1. **Phase Locked Loop Demodulator (quickie):**
   
   (a) Using what you know about phase locked loops (PLLs), show that the PLL can be used as a FM demodulation circuit.
   
   (b) How can you modify your PLL to obtain a PM demodulation circuit?

2. **Hilbert Transform Stuff for Culture:** The Hilbert-transform of a Fourier transformable signal \( m(t) \), denoted by \( m_H(t) \), is defined by

   \[
   m_H(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{m(t)}{t - \tau} d\tau
   \]

   In the frequency domain, we have

   \[
   M_H(f) = -j \text{sgn}(f) M(f)
   \]

   where \( m(t), M(f) \) and \( m_H(t), M_H(f) \) are Fourier transform pairs and \( \text{sgn}(f) \) is the signum function. Using the definition of the Hilbert transform, show that a single-sideband modulated signal resulting from the message signal \( m(t) \) and carrier \( \cos(2\pi f_c t) \) of unit amplitude is given by

   \[
   s(t) = \frac{1}{2} [m(t) \cos(2\pi f_c t) \pm m_H(t) \sin(2\pi f_c t)]
   \]

   where the minus sign corresponds to the transmission of upper sideband and plus to the transmission of lower sideband.

3. **A Little Heterodyne Review:** Section 2.9 in your text describes the superheterodyne receiver. How precise do the RF filters (just after the antenna and after the mixer) need to be to extract a single FM radio station at carrier frequency 99.1MHz and no others just before the envelope detector of FIGURE 2.32 in your text? What is the primary benefit of using the superheterodyne structure?

4. **Problem 2.40 in Haykin**

5. **Cora the Advertising Maven:**

   WRUR, an FM radio station, has hired the famous Dr. Cora C. Enginoire to run advertising. Cora has decided to boost advertising revenue by playing the commercials louder!

   Assume that under normal circumstances WRUR has a carrier frequency of 98.7MHz and the output spectrum has significant components between 98.69MHz and 98.71MHz. The next nearest stations operate at 98.9MHz and 98.5MHz.
By approximately what factor can Cora make the commercials louder before the neighboring stations complain to the FCC of radio interference. You must justify you answer.

HINT: You may assume the following simplified form of Carson’s rule: \( \text{BW} = 2\beta \omega_m \).

6. **Evil in AM Systems:** A baseband signal \( m(t) \) is used to produce a modulated signal \( m(t) \cos 2\pi f_c t \). In the passband it is corrupted by two the addition of Evil signals so that the received signal is \( r(t) = e_1(t) \cos 2\pi f_c t - e_2(t) \sin 2\pi f_c t + m(t) \cos 2\pi f_c t \). The Evil signals, \( e_i(t) \), each have power density \( N_0/2 \) per Hz in double-sided band of width \( 2W \), and \( m(t) \) has power \( P \) in the same band. (double-sided).

(a) What is the total power in each of the \( e_i(t) \)

(b) Please design a coherent receiver to obtain \( m(t) \) and calculate the signal to noise ratio at the output of your receiver.

HINT: You can do this the easy way or the hard way, depending on what you remember about synchronous demodulation.

(c) Suppose you only have an envelope detector available and value of \( WN_0 \) is small relative \( P \). What is the signal to noise at the output of your envelope detector.

HINT: Though you could probably copy a version of the answer from the text, you’ll have more fun (and learn more) if you start from first principles).

7. **Cold Water Bucket at 3 AM:** QUICK! What is the effect on output signal to noise ratio of raising the modulation index for an FM signal. TICK, TICK, TICK, TICK, .... SPLASH!