OPTIMAL WAVEFORM DESIGN FOR MIMO RELAYING

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ABSTRACT

Non-regenerative MIMO relaying between a source and a destination is studied in this paper. Optimal weighting matrix operating on baseband signals (waveforms) at the relay is found that maximizes the capacity between the source and the destination. Relay channels with or without direct link between the source and the destination are considered. The optimal relay exploits the knowledge of the channel matrix between the source and the relay and the channel matrix between the relay and the destination. The optimal weighting matrix can be considered as a matched filter along the singular vectors of the channel matrices. Such a non-regenerative relay can be deployed in mobile ad hoc networks as well as in cellular networks.

1. INTRODUCTION

MIMO relaying is useful for a variety of applications. In a cellular environment, a MIMO relay can be deployed in areas where there are strong shadowing effects, such as inside buildings and tunnels. For mobile ad hoc networks, MIMO relaying is essential not only to overcome shadowing due to obstacles but also to reduce transmission power and RF interference. Capacity analysis of a generic MIMO relay is recently reported in [1, 2].

In this paper, we study non-regenerative MIMO relaying where the information data are not regenerated at the relay except that the baseband symbols are reproduced, weighted and then retransmitted. As for any relay, two orthogonal channels are required: one for the received signal and the other for the retransmitted signal. The two channels required can be implemented through either time-division or frequency-division. But the latter seems much easier to do in practice as no additional synchronization requirement between the source and the destination is necessary. A non-regenerative relay may be easier to install (especially for two-hop relaying) than a regenerative relay as the former needs no high level codes. We note here however that for many-hops relaying, non-regenerative relays may not be as networking-friendly as regenerative relays.

The previously reported schemes of non-regenerative MIMO relays such as in [3] are not optimal, and in fact they are ad hoc. The optimal relay matrix shown in this paper maximizes the capacity between the source and the destination, where the knowledge of the channel matrix between the source and the relay and the channel matrix between the relay and the destination are assumed.

2. OPTIMAL MIMO RELAYING WITHOUT DIRECT LINK

Consider a relay channel as shown in Figure 1, where a relay is used to assist the transmission from a source to a destination. In this section, we do not consider the direct link between the source and the destination.

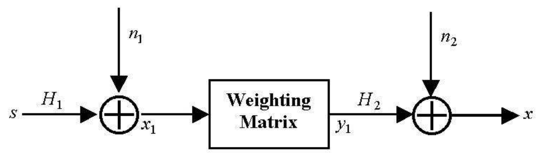

Fig. 1. A two-hop MIMO relay channel without direct link

Conceptually, all terminals can be treated as half-duplex (although it is not necessary when two orthogonal frequency channels are available). Namely, a transmission is done over two time slots using a "listen-and-transmit" protocol. At the first time slot, the source terminal transmits to the relay terminal. At the second time slot, the relay terminal forwards signals to the destination terminal. As long as the channel coherence time is larger than the reciprocal of the channel coherence bandwidth, all channels may be modeled as frequency flat through use of multiple narrow-band carriers (such as OFDM). This is the case for most practical environments. Therefore, we will assume that the channel between the source and the relay and the channel between the relay and the destination are represented by the memoryless channel matrices H_1 and H_2, respectively. Consequently, the function of the (non-regenerative) relay is equivalent to a memoryless weighting matrix F that transforms the (baseband) waveform received at the relay to the (baseband) waveform transmitted from the relay. Furthermore, for each packet of data, H_1, H_2 and F remain constant.

For a simpler treatment, we assume that all terminals are equipped with the same number M of antennas, and there are N sub-carriers. Then, H_1, H_2 and F are each of $L \times L$ where $L = MN$. We also assume that H_1 and H_2 are of full rank. When we evaluate the statistics (such as ergodic capacity) of the channel capacity, we will assume that H_1 and H_2 have i.i.d. $\mathcal{CN}(0, 1)$ entries.

