Chapter 10

Security Protocols

Why do we need security protocols? Aren't public key methods sufficient to establish secure communications? The answer is definitely no. There are major problems with authentication, for example. How does Bob know that it is actually Alice he is communicating with, and not her evil twin, Malice? How can you be sure a web site is legitimate? When you use a credit card electronically, how is your information kept secure? These are questions that are addressed in this chapter.

10.1 Intruders-in-the-Middle and Impostors

If you receive an email asking you to go to a web site and update your account information, how can you be sure that the web site is legitimate? An impostor can easily set up a web page that looks like the correct one, but which simply records sensitive information and forwards it to Eve. This is an important authentication problem that must be addressed in real-world implementations of cryptographic protocols. One standard solution uses certificates and a trusted authority and will be discussed in Section 10.7. Authentication will also play an important role in the protocols in many other sections of this chapters.

Another major consideration that must be addressed in communications over public channels is the intruder-in-the-middle attack, which we'll discuss shortly. It is another cause for several of the steps in the protocols we discuss.

Intruder-in-the-Middle Attacks

Eve, who has recently learned the difference between a knight and a rook, claims that she can play two chess grandmasters simultaneously and either win one game or draw both games. The strategy is simple. She waits for the first grandmaster to move, then makes the identical move against the second grandmaster. When the second grandmaster responds, Eve makes that play against the first grandmaster. Continuing in this way, Eve cannot lose both games (unless she runs into time trouble because of the slight delay in transferring the moves).

A similar strategy, called the **intruder-in-the-middle attack**, can be used against the many cryptographic protocols. Many of the technicalities of the algorithms in this chapter are caused by efforts to thwart such an attack.

Let's see how this attack works against the Diffie-Hellman key exchange from Section 7.4.

Let's recall the protocol. Alice and Bob want to establish a key for communicating. The Diffie-Hellman scheme for accomplishing this is as follows:

1. Either Alice or Bob selects a large, secure prime number p and a primitive root $\alpha \pmod{p}$. Both p and α can be made public.

2. Alice chooses a secret random x with $1 \leq x \leq p-2$, and Bob selects a secret random y with $1 \leq y \leq p-2$.

3. Alice sends $\alpha^x \pmod{p}$ to Bob, and Bob sends $\alpha^y \pmod{p}$ to Alice.

4. Using the messages that they each have received, they can each calculate the session key K. Alice calculates K by $K \equiv (\alpha^y)^x \pmod{p}$, and Bob calculates K by $K \equiv (\alpha^x)^y \pmod{p}$.

Here is how the intruder-in-the-middle attack works.

1. Eve chooses an exponent z.

2. Eve intercepts α^x and α^y.

3. Eve sends α^z to Alice and to Bob (Alice believes she is receiving α^x and Bob believes he is receiving α^y).

4. Eve computes $K_{AO} \equiv (\alpha^x)^z \pmod p$ and $K_{OB} \equiv (\alpha^x)^z \pmod p$. Alice, not realizing that Eve is in the middle, also computes K_{AO}, and Bob computes K_{OB}.

5. When Alice sends a message to Bob, encrypted with K_{AO}, Eve intercepts it, deciphers it, encrypts it with K_{OB}, and sends it to Bob. Bob decrypts with K_{OB} and obtains the message. Bob has no reason to believe the communication was insecure. Meanwhile, Eve is reading the juicy gossip that she has obtained.

To avoid the intruder-in-the-middle attack, it is desirable to have a procedure that authenticates Alice's and Bob's identities to each other while the key is being formed. A protocol that can do this is known as an **authenticated key agreement protocol**.

The standard way to stop the intruder-in-the-middle attack is the **Station-to-Station (STS) Protocol**, which uses digital signatures. Each user U has a digital signature function sig_U with verification algorithm ver_U. For example, sig_U could produce an RSA or ElGamal signature, and ver_U checks that it is a valid signature for U. The verification algorithms are compiled and made public by the trusted authority Trent, who certifies that ver_U is actually the verification algorithm for U, and not for Eve.

