Wireless Communications Technologies  
Course No: 16332:559 - (Spring 2002)  
Midterm Exam Solutions

Instructions: Answer all questions. Maximum Marks: 90. The points for each question are listed below in parentheses. You are allowed a total time of 1 hour and 30 minutes.

1. Read the following statements and state if they are “true” or “false’. Give precise reasons for your answer  

(a) Decision Feedback equalizers improve performance over MLSE equalizers  
False, MLSE is the optimum equalizer.

(b) FM Radio operates in the VHF bands.  
True, FM (88-108 MHz) lies in the VHF (30-300 MHz) band.

(c) Path loss in cellular systems is unaffected by the carrier frequency.  
False, path loss formulas under propagation close to the earth’s surface depend on wavelength of carrier.

(d) Error correction coding improves bandwidth efficiency.  
False, coding increases bits to be transmitted.

(e) GMSK modulation is a form of partial response signaling.  
True, the Gaussian filter in GMSK shapes the response of CPM signals to be partial response.

2. In $M$-ary Frequency Shift Keying ($M$-FSK), the $M$ consecutive signal frequencies are separated by $\frac{f_s}{2T_s}$, where $T_s$ is the symbol duration.

(a) Due to imperfections in frequency synthesizers, the bands at each edge (edge frequencies) of the signal spectrum have signal energy that extends to 50% beyond each edge. In this situation, if we assume the signal energy at the end frequencies extends to $\frac{0.5}{f_s}$ Hz, the bandwidth occupied by the signal is given as

$$B = \frac{M + 1}{2T_s}$$

Observe that $T_s = T_b \log_2(M) = \log_2(M) / R_b$. Therefore, the bandwidth efficiency $\eta_B$ of $M$-FSK is

$$\eta_B = \frac{R_b}{B} = \frac{2 \log_2(M)}{M + 1}$$

(b) $M = 4$ maximizes $\eta_B$
<table>
<thead>
<tr>
<th>( M )</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>16</th>
<th>32</th>
<th>64</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta_B = R_b/B )</td>
<td>2/3</td>
<td>4/5</td>
<td>2/3</td>
<td>8/17</td>
<td>10/33</td>
<td>12/65</td>
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(c) For perfect frequency synthesis,

\[
\eta_B = \frac{R_b}{B} = \frac{2 \log_2(M)}{M}
\]

\( M = 2 \) and \( M = 4 \) both achieve the highest bandwidth efficiency possible.

(d) The power efficiency \( \eta_P \) increases with increasing \( M \)

3. Consider independent lognormally shadowed signals received at a base station from (30) two mobiles at identical distances from the base station. Assume the paths seen by the two mobiles are such that the standard deviation of the shadow fading in each case is 6 dB, but the means are 10 dB and 20 dB respectively.

(a) Using the Fenton-Wilkinson method, find the probability that the received signal at the base station is greater than 20.5 dB.

The threshold \( x \) (dB) is 20.5 \( \Rightarrow \) on an absolute scale is \( x = 112.2 \). Let \( L \) be the sum of the two lognormal signals received at the base station. Then

\[
P(L \geq x) \approx P(e^{\bar{Z}} \geq x) = Q\left(\frac{\ln(x) - \mu_Z}{\sigma_Z}\right) = Q(-0.04) \approx 0.5
\]

where

\[
\mu_Z = 4.78
\]
\[
\sigma_Z^2 = 1.75
\]

(b) Consider a third independent lognormally shadowed signal received at the same base station (with mean 10 dB and standard deviation 6 dB). Find the probability that the signal-to-interference ratio seen by the third mobile (at the base station) is less than 10 dB.

\[
P[SIR(dB) < 10dB] = Q\left(\frac{\mu_3 - \mu_Z - 10}{\sqrt{\sigma_3^2 + \sigma_Z^2}}\right) = Q(-2.5)
\]

where

\[
\mu_Z = \mu_3/0.23026 = 20.75
\]
\[
\sigma_Z^2 = \sigma_3^2/(0.23026)^2 = 33
\]
\[
\mu_3 = 10
\]
\[
\sigma_3 = 6
\]
4. Consider two independently Rayleigh faded signals \( s_1(t) \) and \( s_2(t) \), received at a mobile terminal. Let the mean power in each of these signals be \( \Omega_1 \) and \( \Omega_2 \), respectively.

The instantaneous signal powers \( s_1 \) and \( s_2 \) have the probability density function

\[
p_{s_i}(x) = \frac{1}{\Omega_i} \exp\left(-\frac{x}{\Omega_i}\right), \quad i = 1, 2
\]

The probability that the ratio of the instantaneous signal power of \( s_1(t) \) to interference power \( s_2(t) \) is greater than 2 is

\[
P\left[\frac{s_1}{s_2} > 2\right] = \int_0^{\infty} \exp\left(-\frac{2x}{\Omega_1}\right)p_{s_2}(x)dx = \frac{1}{1 + \frac{2\Omega_2}{\Omega_1}}
\]

5. A Rayleigh fading channel has a delay spread of \( 1 \) ms and a Doppler spread of \( 10 \) Hz and is used for a communication system transmitting signals of bandwidth 25 KHz.

(a) The coherence time of the channel is 0.1 s
(b) The coherence bandwidth of the channel is 1 KHz
(c) Slow fading (since coherence time is greater than symbol duration), Frequency selective fading (since coherence bandwidth is smaller than signal bandwidth)
(d) CPM for spectral efficiency; Whitening Filter and Equalizer for frequency selective channel