# ECE Communication Networks 1
# Homework 1 Solutions
Fall 2006

1. The following is a parity check matrix for a binary $[n, k]$ code $C$:

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

   (a) Find $n$ and $k$.
   (b) Find the generator matrix for $C$.
   (c) List the codewords for $C$.
   (d) What is the code rate for $C$?

   Solution:

   (a) $n = 5$, $k = 2$.

   (b)

$$G = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

   (c) The codewords in C are linear combinations of the rows of G, namely (0, 0, 0, 0, 0), (1, 1, 0, 1, 0), (1, 0, 1, 0, 1), (0, 1, 1, 1, 1).

2. Let $C$ be the binary code $\{(0, 0, 1), (1, 1, 1), (1, 0, 0), (0, 1, 0)\}$.

   (a) Show that $C$ is not linear.
   (b) What is $d(C)$? (note: since $C$ is not linear, this cannot be found by calculating the minimum weight).

   Solution: (a) $C$ does not contain $(0, 0, 0)$, so it is not a subspace. (b) Calculate the Hamming distances from the first vector to the other three, the second vector to the last two, and the third to the fourth. These distances are all 2, so the minimum distance is $d(C) = 2$.

3. Show that $g(X) = 1 + X + X^2 + \cdots + X^{n-1}$ is the generating polynomial for the $[n, 1]$ repetition code.

   Solution: The polynomials obtained from the [n, 1] repetition code are of those of the form $a+aX+$ $+\, aX^{n-1} = ag(X)$ with $a$ a coefficient. Clearly $g(X)$ is the polynomial with the smallest degree (also the same degree) among the nonzero polynomials of this form. Since the leading coefficient of $g(X)$ is 1, it is the generating polynomial.

4. In CRC, the bits that are appended for "error detection" are called the frame check sequence (FCS), or the CRC digits. Suppose that we have a frame with bits $(1, 1, 1, 0, 0, 1, 1, 0)$ and suppose that the generator polynomial for our CRC code is $(1, 1, 0, 0, 1)$.

   (a) Calculate the FCS for this example. (Hint: it should only be 4 bits long)
   (b) Suppose that the received frame is $(1, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0)$. Was there an error in transmission?

   Solution:

   (a) First, start the division process 11001 into 111001100. Do this long-hand style, and you will get a quotient 1011 0110 and a remainder 0110. The remainder is the FCS.

   (b) No, there are no errors in the transmission. Take the quotient above and multiply it by 11001, and you have the received frame.