2.23

After a given packet is transmitted from node A, the second subsequent frame transmission termination from B carries the acknowledgement (recall that the frame transmission in progress from B when A finishes its transmission cannot carry the ack for that transmission; recall also that propagation and processing delays are negligible. Thus q is the probability of n-1 frame terminations from A before the second frame termination from B. This can be rephrased as the probability that out of the next n frame terminations from either node, either n-1 or n come from node A. Since successive frame terminations are equally likely and independently from A or B, this probability is

$$q = \sum_{i=n-1}^{n} \frac{n!}{i!(n-i)!} 2^{-n} = (n+1)2^{-n}$$

2.24

If an isolated error in the feedback direction occurs, then the ack for a given packet is held up by one frame in the feedback direction (i.e., the number RN in the feedback frame following the feedback frame in error reacknowledges the old packet as well as any new packet that might have been received in the interim). Thus q is now the probability of n-1 frame terminations from A before 3 frame terminations from B (one for the frame in progress, one for the frame in error, and one for the frame actually carrying the ack; see the solution to problem 2.23). This is the probability that n-1 or more of the next n+1 frame terminations come from A; since each termination is from A or B independently and with equal probability,

$$q = \sum_{i=n-1}^{n} \left(\frac{(n+1)!}{i!(n+1-i)!}\right) 2^{-n-1} = [n+2+(n+1)n/2]2^{-n-1}$$

2.25

As in the solution to problem 2.23, q is the probability of n-1 frame terminations coming from node A before two frame terminations come from node B. Frame terminations from A (and similarly from B) can be regarded as alternate points in a Poisson point process from A (or from B). There are two cases to consider. In the first, the initial frame is received from A after an even numbered point in the Poisson process at B, and in the second, the initial frame is received after an odd numbered point. In the first case, q is the probability that 2n-2 Poisson events from A occur before 4 Poisson events occur from B. This is the probability, in a combined Poisson point process of Poisson events from A and B, that 2n-2 or more Poisson events come from A out of the next 2n+1 events in the combined process. In the second case, q is the probability that 2n-2 Poisson events from A occur before 3 events occur from B. Since these cases are equally likely,

$$q = \frac{1}{2}\sum_{i=2n-2}^{2n+1} \left(\frac{(2n+1)!}{i!(2n+1-i)!}\right) 2^{-2n-1} + \frac{1}{2}\sum_{i=2n-2}^{2n} \left(\frac{(2n)!}{i!(2n-i)!}\right) 2^{-2n}$$

2.26

We view the system from the receiver and ask for the expected number of frames, γ, arriving at the receiver starting immediately after a frame containing a packet that is accepted

$$A \le \log_2 E\{K\} + 2$$

Finding the minimum of $2^{\gamma} - \gamma + 1$ by differentiation, the minimum occurs at

$$\gamma = -\log_2(\ln 2)$$

The value of $2^{\gamma} - \gamma + 1$ at this minimizing point is $[\ln 2]^{-1} + \log_2(\ln 2) + 1 = 1.914...$, so

$$A \ge \log_2 E\{K\} + (\ln 2)^{-1} + \log_2(\ln 2) + 1$$

2.35

Stuffed bits are always 0's and always follow the pattern 01^5. The initial 0 in this pattern could be a bit in the unstuffed data string, or could itself be a stuffed bit. As in the analysis of subsection 2.5.2, we ignore the case where this initial 0 is a stuffed bit since it is almost negligible compared with the other case (also a well designed flag detector would not allow a stuffed bit as the first bit of a flag). If a stuffed bit (preceded by 01^5 in the data) is converted by noise into a 1, then it is taken as a flag if the next bit is 0 and is taken as an abort if the next bit is 1. Thus an error in a stuffed bit causes a flag to appear with probability 1/2 and the expected number of falsely detected flags due to errors in stuffed bits is $K2^{-7}$. If one is less crude in the approximations, one sees that there are only K-6 places in the data stream where a stuffed bit could be inserted following 01^5 in the data; thus a more refined answer is that the expected number of falsely detected flags due to errors in stuffed bits is $(K-6)2^{-7}$.

There are eight patterns of eight bits such that an error in one of the eight bits would turn the pattern into a flag. Two of these patterns, 01^7 and 1^70, cannot appear in stuffed data. Another two of the patterns, 01^500 and 001^50, can appear in stuffed data but must contain a stuffed bit (i.e. the 0 following 1^5). The first of these cases corresponds to the case in which an error in a stuffed bit causes a flag to appear, and we have already analyzed this. The second corresponds to a data string 001^5. Thus the substrings of data for which a single error in a data bit can cause a flag to appear are listed below; the position in which the error must appear is shown underlined:

$0\ \underline{0}\ 1\ 1\ 1\ 1\ 1$
$0\ 1\ \underline{0}\ 1\ 1\ 1\ 1\ 0$
$0\ 1\ 1\ \underline{0}\ 1\ 1\ 1\ 0$
$0\ 1\ 1\ 1\ \underline{0}\ 1\ 1\ 0$
$0\ 1\ 1\ 1\ 1\ \underline{0}\ 1\ 0$

For any given bit position j in the K bit data string ($j \le K-7$), the probability that one of these patterns starts on bit j is $2^{-7} + 4.2^{-8} = 3.2^{-7}$. Thus the probability of a false flag being detected because of an error on a data bit, starting on bit j of the data is $3p2^{-7}$. This is also the expected number of such flags, and summing over the bits of the data stream, the expected number is $(K-7)3p2^{-7}$. Approximating by replacing K-7 by K, and adding this to the expected number of false flags due to errors in stuffed bits, the overall probability of a false flag in a frame of length K is (1/32)Kp. If K-7 is not approximated by K, and if we recognize that the first pattern above can also appear starting at j=K-6, then the overall probability of a false flag is approximated more closely by (1/32)(K-6.5)p.