Suppose now that Alice and Bob want to establish a key to use in an encryption function E_K. They proceed as in the Diffie-Hellman key exchange, but with the added feature of digital signatures:

1. They choose a large prime p and a primitive root α.

2. Alice chooses a random x and Bob chooses a random y.

3. Alice computes $\alpha^x \pmod p$, and Bob computes $\alpha^y \pmod p$.

4. Alice sends α^x to Bob.

5. Bob computes $K \equiv (\alpha^x)^y \pmod p$.

6. Bob sends α^y and $E_K(sig_B(\alpha^y, \alpha^x))$ to Alice.

7. Alice computes $K \equiv (\alpha^y)^x \pmod p$.

8. Alice decrypts $E_K(sig_B(\alpha^y, \alpha^x))$ to obtain $sig_B(\alpha^y, \alpha^x)$.

9. Alice asks Trent to verify that ver_B is Bob's verification algorithm.

10. Alice uses ver_B to verify Bob's signature.

11. Alice sends $E_K(sig_A(\alpha^x, \alpha^y))$ to Bob.

12. Bob decrypts, asks Trent to verify that ver_A is Alice's verification algorithm, and then uses ver_A to verify Alice's signature.

An enhanced version of this, due to Diffie, van Oorschot, and Wiener, is known as the Station-to-Station protocol. Note that Alice and Bob are also certain that they are using the same key K, since it is very unlikely that an incorrect key would give a decryption that is a valid signature.

Note the role that trust plays in the protocol. Alice and Bob must trust Trent's verification if they are to have confidence that their communications are secure. throughout this chapter, a trusted authority such as Trent will be an important participant in many protocols.

10.2 Key Distribution

So far in this book we have discussed various cryptographic concepts and focused on developing algorithms for secure communication. But a cryptographic algorithm is only as strong as the security of its keys. If Alice were to announce to the whole world her key before starting a DES session with Bob, then anyone could eavesdrop. Such a scenario is absurd, of course. But it represents an extreme version of a very important issue: If Alice and Bob are unable to meet in order to exchange their keys, can they still decide on a key without compromising future communication?

In particular, there is the fundamental problem of sharing secret information for the establishment of keys for symmetric cryptography. By symmetric cryptography, we mean a system such as DES where both the sender and the recipient use the same key. This is in contrast to public key methods such as RSA, where the sender has one key (the encryption exponent) and the receiver has another (the decryption exponent).

In key establishment protocols, there is a sequence of steps that take place between Alice and Bob so that they can share some secret information needed in the establishment of a key. Since public key cryptography methods employ public encryption keys that are stored on public databases, one might think that public key cryptography provides an easy solution to this problem. This is partially true. The main downside to public key cryptography is that even the best public key cryptosystems are computationally slow when compared with the best symmetric key methods. RSA, for example, requires exponentiation, which is not as fast as the mixing of bits that takes place in DES. Therefore, sometimes RSA is used to transmit a DES key that will then be used for transmitting vast amounts of data. However, a central server that needs to communicate with many clients in short time intervals sometimes needs key establishment methods that are faster than current versions of public key algorithms. Therefore, in this and in various other situations, we need to consider other means for the exchange and establishment of keys for symmetric encryption algorithms.

There are two basic types of key establishment. In **key agreement** protocols, neither party knows the key in advance; it is determined as a result of their interaction. In **key distribution** protocols, one party has decided on a key and transmits it to the other party.

Diffie-Hellman key exchange (see Sections 7.4 and 10.1) is an example of key agreement. Using RSA to transmit a DES key is an example of key distribution.

In any key establishment protocol, authentication and intruder-in-the-middle attacks are security concerns. Pre-distribution, which will be discussed shortly, is one solution. Solutions that are more practical for Internet communcations are treated in later sections of this chapter.

Key Pre-Distribution

In the simplest version of this protocol, if Alice wants to communicate with Bob, the keys or key schedules (lists describing which keys to use at which times) are decided upon in advance and somehow this information is sent securely from one to the other. For example, this method was used by the German navy in World War II. However, the British were able to use codebooks from captured ships to find daily keys and thus read messages.

