The following analysis in the next page is extracted from the paper:

L. Kleinrock and F. A. Tobagi, "Packet Switching in Radio Channels: Part I - Carrier Sense Multiple Access Modes and Their Throughput-Delay Characteristics" IEEE transactions on communications, December, 1975

Analysis of Non-persistent CSMA



Consider a time cycle as a busy period plus the following idle period. In each this cycle, it could be either a successful transmission period following by an idle period or an unsuccessful transmission period following by an idle period.

Let us define following parameters:

B: mean length of busy period

I: mean length of idle period

T: length of a frame transmission, T=1

U: probability of a successful transmission in a busy period

Then, (B+I) is the average length of a time cycle.

In general, the expected channel throughput can be regarded as the overall output by averaging all cycles:

$$S = \frac{TU}{B+I} = \frac{U}{B+I}$$

Now, let us view each transmission period from the first transmitter X's perspective. If X is the first one to send a frame at time, it will succeed because no one else access the channel in (t, t+a). This occur with a probability $U = e^{-aG}$. Note that in this case, the busy period B is 1+a, but we cannot regard the whole "1+a" as the channel output, because only the "1" portion can be counted as throughput.

However, if during time (t, t+a), there are other access events. X will experience a busy period as long as 1+a+Y, in which Y is a random variable between (0, a). The CDF of **Y** can be found by:

 $F_{\mathbf{Y}}(\mathbf{y}) = P\{\text{no packet occur in an duration of } a - \mathbf{y}\} = e^{-G(a - \mathbf{y})}$

Then, we can have the PDF and calculate the mean of Y is $E(Y) = a - \frac{1}{G}(1 - e^{-aG})$

From, B, I, U, we can have $S = \frac{Ge^{-aG}}{G(1+2a) + e^{-aG}}$