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The received signal at the relay can be written as

$$x_1 = H_1 s + n_1 \tag{1}$$

and the received signal at the destination is therefore

$$x = H_2 F H_1 s + H_2 F n_1 + n_2 \tag{2}$$

where s is assumed to be a $L \times 1$ zero mean circularly symmetric complex Gaussian signal transmitted by the source terminal. Also assume that the source works in spatial/temporal multiplexing mode, i.e., the source transmits independent data streams from different antennas and over different sub-carriers. So, we have $E\{ss^\dagger\} = \frac{P_1}{L} I_L$, where P_1 is the transmission power used by the source. (The superscript † denotes complex conjugation.) n_1 and n_2 are independent spatio-temporally white circularly symmetric complex Gaussian noise vectors with $n_1 \sim \mathcal{CN}(0, \sigma_1^2 I_L)$ and $n_2 \sim \mathcal{CN}(0, \sigma_2^2 I_L)$.

Let R denote the covariance matrix of the noise term in (2). Then, we have

$$R = \sigma_2^2 (I_L + \frac{\sigma_1^2}{\sigma_2^2} H_2 F F^\dagger H_2^\dagger) \tag{3}$$

By applying $R^{-1/2}$ at both sides of (2) , we have

$$y = \mathcal{H} s + n \tag{4}$$

where $y = R^{-1/2} x$, $\mathcal{H} = R^{-1/2} H_2 F H_1$, and $n = R^{-1/2}(H_2 F n_1 + n_2)$. The "instantaneous" channel capacity between the source and the destination is therefore given by [4]

$$C_I = \frac{1}{2} \log_2 \det(I_L + \frac{P_1}{L} \mathcal{H}^\dagger \mathcal{H}) \tag{5}$$

The factor $1/2$ here accounts for the half duplexity. It then follows that

$$C_I = \frac{1}{2} \log_2 \det \left(I_L + \frac{P_1}{L\sigma_1^2} H_1 H_1^\dagger - \frac{P_1}{L\sigma_1^2} H_1 H_1^\dagger Q^{-1} \right) \tag{6}$$

with $Q = I_L + \frac{\sigma_1^2}{\sigma_2^2} F^\dagger H_2^\dagger H_2 F$, where we applied the matrix inversion lemma to R^{-1} and the property that $\det(I + AB) = \det(I + BA)$ where AB is complex conjugate symmetric. The so called ergodic capacity is simply the mean of C_I, i.e., $C_e = \mathcal{E}_{H_1, H_2}\{C_I\}$.

If the relay does not know any channel state information (CSI), the weighting matrix F may be chosen by maximizing the ergodic capacity C_e. It can be shown that the maximal ergodic capacity can be achieved by just using a diagonal weighting matrix. Since F is diagonal, we do not have to look for the optimal matrix structure for F. To maximize the capacity becomes to optimize the power allocation among antennas/subcarriers at the relay.

Throughout this paper, however, we assume that the relay knows both the channel matrix H_1 from the source to the relay, and the channel matrix H_2 from the relay to the destination. In addition, we assume that the source does not know CSI and the destination knows CSI. Since the relay knows H_1 and H_2, the weighting matrix F should be a function of H_1 and H_2. We will find the optimal F to maximize the instantaneous capacity C_I. When the instantaneous capacity C_I is optimized, the ergodic capacity C_e is also optimized.

To find the optimal matrix structure for F, we need the following matrix inequality (Theorem 16.8.7 [5]):

Minkowski inequality: If K_1 and K_2 are $L \times L$ non-negative definite symmetric matrix, then

$$\det(K_1 + K_2)^{1/L} \geq \det(K_1)^{1/L} + det(K_2)^{1/L} \tag{7}$$

where the equality holds when $K_1 = cK_2$ and $c \geq 0$.

From (7), we have

$$\det(K_1 - K_2)^{1/L} \leq \det(K_1)^{1/L} - \det(K_2)^{1/L} \tag{8}$$

where $K_1 - K_2$ and K_2 are non-negative definite and symmetric, and the equality holds when $K_1 = cK_2$ and $c \geq 1$.

