Maintaining Information Freshness under Jamming

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***Abstract*—In UAV communication with a ground control station, mission success requires maintaining the freshness of the received information, especially when the communication faces hostile interference. We model this problem as a game between a UAV transmitter and an adversarial interferer. We prove that in contrast with the Nash equilibrium, multiple Stackelberg equilibria could arise. This allows us to show that reducing interference activity in the Stackelberg game is achieved by higher sensitivity of the transmitter in the Stackelberg equilibrium strategy to network parameters relative to the Nash equilibrium strategy. All the strategies are derived in closed form and we establish the condition for when multiple strategies arise.**

***Index Terms*—Age of Information, Equilibrium, Jamming**

I. Introduction

In many applications, Unmanned Aerial Vehicles (UAV) communicate with a Ground Control Station (GCS) to receive instructions or accurate positioning. However when such active communication poses a threat, hostile interference may cause delay or even interruption in getting such instructions. Larger delay or interruption, and thus reduced freshness of received instructions, can decrease the probability of mission success. Thus we model the probability of mission success as a function of the age of received information. Note that age-of-information (AoI) is a new system delay performance metric that has been widely employed in different scenarios [1]–[8].

In our model, the UAV transmitter and the interferer each seek a tradeoff between the probability of mission success and the involved cost of its effort. Thus we apply a game-theoretic approach [9] and examine such multi-objective solution concepts as Nash and Stackelberg equilibria. These concepts have been widely used to deal with various jamming problems where power levels are controlled by the agents; see, for example, for Nash equilibrium [10]–[14] and Stackelberg equilibrium [15]–[19].

This work, however, builds on [20], the first paper to study the impact of hostile interference on age of information. Our model of iteration between the UAV and the interferer extends the data updating model in [20] and addresses some complementary problems associated with [20]. In particular,

- We design Nash equilibrium strategies in the presence of background noise. In [20], Nash equilibrium strategies were derived in the absence of background noise.
- We reveal how Nash equilibrium strategies depend on the transmitter's packet updating rate. In the absence of background noise, the Nash equilibrium is insensitive to this rate [20].
- We stabilize hierarchal relations between the transmitter and the interferer when the transmitter is the leader. This stabilization is reflected by the existence of Stackelberg equilibrium, in contrast to the case of zero background noise where such an equilibrium does not exist [20].

The organization of this paper is as follows: in Section II, we present our basic communication model involving hostile interference. In Section III, we incorporate the age of packets updates in the model. In Section IV, the Nash equilibrium is characterized, while in Section V, the Stackelberg equilibrium is found. Finally, in Section VI, a discussion of the results is provided.

II. Basic model

We assume that the UAV in following its route/mission has to communicate with the GCS to get/verify its position and mission data. This communication can be damaged by hostile interference that might lead to loss of the UAV coordinates and failure of the mission. This is why it is important to update data in a timely way. As a metric of data updating in this paper we consider AoI, which reflects the time that has passed since the last update. We assume that probability π_F of mission failure is a function of an age of information metric A, and this function is increasing with A such that: (a) $\pi_F(0) = 0$, and (b) $\pi_F(A) \uparrow 1$ as $A \uparrow \infty$. We note that condition (a) says that if the data is always up-to-date then the mission will be successful with certainty while (b) says that if data is never updated, then the mission fails with certainty.

To model the probability of mission success, we will use the ratio form contest success function. This is commonly used to translate involved resources into probability of winning or losing, and has been widely applied in different economic and attack-defense problems in the literature; see, for example, [21]–[24]. In our scenario, the metric that dictates whether the mission is successful is age of information. Specifically, in terms of positive constants$a$ and b, the probability of mission failure is

$$\pi_F(A) = \frac{aA}{b + aA}, \tag{1}$$

and the probability of mission success is

$$\pi_S(A) = 1 - \pi_F(A) = \frac{b}{b + aA}. \tag{2}$$

III. Age of Information

To model age of information, we will employ a generalization of the model introduced in [20]. For convenience of the readers, we give a brief description:

- The UAV can transmit at a rate that is proportional to the signal to interference plus noise ratio (SINR) at the receiver. Following [20], when p and q are the powers of the transmitting and interfering signals, and h and g are the corresponding channel gains to the GCS, the packet transmission rate associated with power profile (p,q) is

$$\mu(p,q) = z\text{SINR}(p,q) = z\frac{gp}{N+hq}, \tag{3}$$

where N is background noise power and z is a positive constant.

