Communications Engineering
Course No: 16:332:421 - (Fall 2007)

Solution to Homework 2

1. 4.1 (a) & (b)

![Signal and Matched Filter Diagram]

(c) The peak value of the output is $A^2T/4$

2. 4.7

$$P_e = \frac{1}{2} \text{erfc}(\sqrt{\frac{E_b}{N_0}}) = \frac{1}{2} \text{erfc}(\sqrt{\frac{A^2T_b}{N_0}}) = 10^{-6}$$

⇒ $\sqrt{\frac{A^2T_b}{N_0}} = 3.30$ (from Table 1)

If the bit-rate is doubled then, the bit duration $T_b$ is halved, then

$\sqrt{\frac{A^2T_b}{N_0}} = 3.30/\sqrt{2} = 2.33$

Therefore the probability of error is now (from Table 1)

$$P_e = 1.27 \times 10^{-3}$$

3. 4.6

The average probability of error is

$$P_e = p_1 \int_{-\infty}^{\lambda} f_Y(y|1)dy + p_0 \int_{\lambda}^{\infty} f_Y(y|0)dy$$
An optimum choice of $\lambda$ corresponds to minimizing $P_e$. Differentiating the above equation with respect to $\lambda$ and setting to zero we get

$$\frac{dP_e}{d\lambda} = 0 = p_1 f_Y(\lambda|1) - p_0 f_Y(\lambda|0)$$

$$\Rightarrow$$

$$\frac{p_0}{p_1} = \frac{f_Y(\lambda_{opt}|1)}{f_Y(\lambda_{opt}|0)}$$

4. MATLAB Exercise: The answer is figure 4.6 on pg. 258 in the book.