Probability and Random Processes

Course No: 14:332:321 (Fall 2000)

Exam 2

Maximum Marks : 30

Total Time : 1 hour & 10 minutes

Instructions: Answer all questions. The points for each question are listed below in parentheses.

1. Fill in the blanks (5)
   (a) If random variable \( X \) is Bernoulli with parameter 0.5, \( E[X] = \) ________
   (b) If random variable \( X \) is Bernoulli with parameter 0.5, \( Var[X] = \) ________
   (c) If random variable \( X \) is geometric with parameter 0.5, \( E[X] = \) ________
   (d) If random variable \( X \) is geometric with parameter 0.5, \( Var[X] = \) ________
   (e) If random variable \( X \) is Poisson with parameter 0.5, \( Var[X] = \) ________

2. \( X \) is a uniform random variable taking values over the interval \([1, 5]\). Consider \( K = [X] \). (8)
   (a) What is the probability mass function (PMF) of \( K \)?
   (b) What is \( E[K] \)?
   (c) What is \( Var[K] \)?

3. Flip a fair coin until heads occurs twice. Let \( X_1 \) equal the number of flips up to and including the first head. Let \( X_2 \) equal the number of additional flips up to and including the second head. (6)
   (a) What is \( P_{X_1}(x_1) = ? \)
   (b) What \( P_{X_2}(x_2) = ? \)
   (c) Are \( X_1 \) and \( X_2 \) independent? Give reasons for your answer.
   (d) What is \( P_{X_1,X_2}(x_1,x_2) = ? \)

4. If \( X \) and \( Y \) are random variables such that \( Y = aX + b \), prove that (6)
   \[ \rho_{X,Y} = \begin{cases} 
   -1 & a < 0 \\
   0 & a = 0 \\
   1 & a > 0 
   \end{cases} \]

5. \( X \) is a random variable with PDF given as (5)
   \[ f_X(x) = \frac{1}{\sqrt{18\pi}}e^{-\frac{(x-3)^2}{18}} \quad -\infty \leq x \leq \infty \]

Find the second moment of \( X \)?