- Depending on the model for how update packets are delivered to the GCS, the age of information metric A takes on the form

$$A(p,q) = \frac{c}{\lambda} + \frac{d}{\mu(p,q)} \tag{4}$$

for packet arrival rate λ and constants $c \geq 0$ and $d > 0$. In particular, when $(c,d) = (1,2)$ and fresh update packets are generated at the UAV as a rate λ Poisson process, the age metric $A(p,q)$ corresponds to the average peak age of an M/G/1/1 queue [25], [26]. This is the age metric employed in [20].

We note that various other age metrics can be modeled by specifying (c,d) in (4). For example, with $(c,d) = (1,1)$, $A(p,q)$ is the average AoI of an M/M/1/1 server supporting preemption in service [27]. Furthermore, with $(c,d) = (0,2)$ and just-in-time arrivals (i.e., a fresh update goes into service precisely when the server would become idle) at a rate $\mu(p,q)$ memoryless server, $A(p,q)$ is again the average AoI [28]. Finally, with $(c,d) = (0,3/2)$, $A(p,q)$ corresponds to just-in-time updates transmitted with deterministic service times at rate $1/\mu(p,q)$ [28]. In the following, we refer to $A(p,q)$ as the AoI for any $c \geq 0$ and $d > 0$.

In [20], it was proposed that the transmitter and interferer are active agents with p and q as strategies. The sum of the AoI and cost of the involved effort is the transmitters's cost function

$$u_T(p,q) = A(p,q) + w_T p, \tag{5}$$

while the difference between the AoI and cost of the involved effort is the interferer's payoff function

$$u_I(p,q) = A(p,q) - w_I q, \tag{6}$$

where w_T and w_I are the respective costs per unit of transmitted power.

The transmitter wants to minimize its cost function while the interferer wants to maximize its payoff function. In [20, Theorem 3], it was shown that for $c = 1$ and $d = 2$ this problem has a unique equilibrium under assumption that $N = 0$. In particular, these equilibrium strategies are $p = 2h/(gzw_I)$ and $q = 2hw_T/(gzw_I^2)$. An interesting feature of these strategies is that they are indifferent to the update rate λ of the transmitter.

IV. Nash equilibrium

In this section we model the difference between the probability of mission success and the involved cost of efforts as the transmitter's payoff function

$$\begin{aligned} v_T(p,q) &= \pi_S(A(p,q)) - w_T p \\ &= \frac{bzgp}{(b+a/\lambda)zgp + 2a(N+hq)} - w_T p. \end{aligned} \tag{7}$$

For the interferer, the payoff function is the difference

$$\begin{aligned} v_I(p,q) &= \pi_F(A(p,q)) - w_I q \\ &= \frac{a(zgp/\lambda + 2(N+hq))}{(b+a/\lambda)zgp + 2a(N+hq)} - w_I q. \end{aligned} \tag{8}$$

between the probability mission failure and the involved cost of the effort.[1] Thus, $\mathbb{R}_+$ is the set of feasible strategies for the transmitter as well as for the interferer. We also note that for $b = 1$ and sufficiently small a, $u_I(p,q)$ given by (6) is a linear approximation of $v_I(p,q)$ given by (8).