Let $K_1 = I_L + \frac{P_1}{L\sigma_1^2} H_1 H_1^\dagger$ and $K_2 = \frac{P_1}{L\sigma_1^2} H_1 H_1^\dagger (I_L + \frac{\sigma_1^2}{\sigma_2^2} F^\dagger H_2^\dagger H_2 F)^{-1}$. It can be verified that $K_1 - K_2$ and K_2 are non-negative definite and symmetric. Then, applying (8) to (6), we have

$$C_I \leq \frac{L}{2} \log_2 \left(\det(I_L + \frac{P_1}{L\sigma_1^2} H_1 H_1^\dagger)^{\frac{1}{L}} - \frac{\det(\frac{P_1}{L\sigma_1^2} H_1 H_1^\dagger)^{\frac{1}{L}}}{\det(I_L + \frac{\sigma_1^2}{\sigma_2^2} F^\dagger H_2^\dagger H_2 F)^{\frac{1}{L}}} \right) \tag{9}$$

where the equality holds when

$$I_L + \frac{P_1}{L\sigma_1^2} H_1 H_1^\dagger = c \frac{P_1}{L\sigma_1^2} H_1 H_1^\dagger \left(I_L + \frac{\sigma_1^2}{\sigma_2^2} F^\dagger H_2^\dagger H_2 F \right)^{-1} \tag{10}$$

Using (10) in (9) leads to

$$C_I \leq \frac{1}{2} \log_2 \det \left(I_L + \frac{P_1}{L\sigma_1^2} H_1 H_1^\dagger \right) - \frac{L}{2} \log_2 \left(1 + \frac{1}{c-1} \right) \tag{11}$$

The first term on the right hand side of (11) is the capacity of the channel from the source to the relay. We can think of the second term $\frac{L}{2} \log_2(1 + \frac{1}{c-1})$ as a capacity loss due to the second hop. Because the upper bound of C_I is a uniformly increasing function of c, to maximize C_I is to maximize c which is however upper bounded by the power constraint as shown later.

Let the eigenvalue decompositions of $H_1 H_1^\dagger$ and $H_2^\dagger H_2$ be

$$H_1 H_1^\dagger = U_1 \Sigma_1 U_1^\dagger \tag{12}$$

$$H_2^\dagger H_2 = V_2 \Sigma_2 V_2^\dagger \tag{13}$$

where $H_1 H_1^\dagger$ and $H_2^\dagger H_2$ are both assumed to be of full rank, with eigenvalues (in descending order) given by $\Sigma_1 = diag\{\alpha_1, \alpha_1, \ldots, \alpha_L\}$ and $\Sigma_2 = diag\{\beta_1, \beta_1, \ldots, \beta_L\}$, respectively. Combining (12), (13) and (10) yields

$$\frac{\sigma_1^2}{\sigma_2^2} F^\dagger V_2 \Sigma_2 V_2^\dagger F = U_1 \Sigma U_1^\dagger \tag{14}$$

$$\Sigma = (c-1) I_L - c(I_L + \frac{P_1}{L\sigma_1^2} \Sigma_1)^{-1} \tag{15}$$

Since $F^\dagger H_2^\dagger H_2 F$ is nonnegative definite, all the diagonal elements of Σ must be nonnegative. This sets a lower bound on c, i.e.,

$$c \geq 1 + \frac{L\sigma_1^2}{P_1 \alpha_L} \tag{16}$$

In other words, if (16) is not satisfied, the upper bound shown in (11) may not be achievable.

From (14), we have the following optimal structure for F:

$$F = \frac{\sigma_2}{\sigma_1} V_2 \Sigma_2^{-1/2} X \Sigma^{1/2} U_1^\dagger \tag{17}$$

where X is any unitary matrix.

The power used by the relay is no greater than P_2, i.e.,

$$\sigma_1^2 Tr\{F(I_L + \frac{P_1}{L\sigma_1^2} H_1 H_1^\dagger) F^\dagger\} \leq P_2 \tag{18}$$

Using (17) in (18), we have

$$c \leq 1 + \frac{\frac{P_2}{\sigma_2^2} + Tr\{\Sigma_2^{-1}\}}{\frac{P_1}{L\sigma_1^2} Tr\{X^\dagger \Sigma_2^{-1} X \Sigma_1\}} \tag{19}$$

To maximize c, we need to minimize $Tr\{X^\dagger \Sigma_2^{-1} X \Sigma_1\}$, i.e.,

$$\min \quad J = Tr\{X^\dagger \Sigma_2^{-1} X \Sigma_1\} \tag{20}$$

$$s.t. \qquad X^\dagger X = I_L$$

We defer a further discussion of this problem to Section 4.

