ECE 332:202, Discrete Mathematics
Exam 1
Spring 2007

Exam Type: B

Instructions: The exam consists of two parts. The first part is 20 multiple choice questions (at 4 points each). The second part is a single "long" question (worth 20 points). Fill in the bubble answer sheet for the first part, and write your answer to the long question following that problem. You will turn in the bubble answer sheet and the answer to the long question.

Multiple Choice

1. Recall that if p and q are binary propositions, we may define a new proposition p *if and only if* q, denoted $p \leftrightarrow q$. $p \leftrightarrow q$ is equal to:
   (a) $(p \rightarrow q) \vee (q \rightarrow p)$.
   (b) $(p \oplus q)$
   (c) $\overline{(p \oplus q)}$
   (d) None of these.

2. Consider the function $f(x) = 3x^2 + 8x\ lgx$. Which of the following describes the complexity of $f(x)$?
   (a) $\Theta(1)$
   (b) $\Theta(x\ lgx)$
   (c) $\Theta(x^2)$
   (d) $\Theta(x^3)$

3. Identify the complexity of the following pseudo-code by determining the amount of times $x = x + 1$ occurs:

```
for i = 1 to n,
   for j = 1 to n,
     for k = 1 to n,
       x = x+1
     endfor
   endfor
 endfor
```

   (a) $\Theta(n^3)$
   (b) $\Theta(n\ lgn)$
   (c) $\Theta(1)$
   (d) $\Theta(2^n)$

4. What is the value of $\sum_{k=4}^{8}(-1)^k$?
   (a) 1
   (b) -1
   (c) 0
   (d) 2

5. Let $b_n = n(-1)^n$. Which of the following is true about

$$c = \sum_{i=1}^{n} b_i?$$

   (a) $c_n = n/2$ when n is odd.
   (b) $c_n = (n-1)/2 + n$ when n is odd
   (c) $c_n = (n-1)/2 - n$ when n is odd.
   (d) None of these.

6. Define the function $f(n)$ by $f(1) = 1$ and $f(n) = n + f(n-1)$ for $n > 1$. What is a non-inductive representation for $f(n)$?

(a) $f(n) = n + 1$.
(b) $f(n) = n(n + 1)/2$.
(c) $f(n) = n$
(d) $f(n) = 2n - 1$.

7. Assume that p and r are false and q is true, and let F stand for false while T stand for true. Find the truth values of the following four propositions:

$$\neg(p \to q), \quad (p \to q) \wedge (q \to r), \quad (p \to q) \to r, \quad p \to (q \to r)$$

(a) TFTT
(b) FFFT
(c) FTTF
(d) TFFT

8. Choose the function that is both one to one and onto the set of real numbers. Note the domain of each function is the set of all real numbers.

(a) $3^x - 6$
(b) $\frac{x}{1+x^2}$
(c) $3x^2 - 3x + 1$
(d) $2x^3 - 4$

9. Using only NAND gates, xyz is

(a) $(x \uparrow y \uparrow z) \uparrow (x \uparrow y \uparrow z) \uparrow (x \uparrow y \uparrow z)$
(b) $(x \uparrow x \uparrow x) \uparrow (y \uparrow y \uparrow y) \uparrow (z \uparrow z \uparrow z)$
(c) $((x \uparrow y) \uparrow (x \uparrow y) \uparrow z) \uparrow ((x \uparrow y) \uparrow (x \uparrow y) \uparrow z)$
(d) $((x \uparrow y) \uparrow (x \uparrow z) \uparrow x) \uparrow ((x \uparrow y) \uparrow (x \uparrow z) \uparrow x)$

10. Select the expression which has different Θ estimates with others

(a) $9n^2 + 6n + 4$
(b) $2 + 3 + \cdots + n$
(c) $7n^2 + nlgn$
(d) $lgn!$

11. Suppose $f(n) = 2^n$, $g(n) = \log_2 n$, and $h(n) = 2n$. Which of the following is true? (Hint: Be careful!)

(a) $f \circ g = g \circ f$
(b) $f(g(n)) = h(n)$
(c) Both (a) and (b) are true.
(d) Neither (a) nor (b) are true.

12. Consider a universe $U = \{1, 2, 3, \cdots, 10\}$. Let $A = \{1, 4, 7, 10\}$, $B = \{1, 2, 3, 4, 5\}$, and $C = \{2, 4, 6, 8\}$. Which of the following is correct?