We now introduce the auxiliary notations:

$$\alpha = bgz,\ \beta = acgz/\lambda,\ \gamma = dah,\ \text{and } \delta = daN. \tag{9}$$

With this notation, (7) and (8) become

$$v_T(p,q) = \frac{\alpha p}{(\alpha+\beta)p + \gamma q + \delta} - w_T p, \tag{10}$$

$$v_I(p,q) = \frac{\beta p + \gamma q + \delta}{(\alpha+\beta)p + \gamma q + \delta} - w_I q. \tag{11}$$

We are looking for Nash equilibrium (NE). Recall that (p,q) is a Nash equilibrium [9] if and only if for any $(\tilde{p},\tilde{q})$,

$$v_T(\tilde{p},q) \leq v_T(p,q) \text{ and } v_I(p,\tilde{q}) \leq v_I(p,q). \tag{12}$$

Denote this game by Γ_N. By (12), (p,q) is Nash equilibrium if and only if each of these strategies is the best response to the other, i.e.,

$$\begin{aligned} p &= \text{BR}_T(q) = \operatorname*{argmax}_{p\in\mathbb{R}_+} v_T(p,q), \\ q &= \text{BR}_I(p) = \operatorname*{argmax}_{q\in\mathbb{R}_+} v_I(p,q). \end{aligned} \tag{13}$$

To derive the best response strategies in a simplified non-bulky closed form, we introduce the following auxiliary notations:

$$\overline{\beta} = 1 + \beta/\alpha,\ \overline{\gamma} = \gamma/\alpha \text{ and } \overline{\delta} = \delta/\alpha. \tag{14}$$

Thus, $\overline{\beta} \geq 1$ while $\overline{\gamma} > 0$ and $\overline{\delta} \geq 0$.

Theorem 1: *(a) $v_T(p,q)$ is concave in p.*

(b) For the transmitter, the best response strategy is

$$\begin{aligned} p &= BR_T(q) \\ &= \begin{cases} \dfrac{1}{\overline{\beta}}\left(\sqrt{\dfrac{\overline{\gamma}q+\overline{\delta}}{w_T}} - \overline{\gamma}q - \overline{\delta}\right), & w_T < \dfrac{1}{\overline{\gamma}q+\overline{\delta}}, \\ 0, & w_T \geq \dfrac{1}{\overline{\gamma}q+\overline{\delta}}. \end{cases} \end{aligned} \tag{15}$$

[1] w_T and w_I in (5) and (6) will have different values than in (7) and (8) because the players' utilities are different.

(c) $v_I(p,q)$ is concave in q.
(d) For the interferer, the best response strategy is

$$q = BR_I(p) = \begin{cases} \frac{1}{\overline{\gamma}}\left(\sqrt{\frac{\overline{\gamma}p}{w_I}} - \overline{\beta}p - \overline{\delta}\right), & w_I < \frac{\overline{\gamma}p}{(\overline{\beta}p+\overline{\delta})^2}, \\ 0, & w_I \geq \frac{\overline{\gamma}p}{(\overline{\beta}p+\overline{\delta})^2}. \end{cases} \quad (16)$$

PROOF: It follows from (10) that

$$\frac{\partial^2 v_T(p,q)}{\partial p^2} = -\frac{2\overline{\beta}(\overline{\gamma}q+\overline{\delta})}{(\overline{\beta}p+\overline{\gamma}q+\overline{\delta})^3} < 0. \quad (17)$$

Thus, $v_T(p,q)$ is concave on p and $p \in \mathbb{R}_+$ is the best response strategy to q if and only if

$$\frac{\partial v_T(p,q)}{\partial p} = -w_T + \frac{\overline{\gamma}q+\overline{\delta}}{(\overline{\beta}p+\overline{\gamma}q+\overline{\delta})^2} \begin{cases} = 0, & p > 0, \\ \leq 0, & p = 0. \end{cases} \quad (18)$$

It follows from (18) that (15) holds. Note that

$$\frac{\partial^2 v_I(p,q)}{\partial q^2} = -\frac{2\overline{\gamma}^2 p}{(\overline{\beta}p+\overline{\gamma}q+\overline{\delta})^3} < 0. \quad (19)$$