3. OPTIMAL MIMO RELAYING WITH DIRECT LINK

Before we try to solve (20), we show that the above analysis can be readily extended to the case when there exists a direct link between the source and the destination.

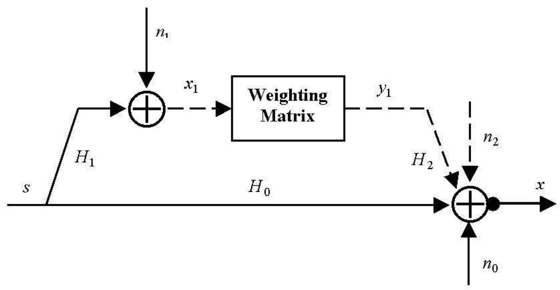

Fig. 2. A two-hop MIMO relaying channel with direct link

As shown in figure 2, the $L \times L$ channel matrix of the direct link between the source and the destination is denoted by H_0, which is also assumed to be of full rank for simplicity. n_0 is a spatio-temporally white circularly symmetric complex Gaussian noise vector with $n_0 \sim \mathcal{CN}(0, \sigma_0^2 I_L)$. Other parameters have the same definitions as in the non-line-of-sight case.

Because of the half duplexity, a transmission takes place over two time slots using "listen-and-transmit" protocol. At the first time slot, the source broadcasts signals to the relay and the destination. At the second time slot, the relay forwards the signals to the destination. Although the source may also send signals to the destination in the second time slot, we will not consider that option. We assume that the source will be silent in the second time slot.

The compound signal vector received at the destination during two time slots can be modelled as

$$\begin{bmatrix} \sigma_0^{-1} n_0 \\ R^{-1/2} x_2 \end{bmatrix} = \begin{bmatrix} \sigma_0^{-1} H_0 \\ R^{-1/2} H_2 F H_1 \end{bmatrix} s + \begin{bmatrix} \sigma_0^{-1} n_0 \\ R^{-1/2} (H_2 F n_1 + n_2) \end{bmatrix} \tag{21}$$

The instantaneous capacity between the source and the destination is now given by

$$C_I = \frac{1}{2} \log_2 \det \left(I_L + \frac{P_1}{L\sigma_0^2} H_0^\dagger H_0 + \frac{P_1}{L\sigma_1^2} H_1^\dagger H_1 - \frac{P_1}{L\sigma_1^2} H_1^\dagger \left(I_L + \frac{\sigma_1^2}{\sigma_2^2} F^\dagger H_2^\dagger H_2 F \right)^{-1} H_1 \right) \tag{22}$$

Applying the Minkowski's inequality, we have

$$C_I \leq \frac{L}{2} \log_2 \left(\det \left(I_L + \frac{P_1}{L\sigma_0^2} H_0^\dagger H_0 + \frac{P_1}{L\sigma_1^2} H_1 H_1^\dagger \right)^{\frac{1}{L}} - \frac{\det(\frac{P_1}{L\sigma_1^2} H_1 H_1^\dagger)^{\frac{1}{L}}}{\det(I_L + \frac{\sigma_1^2}{\sigma_2^2} F^\dagger H_2^\dagger H_2 F)^{\frac{1}{L}}} \right) \tag{23}$$

where the equality holds when

$$\frac{\sigma_1^2}{\sigma_2^2} F^\dagger H_2^\dagger H_2 F = (c-1) I_L - c \left(I_L + \frac{P_1}{L\sigma_1^2} H_3 H_3^\dagger \right)^{-1} \tag{24}$$

with

$$H_3 = H_1 (I_L + \frac{P_1}{L\sigma_0^2} H_0^\dagger H_0)^{-1/2} \tag{25}$$

Furthermore, (23) becomes

$$C_I \leq \frac{1}{2} \log_2 \det \left(I_L + \frac{P_1}{L\sigma_0^2} H_0^\dagger H_0 + \frac{P_1}{L\sigma_1^2} H_1^\dagger H_1 \right) - \frac{L}{2} \log_2 \left(1 + \frac{1}{c-1} \right) \tag{26}$$

This bound is similar to (11) except that now the first term is the capacity of the broadcast channel from the source to both the relay and the destination (as if the relay and the destination are wired together). We need to maximize c to minimize the capacity loss represented by the second term.