(a) $A \cup B = \{1, 4, 5, 7, 10\}$
(b) $B \cap C = \{2, 3, 4\}$
(c) $\overline{A \cap B} \cup C = \{2, 3, 4, 5, 6, 8, 9, 10\}$
(d) $(A \cap B) - C = \{1\}$

13. Suppose that we have three bits x, y and z. The full-adder calculates the sum cs of the three bits, where c corresponds to the carry bit and s corresponds to the least-significant bit. Which is the following is a correct statement about the full-adder?

(a) $c = xy \wedge zxy$.
(b) $c = xy \vee z(x \vee y)$
(c) $s = x \oplus y$, $c = z$
(d) None of these.

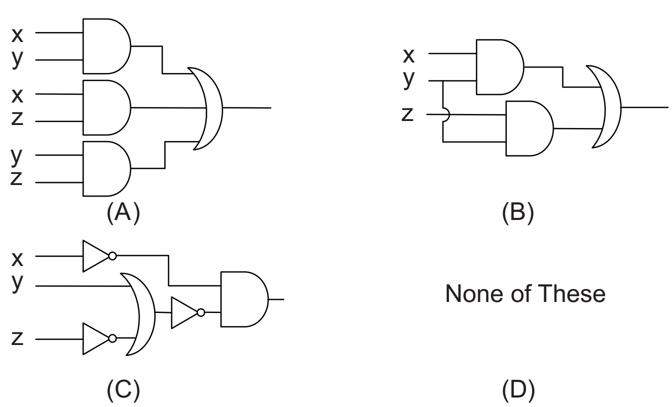
14. Suppose that we have a matrix $A = (A_{ij})$ which is $m \times p$ and a matrix $B = (A_{ij})$ that is $p \times n$. The following is an algorithm for calculating $C = AB$.

```
for i = 1 to m,
   for j = 1 to n,
     C_ij = 0
     for k = 1 to p,
        C_ij = C_ij + A_ik B_kj
     endfor
   endfor
 endfor
```

Which of the following is a correct statement of the complexity for the amount of multiplications performed by this algorithm?

(a) The algorithm is $O(n^3)$.
(b) The algorithm is $O(mnp)$.
(c) The algorithm is $O(mnp^2)$.
(d) None of these.

15. Suppose that we have three politicians who must vote on whether or not to accept a law. Let x, y and z be binary variables describing the votes for each of these three politicians (e.g. if the first politician votes yes, then $x = 1$ else $x = 0$). The majority voting rule will decide that the law is accepted if the law receives at least two yes votes. Which of the following is a circuit that describes the majority voting rule?



16. Which of the following is not correct?

(a) $x_1 \vee x_1 = x_1$
(b) $\overline{\overline{x_1}} = x_1$
(c) $x_1 \vee (x_1 \wedge x_2) = x_1$
(d) $(x_1 \vee x_2) \wedge \overline{(\overline{x_3} \vee x_4)} \wedge (x_3 \wedge \overline{x_2}) = 0$

17. What is the value of

$$\left\lfloor \frac{\lceil 1.71 \rceil + \lceil 2.4 \rceil}{2} \right\rfloor ?$$

(a) 3
(b) 3.1

(c) 2

(d) 1

18. What is the contrapositive of "The home team wins whenever it is raining"?

    (a) If it is raining, then the home team wins.

    (b) If the home team does not win, then it is not raining.

    (c) If it is not raining, then the home team does not win.

    (d) If the home team wins, then it is raining.

19. We now look at a different form of complexity called **little-o**. We say that a function $f(x)$ is little-o $g(x)$, written $f(x) = o(g(x))$ if

    $$\lim_{x \to \infty} \frac{f(x)}{g(x)} = 0.$$

    Which of the following is <u>not</u> true?

    (a) $x^2 = o(x^3)$

    (b) $x^2 + x + 1 = o(x^2)$

    (c) $x^2 = o(2^x)$

    (d) $x\ lgx = o(x^2)$

20. What is the value of $\sum_{k=0}^{8} (0.2)^k$?

    (a) 1

    (b) 1.25

    (c) 1.05

    (d) 0.8

Long Problem:

(20 pts) Use induction to show the following inequality:

$$\frac{1\cdot 3\cdot 5\cdots(2n-1)}{2\cdot 4\cdot 6\cdots(2n)} \leq \frac{1}{\sqrt{n+1}} \quad \text{for } n = 1, 2, \cdots$$