By (19), $v_I(p,q)$ is concave on q. Thus, $q \in \mathbb{R}_+$ is the best response strategy to p if and only if:

$$\frac{\partial v_I(p,q)}{\partial q} = -w_I + \frac{\overline{\gamma}p}{(\overline{\beta}p+\overline{\gamma}q+\overline{\delta})^2} \begin{cases} = 0, & q > 0, \\ \leq 0, & q = 0, \end{cases} \quad (20)$$

and (16) follows. ■

THEOREM 2: *In the game Γ_N, there exists at least one Nash equilibrium.*

PROOF: By the Nash Theorem [9], an equilibrium exists if the payoffs are concave on corresponding strategies and the set of feasible strategies are compact. In our case, by Theorem 1, v_T is concave on p and v_I is concave on q, but the set of feasible strategies $\mathbb{R}_+$ is not compact. By (18) , v_T is strictly decreasing for enough large p, say, for any $p \geq \overline{p}$. By (20), v_I is strictly decreasing for enough large q, say, for any $q \geq \overline{q}$. Thus, if equilibrium (p,q) exits it has to belong to $[0,\overline{p}]\times[0,\overline{q}]$. Since this set is compact, at least one equilibrium exists, and the result follows. ■

To prove uniqueness of the equilibrium, we employ a constructive approach. Specifically, in the proof of the following theorem we first derive properties that each solution of the best response equations has to have, and then show that the only power profile (p,q) satisfies these properties.

THEOREM 3: *In the game Γ_N, the Nash equilibrium (p,q) is unique. Moreover, it is given as follows:*

(a) *If*

$$1/\overline{\delta} \leq w_T \quad (21)$$

then

$$p = q = 0. \quad (22)$$

(b) *If*

$$w_T < 1/\overline{\delta} \quad (23)$$

then

(b-i) *if*

$$w_T \leq \overline{\delta}(w_I\overline{\beta}/\overline{\gamma} + w_T)^2 \quad (24)$$

then

$$p = p_\star \triangleq \frac{1}{\overline{\beta}}\left(\sqrt{\frac{\overline{\delta}}{w_T}} - \overline{\delta}\right) \text{ and } q = 0, \quad (25)$$

(b-ii) *if*

$$w_T > \overline{\delta}(w_I\overline{\beta}/\overline{\gamma} + w_T)^2 \quad (26)$$

then

$$p = \frac{w_I\overline{\gamma}}{(\overline{\beta}w_I + \overline{\gamma}w_T)^2}, \quad (27)$$

$$q = \frac{w_T\overline{\gamma}}{(\overline{\beta}w_I + \overline{\gamma}w_T)^2} - \frac{\overline{\delta}}{\overline{\gamma}}. \quad (28)$$

PROOF: By (20), if $p = 0$ then $q = 0$. Substituting $p = q = 0$ into (18) implies (21), and (a) follows. Thus, we have to consider separately only two cases: (I) $p > 0$ and $q = 0$ and (II) $p > 0$ and $q > 0$.

(I) Let $p > 0$ and $q = 0$. Then, (18) and (20) turn into:

$$\overline{\delta}/(\overline{\beta}p+\overline{\delta})^2 = w_T, \quad (29)$$

$$\overline{\gamma}p/(\overline{\beta}p+\overline{\delta})^2 \leq w_I. \quad (30)$$

Solving (29) for p yields (25). Since $p > 0$ (25) yields (23).
Dividing (30) by (29) we have that $\overline{\gamma}p/\overline{\delta} \leq w_I/w_T$. Substituting (25) into the last inequality yields

$$\frac{\overline{\gamma}}{\overline{\beta}\,\overline{\delta}}\left(\sqrt{\frac{\overline{\delta}}{w_T}} - \overline{\delta}\right) \leq \frac{w_I}{w_T}, \quad (31)$$

and (b-i) follows.

(II) Let $p > 0$ and $q > 0$. Then, (18) and (20) turn into:

$$\frac{\overline{\gamma}q+\overline{\delta}}{(\overline{\beta}p+\overline{\gamma}q+\overline{\delta})^2} = w_T, \quad (32)$$

$$\frac{\overline{\gamma}p}{(\overline{\beta}p+\overline{\gamma}q+\overline{\delta})^2} = w_I. \quad (33)$$

Since the left-side of (32) is decreasing in p to zero as p tends to infinity, by (32) and the fact that p has to be positive, the following inequality has to hold:

$$1/(\overline{\gamma}q+\overline{\delta}) > w_T. \quad (34)$$

Furthermore, the left-side of (34) is decreasing in q to zero as q tends to infinity. Thus, by (32) and (34) for p and q to be positive (23) has to hold.
Dividing (33) by (32) yields

$$(w_T/w_I)\overline{\gamma}p = \overline{\gamma}q+\overline{\delta}. \quad (35)$$