There are some obvious limitations and drawbacks to pre-distribution. First, it requires two parties, Alice and Bob, to have met or to have established a secure channel between them in the first place. Second, once Alice and Bob have met and exchanged information, there is nothing they can do, other than meeting again, to change the key information in case it gets compromised. The keys are predetermined and there is no easy method to change the key after a certain amount of time. When using the same key for long periods of time, one runs a risk that the key will become compromised. The more data that are transmitted, the more data there are with which to build statistical attacks.

Here is a general and slightly modified situation. First, we require a trusted authority whom we call Trent. For every pair of users, call them (A, B), Trent produces a random key K_{AB} that will be used as a key for a symmetric encryption method (hence $K_{BA} = K_{AB}$). It is assumed that Trent is powerful and has established a secure channel to each of the users. He distributes all the keys that he has determined to his users. Thus, if Trent is responsible for n users, each user will be receiving $n-1$ keys to store, and Trent must send $n(n-1)/2$ keys securely. If n is large, this could be a problem. The storage that each user requires is also a problem.

One method for reducing the amount of information that must be sent from the trusted authority is the **Blom key pre-distribution scheme**. Start with a network of n users, and let p be a large prime, where $p \geq n$. Everyone has knowledge of the prime p. The protocol is now the following:

1. Each user U in the network is assigned a distinct public number $r_U \pmod{p}$.
2. Trent chooses three secret random numbers a, b, and c mod p.
3. For each user U, Trent calculates the numbers

   $$a_U \equiv a + br_U \pmod{p} \quad b_U \equiv b + cr_U \pmod{p}$$

   and sends them via his secure channel to U.
4. Each user U forms the linear polynomial

   $$g_U(x) = a_U + b_U x.$$

5. If Alice (A) wants to communicate with Bob (B), then Alice computes $K_{AB} = g_A(r_B)$, while Bob computes $K_{BA} = g_B(r_A)$.
6. It can be shown that $K_{AB} = K_{BA}$ (Exercise 2). Alice and Bob communicate via a symmetric encryption system, for example, DES, using the key (or a key derived from) K_{AB}.

Example. Consider a network consisting of three users Alice, Bob, and Charlie. Let $p = 23$, and let

$$r_A = 11, \quad r_B = 3, \quad r_C = 2.$$

Suppose Trent chooses the numbers $a = 8, \quad b = 3, \quad c = 1$. The corresponding linear polynomials are given by

$$g_A(x) = 18 + 14x, \quad g_B(x) = 17 + 6x, \quad g_C(x) = 14 + 5x.$$

It is now possible to calculate the keys that this scheme would generate:

$$K_{AB} = g_A(r_B) = 14, \quad K_{AC} = g_A(r_C) = 0, \quad K_{BC} = g_B(r_C) = 6.$$

It is easy to check that $K_{AB} = K_{BA}$, etc., in this example. ∎

If the two users Eve and Oscar conspire, they can determine a, b, and c, and therefore find all numbers a_A, b_A for all users. They proceed as follows.

They know the numbers a_E, b_E, a_O, b_O. The defining equations for the last three of these numbers can be written in matrix form as

$$\begin{pmatrix} 0 & 1 & r_E \\ 1 & r_O & 0 \\ 0 & 1 & r_O \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} \equiv \begin{pmatrix} b_E \\ a_O \\ b_O \end{pmatrix} \pmod{p}.$$

The determinant of the matrix is $r_E - r_O$. Since the numbers r_A were chosen to be distinct mod p, the determinant is nonzero mod p and therefore the system has a unique solution a, b, c.

Without Eve's help, Oscar has only a 2×3 matrix to work with and therefore cannot find a, b, c. In fact, suppose he wants to calculate the key K_{AB} being used by Alice and Bob. Since $K_{AB} \equiv a + b(r_A + r_B) + c(r_A r_B)$ (see Exercise 2), Oscar has the matrix equation

$$\begin{pmatrix} 1 & r_A + r_B & r_A r_B \\ 1 & r_O & 0 \\ 0 & 1 & r_O \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} \equiv \begin{pmatrix} K_{AB} \\ a_O \\ b_O \end{pmatrix} \pmod{p}.$$

The matrix has determinant $(r_O - r_A)(r_O - r_B) \not\equiv 0 \pmod{p}$. Therefore, there is a solution a, b, c for every possible value of K_{AB}. This means that Oscar obtains no information about K_{AB}.

