Communications Engineering

Course No: 16:332:421 - (Fall 2007)

Solution to Homework 3

1. 4.13 To find inverse the Fourier transform of \( P(f) \) which is given as

\[
P(f) = \begin{cases} 
1/2W, & 0 < |f| < f_1 \\
\frac{1}{4W}[1 - \cos(\pi(f/f_1))], & f_1 < f < 2W - f_1 \\
0, & \text{otherwise}
\end{cases}
\]

Note that \( P(f) \) is an even function. Therefore its inverse Fourier transform is given as

\[
p(t) = 2 \int_{0}^{\infty} P(f) \cos(2\pi ft) \, df
\]

Using the expression for \( P(f) \) in the above equation and solving for the appropriate integrals results in

\[
p(t) = \text{sinc}(2Wt) \cos(2\pi W\alpha t) \frac{\cos(2\pi W\alpha t)}{1 - 16\alpha^2 W^2 t^2}
\]

2. 4.16

The transmission bandwidth of a raised cosine pulse spectrum is

\[
B = W(1 + \alpha),
\]

where \( W = 1/2T_b \) and \( \alpha \) is the rolloff factor.

For a data rate of 56\,kb/s, \( W = 28\,kHz \).

Then it follows that the transmission bandwidth is given in each case as

(a) \( \alpha = 0.25, \quad B = 35kHz \)

(b) \( \alpha = 0.5, \quad B = 42kHz \)

(c) \( \alpha = 0.75, \quad B = 49kHz \)

(d) \( \alpha = 1.0, \quad B = 56kHz \)

3. 4.17

The use of eight amplitude levels ensures that 3 bits can be transmitted per pulse. Therefore, the symbol period can be increased by a factor of 3 \( \Rightarrow \) all four bandwidths in Problem 7.12 will be reduced by a factor of 1/3.

4. 4.18

(a) For a unity rolloff factor, \( B = 2W = 1/T_b \)

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Since $B = 12 \text{kHz}$, $T_b = 10^{-3}/12$ seconds. Quarternary PAM ⇒ 2 bits per pulse, therefore the information rate transmitted through the channel is

$$R = 2/T_b = 24 \text{ kb/s}$$

(b) For 128 quantizing levels, 7 bits are required to transmit each amplitude. The additional bit for synchronization makes the codeword 8 bits long. Since the signal is transmitted at 24 kb/s (from part (a)), it follows that the sampling rate must be

$$\frac{24 \text{ kb/s}}{8 \text{ bits/sample}} = 3 \text{ kHz}$$

The maximum possible frequency to avoid aliasing is therefore 1.5 kHz.

5. 4.34

Figure 1: Transform domain representation of channel and equalizer

The channel output is

$$x(t) = K_1s(t - t_{01}) + K_2s(t - t_{02})$$

Taking Fourier transforms on both sides, and rearranging terms, we can write the channel transfer function as

$$H_c(f) = \frac{X(f)}{S(f)} = K_1 \exp(-j2\pi ft_{01}) + K_2 \exp(-j2\pi ft_{02})$$

Since we are using a three-tap delay-line-filter, it follows that the transfer function of the equalizer is of the form

$$H_e(f) = w_0 + w_1 \exp(-j2\pi fT) + w_2 \exp(-j4\pi fT)$$

$$\Rightarrow$$

$$H_e(f) = w_0[1 + \frac{w_1}{w_0} \exp(-j2\pi fT) + \frac{w_2}{w_0} \exp(-j4\pi fT)] \quad (1)$$

Ideally, the equalizer $H_e(f)$ should be designed such that the overall transfer function is

$$H_c(f)H_e(f) = K_0 \exp(-j2\pi ft_0),$$

where $K_0$ is a constant gain and $t_0$ is the transmission delay.

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In this case, let us assume that the gain $K_0$ is the same as that of the first path, i.e., $K_0 = K_1$, and also let the transmission delay be the same as that of the first path, i.e., $t_0 = t_{01}$

Therefore,

$$H_e(f) = \frac{K_1 \exp(-j2\pi ft_{01})}{H_e(f)}$$

$$\Rightarrow$$

$$H_e(f) = \frac{1}{1 + \frac{K_2}{K_1} \exp(-j2\pi f(t_{01} - t_{02}))}$$

Using the fact that $K_2 \ll K_1$, we can write

$$H_e(f) = \left\{1 - \frac{K_2}{K_1} \exp(-j2\pi f(t_{02} - t_{01})) + \left(\frac{K_2}{K_1}\right)^2 \exp(-j4\pi f(t_{02} - t_{01}))\right\}$$

(2)

Comparing (1) and (2), we deduce that

$$1 = w_0$$

$$-\frac{K_2}{K_1} = \frac{w_1}{w_0}$$

$$\left(\frac{K_2}{K_1}\right)^2 = \frac{w_2}{w_0}$$

$$T = t_{02} - t_{01}$$

Thus, we find the filter taps to be

$$w_0 = 1, w_1 = -\frac{K_2}{K_1}, w_2 = \left(\frac{K_2}{K_1}\right)^2$$

6. 4.35

The input sequence consists of uniformly sampled samples $\{x(nT)\}$. Therefore, the Fourier transform of the input sequence may be written as

$$X_{in}(f) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(f - \frac{k}{T})$$

The Fourier transform of the tapped-delay-line equalizer output is

$$Y_{out}(f) = X_{in}(f)H(f),$$

where $H(f)$ is the equalizer transfer function. For perfect equalization, we require

$$Y_{out}(f) = 1$$

Therefore the equalizer $H(f)$ is now given as

$$H(f) = \frac{1}{X_{in}(f)} = \frac{T}{\sum_{k=-\infty}^{\infty} X(f - \frac{k}{T})}$$
Let the impulse response sequence of the equalizer be \( \{ w_n \} \). Then for an infinite number of taps, it follows that

\[
H(f) = \sum_{n=-\infty}^{n=\infty} w_n \exp(j2\pi nfT),
\]

which is in the form of a complex Fourier series with real coefficients defined by the tap-weights of the equalizer, i.e.,

\[
w_n = \frac{1}{T} \int_{-T/2}^{T/2} H(f) \exp(-j2\pi nt/T) df = \frac{1}{T} \int_{-1/2T}^{1/2T} \frac{T}{\sum_{k=-\infty}^{k=\infty} X(f-k/T)} \exp(-j2\pi nt/T) df
\]

⇒ the equalizer can approximate any input \( x(t) \) on the interval \((-1/2T, 1/2T)\).