Substituting (35) into (33) yields

$$\frac{\overline{\gamma}}{\left(\overline{\beta}+\overline{\gamma}w_T/w_I\right)^2 p} = w_I. \quad (36)$$

This implies p given by (27). Substituting this p into (32) yields q given by (27). Since $q > 0$ (25) implies (26), and (b-ii) follows. ■

Note that Nash equilibrium found in [20] is a boundary case of equilibrium given by Theorem 3. Namely, the following result holds.

PROPOSITION 1: *For $N = 0$, $c = 1$ and $d = 2$ we have that*

$$p \approx \frac{2h}{gz}\frac{a}{w_I} \text{ and } q \approx \frac{2h}{gz}\frac{w_T}{a}\left(\frac{a}{w_I}\right)^2 \quad (37)$$

for enough small a and $b \approx 1$.

Recalling that w_T and w_I in (7), (8) and (5) and (6) are different because players' utilities are different and assigning a as conversion factor implies that (37) coincides with equilibrium of model derived in [20, Theorem 3].

PROOF: If $N = 0$ then, by (9), (14), $\overline{\delta} = 0$. Then, (23) and (26) hold, and, so, $p > 0$ and $q > 0$. Moreover, for $b \approx 1$ and enough small a we have that

$$\overline{\beta} = 1 + a/(b\lambda) \approx 1 \text{ and } \overline{\gamma} = 2ah/(bzg) \approx 2ah/(zg). \quad (38)$$

Substituting (38) into (27) implies

$$p = \frac{w_I\overline{\gamma}}{(\overline{\beta}w_I + \overline{\gamma}w_T)^2} \approx \frac{\overline{\gamma}}{w_I} \approx \frac{2h}{gz}\frac{a}{w_I}, \quad (39)$$

$$\begin{aligned} q &= \frac{w_T\overline{\gamma}}{(\overline{\beta}w_I + \overline{\gamma}w_T)^2} - \frac{\overline{\delta}}{\overline{\gamma}} = (\text{since } \overline{\delta} = 0) \\ &= \frac{w_T\overline{\gamma}}{(\overline{\beta}w_I + \overline{\gamma}w_T)^2} \approx \frac{\overline{\gamma}w_T}{w_I^2} \approx \frac{2haw_T}{gzw_I^2} = \frac{2h}{gz}\frac{w_T}{a}\left(\frac{a}{w_I}\right)^2, \end{aligned} \quad (40)$$

and the result follows. ■

V. STACKELBERG EQUILIBRIUM

In this section, we consider a hierarchical relationship between the transmitter and the interferer, where the transmitter is the leader and the interferer is the follower. Such a scenario is formulated as a two-level optimization problem, with the transmitter at the top-level and the interferer at the lower-level. The problem can be solved in two steps by backward induction, and the solution is referred to as the Stackelberg equilibrium (SE), while the game is called a Stackelberg game (SG) [9]:

- *In the first step* of the two-level game: for a fixed p, determined by the transmitter, the interferer tries to maximize its payoff. Thus, by Theorem 1, the interferer intends to apply strategy $q = \text{BR}_I(p)$, which is given in closed form by (16).
- *In the second step* of the two-level game: the transmitter selects the optimal p to get a maximal payoff, i.e., to solve the optimization problem

$$p = \text{argmax}\{F_T(p) : p \in \mathbb{R}_+\}, \quad (41)$$

where

$$F_T(p) \triangleq v_T(p, \text{BR}_I(p)). \quad (42)$$

Such $(p, q = \text{BR}_I(p))$ is called a Stackelberg equilibrium. We denote this Stackelberg game by Γ_S.