For each $k \geq 1$, there are Blom schemes that are secure against coalitions of at most k users, but which succumb to conspiracies of $k + 1$ users. See [Blom].

10.3 Kerberos

Kerberos (named for the three-headed dog that guarded the entrance to Hades) is a real-world implementation of a symmetric cryptography protocol whose purpose is to provide strong levels of authentication and security in key exchange between users in a network. Here we use the term *users* loosely, as a user might be an individual, or it might be a program requesting communication with another program. Kerberos grew out of a larger development project at M.I.T. known as Project Athena. The purpose of Athena was to integrate a huge network of computer workstations into the curriculum of the undergraduate student body at M.I.T., allowing students to be able to access their files easily from anywhere on the network. As one might guess, such a development quickly raised questions about network security. In particular, communication across a public network such as Athena is very insecure and it is easily possible to observe data flowing across a network and look for interesting bits of information such as passwords and other types of information that one would wish to remain private. Kerberos was developed in order to address such security issues. In the following, we present the basic Kerberos model and describe what it is and what it attempts to do. For more thorough descriptions, see [Schneier].

Kerberos is based on a client/server architecture. A client is either a user or some software that has some task that it seeks to accomplish. For example, a client might wish to send e-mail, print documents, or mount devices. Servers are larger entities whose function is to provide services to the clients. As an example, on the Internet and World Wide Web there is a concept of a domain name server (DNS), which provides names or addresses to clients such as e-mail programs or Internet browsers.

The basic Kerberos model has the following participants:

- Cliff: a client
- Serge: a server
- Trent: a trusted authority
- Grant: a ticket-granting server

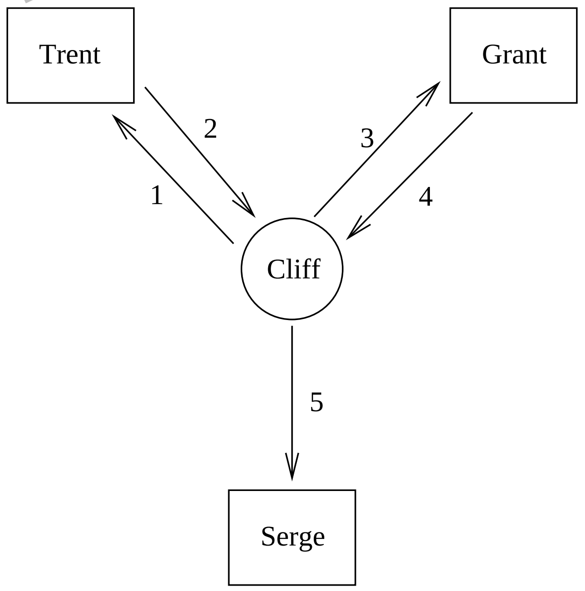


Figure 10.1: Kerberos.

The trusted authority is also known as an authentication server. To begin, Cliff and Serge have no secret key information shared between them, and it is the purpose of Kerberos to give each of them information securely. A result of the Kerberos protocol is that Serge will have verified Cliff's identity (he wouldn't want to have a conversation with a fake Cliff, would he?), and a session key will be established.

The protocol, depicted in Figure 10.1, begins with Cliff requesting a ticket for Ticket-Granting Service from Trent. Since Trent is the powerful trusted authority, he has a database of password information for all the clients (for this reason, Trent is also sometimes referred to as the Kerberos server). Trent returns a ticket that is encrypted with the client's secret password information. Cliff would now like to use the service that Serge provides, but before he can do this, he must be allowed to talk to Serge. Cliff presents his ticket to Grant, the ticket-granting server. Grant takes this ticket, and if everything is OK (recall that the ticket has some information identifying Cliff), then Grant gives a new ticket to Cliff that will allow Cliff to make use of Serge's service (and only Serge's service; this ticket will not be valid with Sarah, a different server). Cliff now has a service ticket, which he can present to Serge. He sends Serge the service ticket as well as an authentication credential. Serge checks the ticket with the authentication credential to make sure it is valid. If this final exchange checks out, then Serge will provide the service to Cliff.

