Procedures

- Have your Rutgers student ID visible. You must put your SIGNATURE and student ID NUMBER on the quiz sheet and on each blue book used.
- You must use ink. PENCIL=ZERO CREDIT.
- Sit every other row with two empty seats between neighbors.
- Answers given without work shown will receive NO credit.
- You must hand in the quiz sheet with the blue book(s).

This quiz has four questions each worth 25 points for a total of 100 points. They are arranged according to my perception of their difficulty. You should read each question in its entirety before you begin your answer; sometimes knowledge of what’s to come shows you an easier path. You are allowed 2 sides of two 8.5 X 11 sheets of notes. A calculator is useless except for its comfort value. Think carefully and don’t panic when you come across something you don’t know. You can work it out from what you DO know if you keep your head.

GOOD LUCK!

NAME: ________________________________

SIGNATURE: __________________________

STUDENT ID: __________________________
1. (25 points) Please prove or disprove the following statements: You may assume that correlation is defined as 
\[ R_{fg}(\tau) = \int_{-\infty}^{\infty} f^*(t)g(t+\tau)dt \]
and the Hilbert transform is defined as 
\[ \hat{f}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{f(\tau)}{t-\tau}d\tau \]
(a) (5 points) 
\[ R_{ff}(\tau) = R_{ff}(-\tau) \]
(b) (5 points) 
\[ R_{ff}(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_f(\omega)d\omega \]
(c) (5 points) Prove that if \( f(t) \) is periodic then \( \hat{f}(t) \) is also periodic.
(d) (10 points) Prove that the Hilbert transform of \( \hat{f}(t) \) is \( -f(t) \).

2. (25 points) This problem carefully explores energy in A.M. signals. Where before we simply waved our hands about “slowly varying \( f(t) \) relative cos \( \omega_c t \)”, here we nail it down quantitatively.
We are given a real energy signal \( f(t) \) which is band-limited to \( \pm W \) radians/sec; i.e., \( F(\omega) = 0 \) for \( |\omega| > W \). We set \( \phi(t) = f(t) \cos \omega_c t \). For each of the following parts an analytic answer is required. No sketches will be accepted.

(a) (10 points) What is \( \Phi(\omega) \) the Fourier transform of \( \phi(t) \) in terms of \( F(\omega) \)?
(b) (15 points) Carefully show that if \( \omega_c > W \) then \( E_\phi = E_f/2 \) where 
\[ E_{\phi(t)} = \int_{-\infty}^{\infty} |\phi(t)|^2dt \]

3. (25 points) Cora the communications engineer is the operator of an early warning sonar system. The system takes sonar input \( f(t) \) and produces a number, \( \Gamma \) as 
\[ \Gamma = \int_{0}^{2} f(t)c(t)dt \]
where \( c(t) \) is some fixed signal. Notice that \( \Gamma \) is simply \( R_{fc}(0) \) on the interval \( (0,2) \) for real signals \( f(t) \) and \( c(t) \).
Incoming enemy subs produce the signal \( e(t) = \sin \pi t \). Incoming allied subs produce \( a(t) = \cos \pi t \). Thus, \( f(t) \) can be the sum of each, both or neither signal; i.e.,

I no subs present \( \rightarrow f(t) = 0 \)
II enemy sub only \( \rightarrow f(t) = e(t) \)
III allied sub only \( \rightarrow f(t) = a(t) \)
IV both subs present \( \rightarrow f(t) = a(t) + e(t) \)
(a) (10 points) Which of the following signals should be chosen for $c(t)$ if Cora wishes to have $\Gamma$ nonzero ONLY when an enemy submarine is present? Assume that both an enemy sub and an allied sub might be present at the same time. You must justify your answer quantitatively.

- $\cos \pi t$
- $\cos(\pi t + \pi/4)$
- $\cos(\pi t - \pi/4)$
- $\sin 8\pi t$
- $u(t) - 2u(t - 1) + u(t - 2)$

(b) (15 points) Suppose Cora wants to be able to distinguish events I through IV based on the value of $\Gamma$: Which $c(t)$ should she use? You must justify your answer quantitatively.

4. (25 points) Cora the communications engineer has been hired by RC (Really Cheap), Inc. to build AM radio receivers. RC’s primary expertise is in building exceptionally good multipliers, filters and a box which takes the square root of the incoming signal. Using ONLY filters and multipliers and “squarerooters”, Cora must devise a way to recover $f(t)$ from a transmitted signal $\phi(t) = f(t) \cos \omega_c t$.

**Note:** Cora does NOT have available a local copy of the carrier nor does she have a signal generator which will allow her to synthesize a periodic wave form. You may assume that $f(t)$ is band-limited to $\pm W$ where $|W| \ll \omega_c$.

(a) (10 points) Show how to use multipliers and filters to recover $f(t)$ from $\phi(t)$. What property must $f(t)$ obey so that it can be recovered unambiguously? **Note:** you can use as many multipliers, filters and squarerooters as you like, but must carefully and quantitatively justify your answer.

(b) (10 points) Will your scheme work for single sideband suppressed carrier AM where $\phi(t) = f(t) \cos \omega_c t - \dot{f}(t) \sin \omega_c t$? Why/why not?

(c) (5 points) What if single sideband large carrier is used; i.e.,

$$\phi(t) = f(t) \cos \omega_c t - \dot{f}(t) \sin \omega_c t + A \cos \omega_c t$$

Can $f(t)$ now be recovered from $\phi(t)$ using only filters, multipliers and squarerooters? **HINT:** You don’t have to use all the devices. Just use enough to get the job done.