1. **Nyquist 101**: Specify the Nyquist rate and Nyquist interval for each of the following signals. Note that \( \text{sinc}(x) \equiv \frac{\sin(\pi x)}{\pi x} \).

(a) \( g(t) = \text{sinc}(200t) \)

**SOLUTION:** This sinc pulse corresponds to a bandwidth of \( W = 100 \text{ Hz} \). Hence, the Nyquist rate is \( 200 \text{ Hz} \), and the Nyquist interval is \( 1/200 \text{ seconds} \).

(b) \( g(t) = \text{sinc}^2(200t) \)

**SOLUTION:** This signal may be viewed as the product of the sinc pulse \( \text{sinc}(200t) \) with itself. Since multiplication in time domain corresponds to convolution in frequency domain, we find that the signal \( g(t) \) has a bandwidth equal to twice that of the sinc pulse \( \text{sinc}(200t) \), that is \( 200 \text{ Hz} \). The Nyquist rate of \( g(t) \) is therefore \( 400 \text{ Hz} \), and the Nyquist interval is \( 1/400 \text{ seconds} \).

(c) \( g(t) = \text{sinc}(200t) + \text{sinc}^2(200t) \)

**SOLUTION:** The bandwidth of \( g(t) \) is determined by the highest frequency content of either \( \text{sinc}(200t) \) or \( \text{sinc}^2(200t) \). From earlier parts, we know that \( \text{sinc}^2(200t) \) has the higher bandwidth equal to \( 200 \text{ Hz} \). Correspondingly, the Nyquist rate is \( 400 \text{ Hz} \) and the Nyquist interval is \( 1/400 \text{ seconds} \).

2. **Nyquist 102**: Suppose we have samples of a signal \( a_k = g(k\Delta) \) where \( \Delta \) is shorter than the Nyquist interval for the bandlimited function \( g(t) \). Derive an explicit time-domain expression for how we recover the function \( g(t) \) from the samples \( \{a_k\} \).

**SOLUTION:** Sampled signal can be expressed as

\[
g_\delta(t) = \sum_{k=-\infty}^{\infty} g(k\Delta)\delta(t - k\Delta)
\]

The Fourier transform of \( g_\delta(t) \) is Now, we apply a perfect low pass filter of bandwidth \( W \) to obtain the original signal \( g(t) \). The inverse Fourier transform of such a filter which is \( 1 \) on \( -W \leq f \leq W \) is

\[
h(t) = \frac{\sin 2\pi W t}{\pi t}
\]

Convolution of \( g_\delta(t) \) with \( h(t) \) yields

\[
g(t) = \sum_{k=-\infty}^{\infty} g(k\Delta)\frac{\sin 2\pi W (t - k\Delta)}{\pi (t - k\Delta)}
\]
so the “sinc” function is the interpolation function for Nyquist sampling.

3. **Nyquist Grad School:** Does the Nyquist Sampling Theorem apply to strictly time limited signals? If not why not? If so, why? This problem is a bit subtle so think carefully and analytically (and justify any assumptions).

**SOLUTION:** The Nyquist Sampling Theorem can’t be directly applied to strictly limited signals, because these signals are not band-limited. The reason is pretty simple. You can think of an time-limited signal $s(t)$ as an unlimited time signal $q(t)$ (that has limited bandwidth) multiplied by a window function $w(t)$ (a unit pulse of a certain duration which is zero everywhere else). So, $s(t) = q(t)w(t)$. Multiplication in time domain implies convolution in frequency domain and since the fourier transform of the window function is a sinc and has infinite extent, when it’s convolved with the finite extent (because $(q(t)$ is bandlimited) fourier transform of $q(t)$, the result will be of infinite extent too. So time-limited means band-unlimited. And by duality, band-limited means time-unlimited.

In practice, we may take the following two measures:

(a) Prior to sampling, a low-pass filter is used to limit the signal bandwidth in which high-frequency components are not essential to the information being conveyed by the signal.

(b) The filtered signal is sampled at a rate slightly higher than the Nyquist rate just to be sure that nothing really significant slips through the filter.

But not matter how you slice it, it’s just an approximation to the theory (or the theory is an approximation to the reality).

4. **Pulse Modulation**

(a) What is Pulse Amplitude Modulation? Provide a pictorial example.

**SOLUTION:** The amplitude of regularly spaced pulses are varied in proportion to the corresponding sample values of a continuous message signal, as illustrated in figure 3.5 in text.

(b) What is Pulse Position Modulation? Provide a pictorial example.

**SOLUTION:** The position of a pulse relative to its unmodulated time of occurrence is varied in accordance with the message signal, as illustrated in figure 3.8(d) in text.

(c) What is Pulse Frequency Modulation? Provide a pictorial example.

**SOLUTION:** The pulse repetition rate is varied in accordance with the modulating signal.

(d) What is Pulse Width Modulation? Provide a pictorial example.

**SOLUTION:** Samples of the message signal are used to vary the duration of the individual pulses in the carrier, as illustrated in figure 3.8(c) in text.

(e) Consider a full wave rectified AM signal $r(t) = m(t)\cos 2\pi f_c t$ where we assume $m(t) \geq 0 \forall t$. Assuming the highest frequency content of $m(t)$ is much less than $f_c$, can $r(t)$ be considered the approximate result of a pulse modulation method applied to $m(t)$? If so, which one?

**SOLUTION:** Yes, $r(t)$ can be considered the approximation result of PAM, in which the pulse is not a rectangle, but $\cos 2\pi f_c t (0 \leq t \leq T/2)$, where $T = \frac{1}{f_c}$ instead.
5. Problem 3.5 in Haykin

**SOLUTION:** We can determine the spectrum of the resulting PAM signal according to

\[
S(f) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s)H(f)
\]

Where \( f_s \) is the sampling rate and \( H(f) \) is the spectrum of the pulse.

\[
H(f) = T \text{sinc}(fT)e^{-j\pi fT}
\]

\[
= 10^{-4} \text{sinc}(10^{-4} f)e^{-j\pi 10^{-4} f}
\]

where \( T \) is the pulse duration, which is 0.1ms. So,

\[
S(f) = \frac{1}{10} \sum_{k=-\infty}^{\infty} M(f - 10^3 k)\text{sinc}(10^{-4} f)e^{-j\pi 10^{-4} f}
\]