Substituting (16) into (42) implies that

$$F_T(p) = \begin{cases} M_\#(p), & p \in I_\#, \\ M_0(p), & p \in I_0, \end{cases} \quad (43)$$

where

$$M_\#(p) \triangleq \sqrt{w_I p/\overline{\gamma}} - w_T p, \quad (44)$$

$$M_0(p) \triangleq p/(\overline{\beta}p + \overline{\delta}) - w_T p, \quad (45)$$

$$I_\# \triangleq \left\{p \in \mathbb{R}_+ : w_I < \overline{\gamma}p/(\overline{\beta}p + \overline{\delta})^2\right\}, \quad (46)$$

$$I_0 \triangleq \left\{p \in \mathbb{R}_+ : w_I \geq \overline{\gamma}p/(\overline{\beta}p + \overline{\delta})^2\right\}. \quad (47)$$

THEOREM 4: *In the game Γ_S there exists at least one Stackelberg equilibrium.*

PROOF: By Theorem 1, $\text{BR}_I(p)$ is continuous on $p \geq 0$. Moreover, by (42), $F_T(p)$ is continuous in $\mathbb{R}_+$, and, by (43), it tends to $-\infty$ as p tends to infinity. Thus, $F_T(p)$ achieves its maximum, and the result follows. ■

Note that, for the model considered in [20, Theorem 4], the Stackelberg equilibrium does not exist for such hierarchical relation between the rivals.

To characterize the Stackelberg equilibrium we first present the following auxiliary result.

PROPOSITION 2:

(a) If

$$w_I \geq \overline{\gamma}/(4\overline{\beta}\overline{\delta}) \quad (48)$$

then $I_\#$ is an empty set.

(b) If

$$w_I < \overline{\gamma}/(4\overline{\beta}\overline{\delta}) \quad (49)$$

then $I_\# = (p_-, p_+)$, where $p_- < p+$ are such that

$$BR_I(p_-) = BR_I(p_+) = 0, \quad (50)$$

i.e.,

$$p_\pm = \left(\sqrt{\overline{\gamma}/w_I} \pm \sqrt{\overline{\gamma}/w_I - 4\overline{\beta}\overline{\delta}}\ \right)/(2\overline{\beta}). \quad (51)$$

(c) $M_\#(p)$ is concave in p and achieves its maximum at

$$p_\# = w_I/(4\overline{\gamma}w_T^2). \quad (52)$$

(d) $M_\#(p_\#) = w_I/(4\overline{\gamma}w_T) > 0$.

(e) $M_0(p)$ is concave in p and achieves its maximum at

$$p_\diamond = \max\{p_\star, 0\} = \max\left\{\frac{1}{\overline{\beta}}\left(\sqrt{\frac{\overline{\delta}}{w_T}} - \overline{\delta}\right), 0\right\}. \quad (53)$$

(f) $F_T(0) = 0$.

(g) If (49) holds then

$$M_\#(p_-) = M_0(p_-) \text{ and } M_\#(p_+) = M_0(p_+). \quad (54)$$

PROOF: Note that $w_I \geq \dfrac{\overline{\gamma}p}{(\overline{\beta}p + \overline{\delta})^2}$ is equivalent to $\overline{\beta}p + \overline{\delta} \geq \sqrt{\overline{\gamma}p/w_I}$. Straightforward solving of this inequality and taking into account (46) yields (a) and (b), where $p_\pm$ are two roots of the equation:

$$\overline{\beta}p_\pm + \overline{\delta} = \sqrt{\overline{\gamma}p_\pm/w_I}, \tag{55}$$

or, by (51), the equivalent, roots of the (50). (c)-(f) also can be shown by straightforward calculations, while (g) follows from (44), (45) and (55). ■

By (43), the form of transmitter payoff $F_T(p)$ depends on whether the set $I_\#$ is empty or non-empty. These two cases are specified by conditions (48) and (49). Moreover, in case (49), we will have consider separately two cases corresponding to the situations whether $M_0(p)$ gets its maximum at the boundary point $p = 0$ (i.e., (21) holds) or at the inner point $p > 0$ (i.e., (24) holds). In the following three theorems corresponding to for each of these cases, we derive the Stackelberg equilibrium as well as establish the condition for the Stackelberg equilibrium to be unique.