We use the following expression of the eigenvalue decomposition of $H_3 H_3^\dagger$:

$$H_3 H_3^\dagger = U_3 \Sigma_3 U_3^\dagger \tag{27}$$

where the eigenvalues (in descending order) are given by $\Sigma_3 = diag\{\gamma_1, \gamma_1, \ldots, \gamma_L\}$. Combining (25), (26), (27) and (13), we eventually have the optimal matrix structure of F:

$$F = \frac{\sigma_2}{\sigma_1} V_2 \Sigma_2^{-1/2} X \Sigma^{1/2} U_3^\dagger \tag{28}$$

where

$$\Sigma = (c-1) I_L - c \left(I_L + \frac{P_1}{L\sigma_1^2} \Sigma_3 \right)^{-1} \tag{29}$$

We see that (28) is similar to (17), and (29) is similar to (15). Furthermore, we need to solve the same optimization problem as given by (20) but with Σ_1 replaced by Σ_3.

We note that the exact expressions of F, Σ and c in this section are different from the previous section although it does not affect the method of the analysis.

4. OPTIMAL WEIGHTING MATRIX

We now go back to the case where there is no direct link between the source and the relay. Given two $L \times L$ Hermitian matrices A and B with eigenvalues $diag\{\alpha_1, \alpha_2, \cdots, \alpha_L\}$ and $diag\{\beta_1, \beta_2, \cdots, \beta_L\}$, assuming that $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_L$ and $\beta_1 \geq \beta_2 \geq \cdots \geq \beta_L$, we have the following inequality [6]:

$$\sum_{l=1}^{L} \alpha_l \beta_{L-l+1} \leq tr(AB) \leq \sum_{l=1}^{L} \alpha_l \beta_l \tag{30}$$

Based on this inequality, We have

$$\sum_{l=1}^{L} \frac{\alpha_l}{\beta_l} \leq tr(X^\dagger \Sigma_2^{-1} X \Sigma_1) \leq \sum_{l=1}^{L} \frac{\alpha_l}{\beta_{L+l-1}} \tag{31}$$

The upper bound in (31) is achieved when $X = J_L$ where J_L has all ones along the anti-diagonal line and zeros elsewhere. The lower bound is achieved when $X = I_L$.

Using the lower bound of (31) in (19), we have the upper bound of c:

$$c \leq 1 + \frac{\rho_2 + \frac{1}{L}\sum_{l=1}^{L} \beta_l^{-1}}{\rho_1 \left(\frac{1}{L}\sum_{l=1}^{L} \alpha_l \beta_l^{-1}\right)} \tag{32}$$

where $\rho_1 = \frac{P_1}{L\sigma_1^2}$ and $\rho_2 = \frac{P_2}{L\sigma_2^2}$ are the signal to noise ratios at the relay and the destination, respectively. Comparing with (16), we have the requirement for ρ_2,

$$\rho_2 \geq \frac{1}{L}\sum_{l=1}^{L} \left(\frac{\alpha_l}{\alpha_L} - 1\right) \beta_l^{-1} \tag{33}$$

Under the condition of (33), combining (32) and (11), we have the maximal instantaneous capacity of the MIMO relaying channel (without the direct link):

$$C_{I,Max} = \frac{1}{2}\log_2 \prod_{l=1}^{L} (1 + \rho_1 \alpha_l) - \frac{L}{2}\log_2 \left(1 + \rho_1 \frac{\frac{1}{L}\sum_{j=1}^{L} \alpha_j \beta_j^{-1}}{\rho_2 + \frac{1}{L}\sum_{j=1}^{L} \beta_l^{-1}}\right) \tag{34}$$