The Kerberos protocol is a formal version of protocols we use in everyday life (for example cashing a check at a bank, or getting on a ride at a fair).

We now look at Kerberos in more detail. Kerberos makes use of a symmetric encryption algorithm. In Version V, Kerberos makes use of DES operating in CBC mode; however, any symmetric encryption algorithm would suffice.

1. Cliff to Trent: Cliff sends a message to Trent that contains his name and the name of the ticket-granting server that he will use (in this case Grant).

2. Trent to Cliff: Trent looks up Cliff's name in his database. If he finds it, he generates a session key K_{CG} that will be used between Cliff and Grant. Trent also has a secret key K_C with which he can communicate with Cliff, so he uses this to encrypt the Cliff-Grant session key:

$$T = e_{K_C}(K_{CG}).$$

In addition, Trent creates a Ticket Granting Ticket (TGT), which will allow Cliff to authenticate himself to Grant. This ticket is encrypted using Grant's personal key K_G (which Trent also has):

$$TGT =$$
$$\text{Grant's name} \| e_{K_G}(\text{Cliff's name, Cliff's Address, Timestamp1, } K_{CG}).$$

Here $\|$ is used to denote concatenation. The ticket that Cliff receives is the concatenation of these two subtickets:

$$\text{Ticket} = T\|TGT.$$

3. Cliff to Grant: Cliff can extract K_{CG} using the key K_C, which he shares with Trent. Using K_{CG}, Cliff can now communicate securely with Grant. Cliff now creates an authenticator, which will consist of his name, his address, and a timestamp. He encrypts this using K_{CG} to get

$$\text{Auth}_{CG} = e_{K_{CG}}(\text{Cliff's name, Cliff's address, Timestamp2}).$$

Cliff now sends Auth_{CG} as well as TGT to Grant so that Grant can administer a service ticket.

4. Grant to Cliff: Grant now has Auth_{CG} and TGT. Part of TGT was encrypted using Grant's secret key, so Grant can extract this portion and can decrypt it. Thus he can recover Cliff's name, Cliff's address, Timestamp1, as well as K_{CG}. Grant can now use K_{CG} to decrypt Auth_{CG} in order to verify the authenticity of Cliff's request. That is, $d_{K_{CG}}(\text{Auth}_{CG})$ will provide another copy of Cliff's name, Cliff's address, and a different timestamp. If the two versions of Cliff's name and address match, and if Timestamp1 and Timestamp2 are sufficiently close to each other, then Grant will declare Cliff valid. Now that Cliff is approved by Grant, Grant will generate a session key K_{CS} for Cliff to communicate with Serge and will also return a service ticket. Grant has a secret key K_S which he shares with Serge. The service ticket is

$$\text{ServTicket} = \\ e_{K_S}(\text{Cliff's name, Cliff's address, Timestamp3, ExpirationTime}, K_{CS}).$$

Here ExpirationTime is a quantity that describes the length of validity for this service ticket. The session key is encrypted using a session key between Cliff and Grant:

$$e_{K_{CG}}(K_{CS}).$$

Grant sends ServTicket and $e_{K_{CG}}(K_{CS})$ to Cliff.

5. Cliff to Serge: Cliff is now ready to start making use of Serge's services. He starts by decrypting $e_{K_{CG}}(K_{CS})$ in order to get the session key

K_{CS} that he will use while communicating with Serge. He creates an authenticator to use with Serge:

$$\text{Auth}_{CS} = e_{K_{CS}}(\text{Cliff's name, Cliff's address, Timestamp4}).$$

Cliff now sends Serge Auth_{CS} as well as ServTicket. Serge can decrypt ServTicket and extract from this the session key K_{CS} that he is to use with Cliff. Using this session key, he can decrypt Auth_{CS} and verify that Cliff is who he says he is, and that Timestamp4 is within ExpirationTime of Timestamp3. If Timestamp4 is not within ExpirationTime of Timestamp3, then Cliff's ticket is stale and Serge rejects his request for service. Otherwise, Cliff and Serge may make use of K_{CS} to perform their exchange.