THEOREM 5: *Let (48) hold. Then the transmitter Stackelberg equilibrium is unique, and it is $p = p_\diamondsuit$ given by (53).*

Thus, in this case Stackelberg equilibrium coincides with Nash equilibrium.

PROOF: If (48) holds, then, by Proposition 2(a), $I_\#$ is empty set. Thus, the result follows from Proposition 2(e). ■

THEOREM 6: *Let (21) and (49) hold. Then the transmitter Stackelberg equilibrium is unique and given as follows:*

$$p = \begin{cases} p_\#, & p_- < p_\# \le p_+, \\ 0, & \textit{otherwise}. \end{cases} \tag{56}$$

PROOF: Since (49) holds, it follows from Proposition 2(b) that the set $I_\#$ is not empty. Moreover, since (21) holds, (43) and (45) imply, $\frac{dF_T(p)}{dp} < 0$ for $p \in I_0$, and $p_\diamondsuit = 0$. Then, the result follows from Propositions 2(c), (d) and (f). ■

THEOREM 7: *Let (24) and (49) hold. Then the transmitter Stackelberg equilibrium is unique except two particular cases where two transmitter Stackelberg equilibrium arise.*

(a) If

$$p_\star \le p_- \tag{57}$$

then

$$p = \begin{cases} p_\star, & p_\# \le p_-, \\ p_\#, & M_0(p_\star) < M_\#(p_\#) \& p_- < p_\# \le p_+, \\ p_\star, & M_0(p_\star) > M_\#(p_\#) \& p_- < p_\# \le p_+, \\ p_\star \textit{ and } p_\#, & M_0(p_\star) = M_\#(p_\#) \& p_- < p_\# \le p_+. \end{cases} \tag{58}$$

(b) If

$$p_- < p_\star \le p_+ \tag{59}$$

then

$$p = \begin{cases} p_-, & p_\# < p_-, \\ p_\#, & p_- \le p_\# \le p_+, \\ p_+, & p_+ < p_\#. \end{cases} \tag{60}$$

(c) If

$$p_+ < p_\star \tag{61}$$

then

$$p = \begin{cases} p_\#, & M_0(p_\star) < M_\#(p_\#) \& p_- < p_\# \le p_+, \\ p_\star, & M_0(p_\star) > M_\#(p_\#) \& p_- < p_\# \le p_+, \\ p_\star \textit{ and } p_\#, & M_0(p_\star) = M_\#(p_\#) \& p_- < p_\# \le p_+, \\ p_\star, & p_+ < p_\#. \end{cases} \tag{62}$$

PROOF: (a) Let (57) hold. First note that, then $p_\# \le p_+$. Assume that $p_\# > p_+$. Then, by Proposition 2(c), $M_\#$ is increasing in (p_-, p_+), and so,

$$M_\#(p_-) < M_\#(p_+). \tag{63}$$

While, by assumption (57), by Proposition 2(e), M_0 is decreasing in $[p_-, \infty)$. Thus, $M_0(p_-) < M_0(p_+)$, and this leads to contradiction with (63) by Proposition 2(g).

Let $p_\# \le p_-$. Then, by Propositions 2(c) and (e), $F_T(p)$ is decreasing in $[p_-, \infty)$. Thus, $p = p_\star$.

Let $p_- < p_\# \le p_+$. Then, by Propositions 2(c) and (e), $F_T(p)$ is decreasing in $[p_\#, \infty)$. Thus, $p = p_\star$ or $p = p_\#$, and (a) follows.

(b) Let (59) hold. Then, by Proposition 2(c),

$$F_T(p) \text{ is } \begin{cases} \text{increasing in } [0, p_-] \\ \text{decreasing in } [p_+, \infty) \end{cases} \tag{64}$$

and

$$F_T(p) \text{ is } \begin{cases} \text{decreasing in } [p_-, p_+], & p_\# \le p_-, \\ \text{maximum at } p = p_\#, & p_- < p_\# < p_+, \\ \text{increasing in } [p_-, p_+], & p_+ < p_\#. \end{cases} \tag{65}$$

Then, (64) and (65) imply (60). The case (c) can be proven similarly to case (a). ■