In (34), the second term represents the capacity loss due to the second hop. If ρ_2 approaches infinity, the capacity loss approaches zero. (34) can be further written as

$$C_{I,Max} = \frac{1}{2}\sum_{l=1}^{L} \log_2 \left(\frac{(1+\rho_1\alpha_l)(\rho_2 + \phi_1)}{\rho_2 + \phi_1 + \rho_1\phi_2}\right) \tag{35}$$

where

$$\phi_1 = \frac{1}{L}\sum_{j=1}^{L} \alpha_j \beta_j^{-1} \tag{36}$$

$$\phi_2 = \frac{1}{L}\sum_{j=1}^{L} \beta_j^{-1} \tag{37}$$

Combining (32), (19) and (17), the optimal weighting matrix F is given by

$$F = V_2 \Lambda_F U_1^\dagger \tag{38}$$

with $\Lambda_F = diag\{f_1, f_2, \ldots, f_L\}$ and

$$f_l = \sqrt{\frac{\sigma_2^2}{\sigma_1^2} \frac{(\rho_2 + \phi_2)\alpha_l - \phi_1}{\phi_1 \beta_l (1 + \rho_1 \alpha_l)}} \tag{39}$$

It is useful to observe that the right singular vectors of the optimal F match the left singular vectors of H_1, and the left singular vectors of the optimal F match the right singular vectors of H_2.

We note that the optimal weighting matrix derived in this paper is valid only when ρ_2 meets the condition given by (33). This condition requires that the signal to noise ratio at the destination is high, especially when H_1 is ill-conditioned. At this time, we do not know the optimal weighting matrix F when (33) does not hold.

However, even though (33) may not hold, we can still use (38), provided that the power used by the relay does not exceed P_2. In other words, if (33) does not hold, the signal to be transmitted from the relay needs to be normalized to meet the constraint (18). We thus introduce a normalization factor η in the weighting matrix:

$$F^* = \eta V_2 \Lambda_F U_1^\dagger \tag{40}$$

$$\eta = \sqrt{\frac{L\rho_2}{Tr\{\Lambda_F (I_L + \rho_1 \Sigma_1) \Lambda_F^\dagger\}}} \tag{41}$$

When the condition (33) holds, $\eta = 1$ and $F^* = F$ is the optimum.

5. NUMERICAL RESULTS

In this section, we compare the optimal relaying scheme given by (40) with two other relaying schemes in terms of the ergodic capacity of the MIMO relaying channel (without direct link). We note that all schemes compared here (including the one shown earlier) can be classified as ”amplify-forward” although the levels of optimality differ.

(1) (Simplest) Amplify-Forward: This is a relaying scheme where the received signal is simply normalized to meet the power constraint and then forwarded to the destination. In this case, the weighting matrix at the relay is

$$F_1 = \eta_1 I_L \tag{42}$$

The power constraint is given by (18), and hence

$$\eta_1 = \sqrt{\frac{\sigma_2^2}{\sigma_1^2} \frac{L\rho_2}{Tr\{I_L + \rho_1 H_1 H_1^\dagger\}}} \tag{43}$$

(2) Match-Forward: This is another simple scheme where the weighting matrix at the relay as used in [3] is

$$F_2 = \eta_2 H_2^\dagger H_1^\dagger \tag{44}$$

To meet the power constraint, η_2 is given by

$$\eta_2 = \sqrt{\frac{\sigma_2^2}{\sigma_1^2} \frac{L\rho_2}{Tr\{H_1^\dagger H_1 (I_L + \rho_1 H_1^\dagger H_1) H_2 H_2^\dagger\}}} \tag{45}$$

Figure 3 shows the ergodic capacity of the relaying channel as a function of ρ_2. Figure 5 shows the ergodic capacity of the relay channel as a function of L, the number of antennas at each terminal. Figure 4 shows the ergodic capacity of the relay channel