10.4 Public Key Infrastructures (PKI)

Suppose you want to buy something on the Internet. You go to Gigafirm's website, select your items, and then proceed to the checkout page. You are asked to enter your credit card number and other information. The website assures you that it is using secure public key encryption, using Gigafirm's public key, to set up the communications.

Public key cryptography is a powerful tool that allows for authentication, key distribution, and non-repudiation. In these applications, the public key is published, but when you access public keys, what assurance do you have that Alice's public key actually belongs to Alice? Perhaps Eve has substituted her own public key in place of Alice's. Unless confidence exists in how the keys were generated, and in their authenticity and validity, the benefits of public key cryptography are minimal.

In order for public key cryptography to be useful in commercial applications, it is necessary to have an infrastructure that keeps track of public keys. A public key infrastructure, or PKI for short, is a framework consisting of policies defining the rules under which the cryptographic systems operate and procedures for generating and publishing keys and certificates.

All PKIs consist of certification and validation operations. Certification binds a public key to an entity, such as a user or a piece of information. Validation guarantees that certificates are valid.

A **certificate** is a quantity of information that has been signed by its publisher, who is commonly referred to as the **certification authority (CA)**. There are many types of certificates. Two popular ones are identity certificates and credential certificates. Identity certificates contain an entity's identity information, such as an e-mail address, and a list of public keys for the entity. Credential certificates contain information describing access rights. In either case, the data are typically encrypted using the CA's private key.

Suppose we have a PKI, and the CA publishes identity certificates for Alice and Bob. If Alice knows the CA's public key, then she can take the encrypted identity certificate for Bob that has been published and extract Bob's identity information as well as a list of public keys needed to communicate securely with Bob. The difference between this scenario and the conventional public key scenario is that Bob doesn't publish his keys, but instead the trust relationship is placed between Alice and the publisher. Alice might not trust Bob as much as she might trust a CA such as the government or the phone company. The concept of trust is critical to PKIs and is perhaps one of the most important properties of a PKI.

It is unlikely that a single entity could ever keep track of and issue every Internet user's public keys. Instead, PKIs often consist of multiple CAs that are allowed to certify each other and the certificates they issue. Thus, Bob might be associated with a different CA than Alice, and when requesting Bob's identity certificate, Alice might only trust it if her CA trusts Bob's CA. On large networks like the Internet, there may be many CAs between Alice and Bob, and it becomes necessary for each of the CAs between her and Bob to trust each other.

In addition, most PKIs have varying levels of trust, allowing some CAs to certify other CAs with varying degrees of trust. It is possible that CAs may only trust other CAs to perform specific tasks. For example, Alice's CA may only trust Bob's CA to certify Bob and not certify other CAs, while Alice's CA may trust Dave's CA to certify other CAs. Trust relationships can become very elaborate, and, as these relationships become more complex, it becomes more difficult to determine to what degree Alice will trust a certificate that she receives.

In the following two sections, we discuss two examples of PKIs that are used in practice.

10.5 X.509 Certificates

Suppose you want to buy something on the Internet. You go to the website Gigafirm.com, select your items, and then proceed to the checkout page. You are asked to enter your credit card number and other information. The website assures you that it is using secure public key encryption, using Gigafirm's public key, to set up the communications. But how do you know that Eve hasn't substituted her public key? In other words, when you are using public keys, how can you be sure that they are correct? This is the purpose of Digital Certificates.

One of the most popular types of certificate is the X.509. In this system, every user has a certificate. The validity of the certificates depends on a chain of trust. At the top is a Certificate Authority (CA). These are often commercial companies such as VeriSign, GTE, ATT, and others. It is assumed that the CA is trustworthy. The CA produces its own certificate and signs it. This certificate is often posted on the CA's website. In order to ensure that their services are used frequently, various CAs arrange to have their certificates packaged into Internet browsers such as Netscape and Microsoft Internet Explorer.