VI. DISCUSSION OF THE RESULTS

Explicit formulas of the transmitter Stackelberg equilibrium given in Theorem 6 and Theorem 7 imply that such a strategy can have jumps with varying network parameters, i.e., the transmitter Stackelberg equilibrium can be sensitive to network parameters in contrast with the transmitter Nash equilibrium, which is continuous in network parameters, according to the explicit formulas given by Theorem 3. We illustrate this phenomena by the example shown in Fig. 1. An increase in jamming cost w_I makes the interferer reduce its jamming effort and this allows the transmitter to increase its transmission power. All together, this leads to an increase in the transmitter's payoff. When the jamming power q approaches zero level the Nash and Stackelberg behaviours of the transmitter diverge drastically. According to the Nash strategy, to make the interferer to stay at $q = 0$, the transmitter has to keep the same strategy, and this returns the same payoff. However, according to the Stackelberg strategy, to make the interferer to stay at $q = 0$, if this interferer's level was achieved by applying strategy $p = p_+$, by (50), the same result can be achieved by a jump of the transmitter to the less effort expensive strategy $p = p_-$. Such a jump in transmitter's strategy can also lead to an increase by a jump in the transmitter's payoff (Fig. 1(f)).

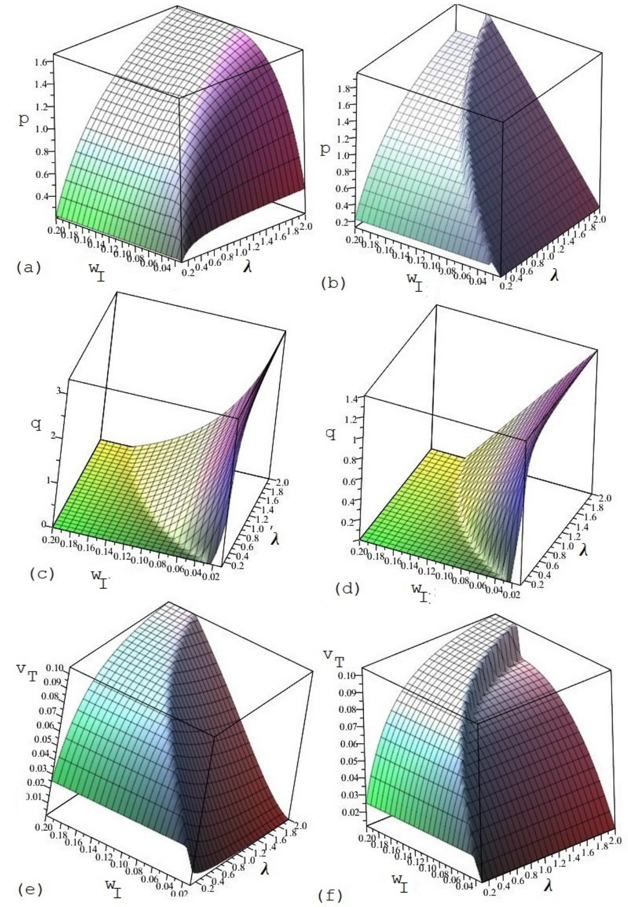

Fig. 1. (a) Nash transmitter strategy, (b) Stackelberg transmitter strategy, (c) Nash interferer strategy, (d) Stackelberg interferer strategy, (e) Payoff to the transmitter in Nash game, (f) payoff to the interferer in Stackelberg game for $a = 1$, $b = 1$, $c = 1$, $d = 2$, $z = 1$, $N = 1$, $g = 1$, $h = 1$, $w_T = 0.1$, .

Note that hierarchical relation between the transmitter and the interferer reduces hostile interference (Fig. 1(c) and Fig. 1(d)). An increase in the update transmission rate at the transmitter λ makes the interferer to increase jamming efforts. In response, the transmitter also has to increase its transmission efforts. Finally, as explanation why in contrast with original model of information freshness under jamming [20], in the suggested extension, equilibrium always exists, we refer to a similarity with matrix games [9] where extension of the set of feasible strategies from pure strategies to mixed strategies (so, payoffs to expected payoffs) leads to existence of equilibrium.

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