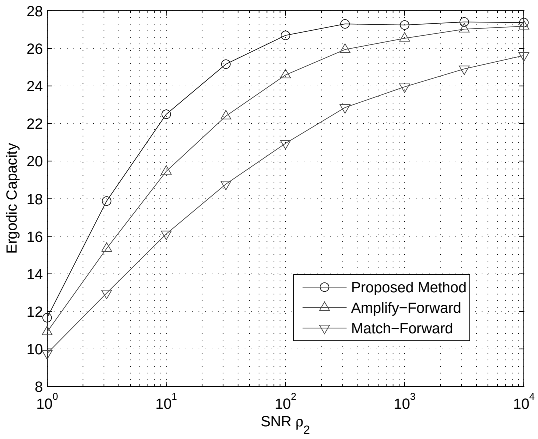

Fig. 3. Ergodic capacity as a function of ρ_2. $L = 10$, $\rho_1 = 10dB$, $0dB \leq \rho_2 \leq 40dB$

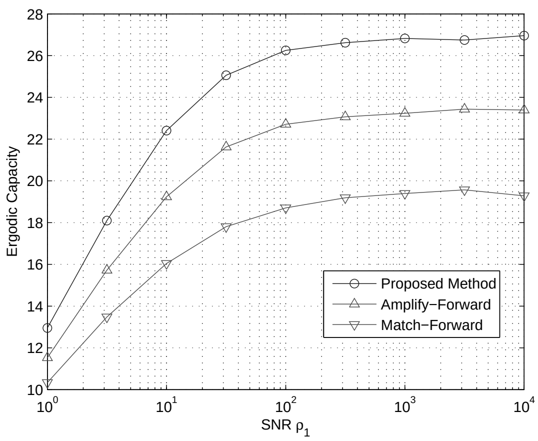

Fig. 4. Ergodic capacity as a function of ρ_1. $L = 10$, $0dB \leq \rho_1 \leq 40dB$, $\rho_2 = 10dB$

as a function of ρ_1. Note that there is no requirement on ρ_1 to ensure the existence of the optimal weighting matrix F.

It is clear that the amplify-forward is far from the optimum. The match-forward method is even worse than the amplify-forward method, and therefore the weighting matrix given by (44) actually does not really "match" the channels.

6. CONCLUSIONS AND FUTURE WORK

In this paper, we have considered the capacity maximizing memoryless non-regenerative MIMO relaying scheme. It is shown that the optimal weighting matrix exists under a certain condition on the power used by the relay terminal. Future work could consider how to relax the condition. We have detailed the optimal MIMO relay channel capacity when there is no direct link between the source and the destination. However, as we also showed, it is easy to extend the analysis to the case where there exists a direct link. We did not consider the power allocation between the source and

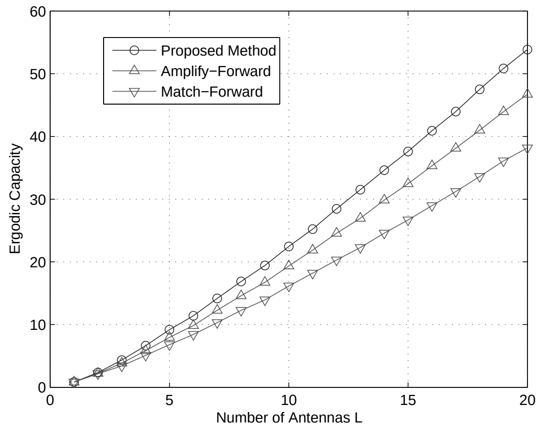

Fig. 5. Ergodic capacity as a function of L, the number of antennas at each terminals. $1 \leq L \leq 20$, $\rho_1 = 10dB$, $\rho_2 = 10dB$

the relay under a total power constraint. Future work will consider the optimal power allocation. We also need to analyze the power and spectral efficiency of different schemes. We assumed that all terminals have the same number of antennas. All channel matrices are of full rank. Future work will consider the case where a different number of antennas is employed at each terminal and/or the channel matrices are singular. We used the ergodic capacity in numerical comparisons. The variance or distribution of the instantaneous capacity could be more meaningful for packet loss rate computations. Numerical comparisons of this type will be shown in our upcoming report.

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