The CA then (for a fee) produces certificates for various clients, such as Gigafirm. Such a certificate contains Gigafirm's public key. It is signed by the CA using the CA's private key. Often, for efficiency, the CA authozizes various Registration Authorities (RA) to sign certificates. Each RA then has a certificate signed by the CA.

A certificate holder can sometimes then sign certificates for others. We therefore get a **certification hierarchy** where the validity of each certificate is certified by the user above it, and this continues all the way up to the CA.

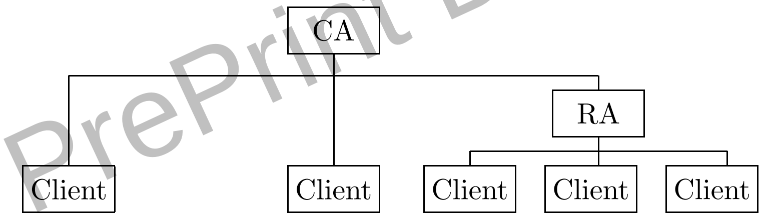
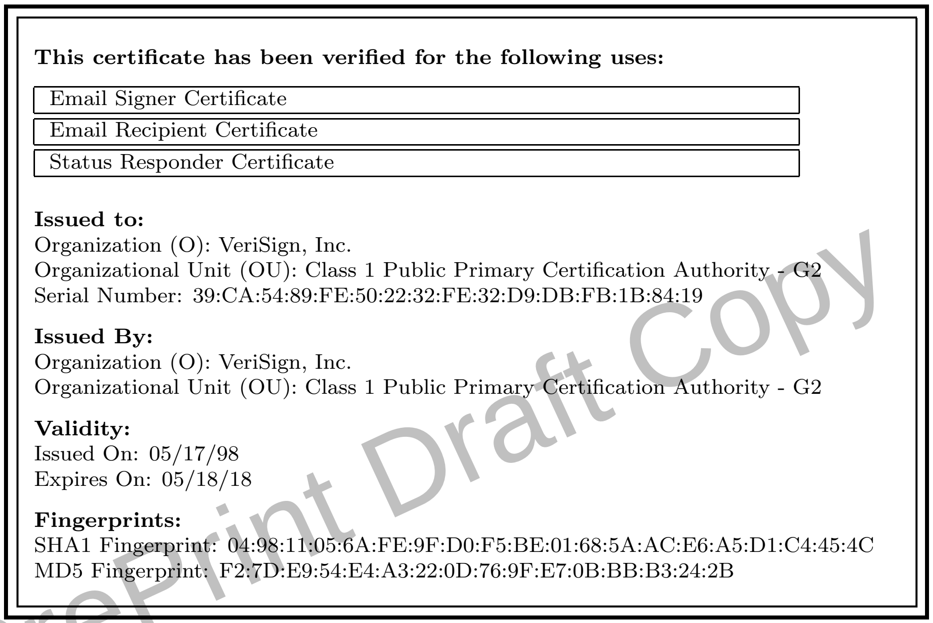


Figure 10.2: A Certification Hierarchy

If Alice wants to verify that Gigafirm's public key is correct, she uses her copy of the CA's certificate (stored in her computer) to get the CA's public key. She then verifies the signature on Gigafirm's certificate. If it is valid, she trusts the certificate and thus has a trusted public key for Gigafirm. Of course, she must trust the CA's public key. This means that she trusts the company that packaged the CA's certificate into her computer. The computer company of course has a financial incentive to maintain a good reputation, so this trust is reasonable. But if Alice has bought a used computer in which Eve has tampered with the certificates, there might be a problem (in other words, don't buy used computers from your enemies, except to extract unerased information).

Figures 10.3, 10.4, and 10.5 show examples of X.509 certificates. The ones in Figures 10.3 and 10.4 are for a CA, namely VeriSign. The part in Figure 10.3 gives the general information about the certificate, including its possible uses. Figure 10.4 gives the detailed information. The one in Figure

10.5 is an edited version of the Details part of a certificate for the bank Wells Fargo.



