are 1; i.e. that $e(D)$ corresponds to an even number of errors. Thus all undetectable error sequences contain an even number of errors; any error sequence with an odd number of errors is detected.

2.15

a) Let D^{i+L}, divided by $g(D)$, have the quotient $z^{(i)}(D)$ and remainder $c^{(i)}(D)$ so that

$$D^{i+L} = g(D)z^{(i)}(D) + c^{(i)}(D)$$

Multiplying by s_i and summing over i,

$$s(D)D^L = \Sigma_i\, s_i z^{(i)}(D) + \Sigma_i\, s_i c^{(i)}(D)$$

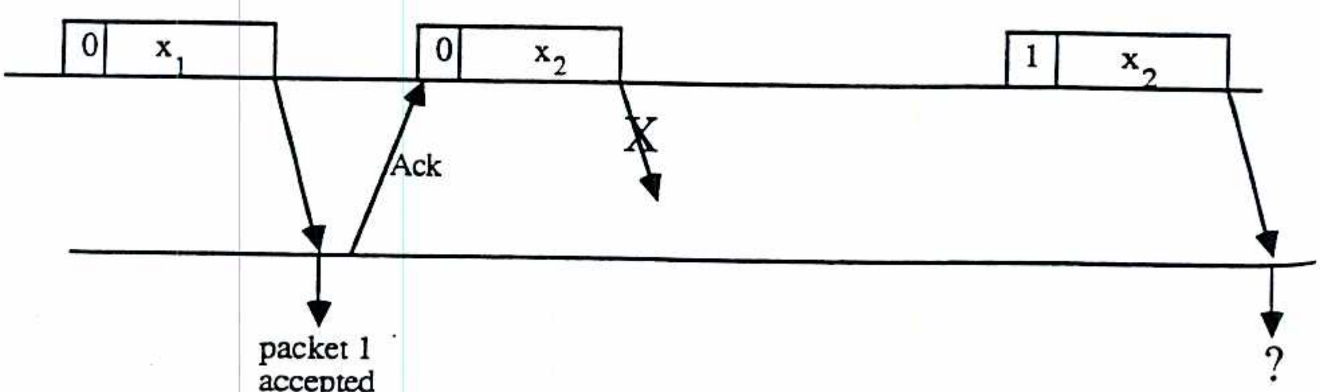
Since $\Sigma_i\, s_i c^{(i)}(D)$ has degree less than L, this must be the remainder (and $\Sigma_i\, s_i z^{(i)}(D)$ the quotient) on dividing $s(D)D^L$ by $g(D)$. Thus $c(D) = \Sigma_i\, s_i c^{(i)}(D)$.

b) Two polynomials are equal if and only if their coefficients are equal, so the above polynomial equality implies

$$c_j = \Sigma_i\, s_i c_j^{(i)}$$

2.16

a) Consider the two scenarios below and note that these scenarios are indistinguishable to the receiver.



Key point: payload x_1 may equal payload x_2 !!

2.20

The simplest example is for node A to send packets 0 through n-1 in the first n frames. In case of delayed acknowledgements (i.e. no return packets in the interim), node A goes back and retransmits packet 0. If the other node has received all the packets, it is waiting for packet n, and if the modulus m equals n, this repeat of packet 0 is interpreted as packet n.

The right hand side of Eq. (2.24) is satisfied with equality if $SN = SN_{min}(t_1)+n-1$. This occurs if node A sends packets 0 through n-1 in the first n frames with no return packets from node B. The last such frame has $SN = n-1$, whereas SN_{min} at that time (say t_1) is 0.

Continuing this scenario, we find an example where the right hand side of Eq. (2.25) is satisfied with equality. If all the frames above are correctly received, then after the last frame, RN becomes equal to n. If another frame is sent from A (now call this time t_1) and if SN_{min} is still 0, then when it is received at B (say at t_2), we have $RN(t_2) = SN_{min}(t_1)+n$.

2.32

The hint shows that the data string $01^501x_1x_2...$ must have a zero stuffed after 01^5, thus appearing as $01^500x_1x_2...$. This stuffed pattern will be indistinguishable from the original string $01^500x_1x_2...$ unless stuffing is also used after 01^5 in the string $01^500x_1x_2...$. Thus stuffing must be used in this case. The general argument is then by induction. Assume that stuffing is necessary after 01^5 on all strings of the form $01^50^kx_1x_2...$. Then such a stuffed sequence is $01^50^{k+1}x_1x_2...$. It follows as before that stuffing is then necessary after 01^5 in the sequence $01^50^{k+1}x_1x_2...$. Thus stuffing is always necessary after 01^5.

2.34

Let γ be $\log_2E\{K\} - j$. Since j is the integer part of $\log_2E\{K\}$, we see that γ must lie between 0 and 1. Expressing $A = E\{K\}2^{-j} + j + 1$ in terms of γ and $E\{K\}$, we get

$$A = 2^\gamma + \log_2E\{K\} - \gamma + 1$$

$$A - \log_2E\{K\} = 2^\gamma - \gamma + 1$$

This function of γ is easily seen to be convex (i.e., it has a positive second derivative). It has the value 2 at $\gamma = 0$ and at $\gamma = 1$ and is less than 2 for $0 < \gamma < 1$. This establishes that

$$A \le \log_2E\{K\} + 2$$

Finding the minimum of $2^\gamma - \gamma + 1$ by differentiation, the minimum occurs at

$$\gamma = -\log_2(\ln 2)$$

The value of $2^\gamma - \gamma + 1$ at this minimizing point is $[\ln 2]^{-1} + \log_2(\ln 2) + 1 = 1.914...$, so

$$A \ge \log_2E\{K\} + (\ln 2)^{-1} + \log_2(\ln 2) + 1$$