( \tilde{\gamma}_1^\text{mr} = 1.992 \). To reduce \( P_b \) to a value of \( 10^{-4} \), solving the above equation again gives

\[ e^{-1/\mu_r} = 0.00265 \Rightarrow \mu_r = 0.1685 \]

2. The variance of the interference can be derived as

\[ \sigma^2 = \sum_{k=2}^{K} (1 + \cos(2\phi_k) N(\psi_k^2 - \psi_k + 1/2) \]

where \( \phi_k \) denotes the phase asynchronism and \( \psi_k \) denoted the chip asynchronism.

(a) For chip synchronous and phase asynchronous, we have \( \psi_k = 0 \) and \( \phi_k \) uniform on \( [0, 2\pi] \). Therefore \( E[\cos(2\phi_k)] = 0 \), \( \forall k \).

Therefore, the decision variable \( \mu_n \) can be interpreted as a Gaussian random variable with mean \( N \) and variance \( N/2(K-1) \). Therefore

\[ P_b = Q\left( \sqrt{\frac{2N}{K-1}} \right) \]

(b) For chip asynchronous and phase synchronous, we have \( \psi_k \) is uniform on \( [0,1] \), and \( \phi_k = 0 \). Therefore, \( E[\psi_k^2 - \psi_k] = -1/6 \).

Therefore, the decision variable \( \mu_n \) can be interpreted as a Gaussian random variable with mean \( N \) and variance \( 3N/2(K-1) \). Therefore

\[ P_b = Q\left( \sqrt{\frac{3N}{2(K-1)}} \right) \]