This certificate has been verified for the following uses:

Email Signer Certificate

Email Recipient Certificate

Status Responder Certificate

Issued to:
Organization (O): VeriSign, Inc.
Organizational Unit (OU): Class 1 Public Primary Certification Authority - G2
Serial Number: 39:CA:54:89:FE:50:22:32:FE:32:D9:DB:FB:1B:84:19

Issued By:
Organization (O): VeriSign, Inc.
Organizational Unit (OU): Class 1 Public Primary Certification Authority - G2

Validity:
Issued On: 05/17/98
Expires On: 05/18/18

Fingerprints:
SHA1 Fingerprint: 04:98:11:05:6A:FE:9F:D0:F5:BE:01:68:5A:AC:E6:A5:D1:C4:45:4C
MD5 Fingerprint: F2:7D:E9:54:E4:A3:22:0D:76:9F:E7:0B:BB:B3:24:2B

Figure 10.3: CA's Certificate; General

Some of the fields in Figure 10.4 are as follows:

1. *Version:* there are three versions, the first being *Version 1* (from 1988) and the most recent being *Version 3* (from 1997).

2. *Serial number:* there is a unique serial number for each certificate issued by the CA.

3. *Signature algorithm:* Various signature algorithms can be used. This one uses *RSA* to sign the output of the hash function *SHA-1.*

4. *Issuer:* The name of the CA that created and signed this certificate. *OU* is the Organizational Unit, *O* is the organization, *C* is the country.

5. *Subject:* The name of the holder of this certificate.

6. *Public key:* Several options are possible. This one uses RSA with a 1024-bit modulus. The key is given in hexadecimal notation. In hexadecimal, the letters *a, b, c, d, e, f* represent the numbers *10, 11,*

Certificate Hierarchy

▷ Verisign Class 1 Public Primary Certification Authority - G2

Certificate Fields

```
Verisign Class 1 Public Primary Certification Authority - G2
  Certificate
    Version: Version 1
    Serial Number: 39:CA:54:89:FE:50:22:32:FE:32:D9:DB:FB:1B:84:19
    Certificate Signature Algorithm: PKCS #1 SHA-1 With RSA Encryption
    Issuer: OU = VeriSign Trust Network
            OU = (c) 1998 VeriSign, Inc. - For authorized use only
            OU = Class 1 Public Primary Certification Authority - G2
            O = VeriSign, Inc.
            C = US
  Validity
    Not Before: 05/17/98 20:00:00 (05/18/98 00:00:00 GMT)
    Not After: 05/18/18 19:59:59 (05/18/18 23:59:59 GMT)
  Subject: OU = VeriSign Trust Network
            OU = (c) 1998 VeriSign, Inc. - For authorized use only
            OU = Class 1 Public Primary Certification Authority - G2
            O = VeriSign, Inc.
            C = US
    Subject Public Key Info: PKCS #1 RSA Encryption
    Subject's Public Key:
            30 81 89 02 81 81 00 aa d0 ba be 16 2d b8 83 d4
            ca d2 0f bc 76 31 ca 94 d8 1d 93 8c 56 02 bc d9
            6f 1a 6f 52 36 6e 75 56 0a 55 d3 df 43 87 21 11
            65 8a 7e 8f bd 21 de 6b 32 3f 1b 84 34 95 05 9d
            41 35 eb 92 eb 96 dd aa 59 3f 01 53 6d 99 4f ed
            e5 e2 2a 5a 90 c1 b9 c4 a6 15 cf c8 45 eb a6 5d
            8e 9c 3e f0 64 24 76 a5 cd ab 1a 6f b6 d8 7b 51
            61 6e a6 7f 87 c8 e2 b7 e5 34 dc 41 88 ea 09 40
            be 73 92 3d 6b e7 75 02 03 01 00 01
  Certificate Signature Algorithm: PKCS #1 SHA-1 With RSA Encryption
  Certificate Signature Value:
            8b f7 1a 10 ce 76 5c 07 ab 83 99 dc 17 80 6f 34
            39 5d 98 3e 6b 72 2c e1 c7 a2 7b 40 29 b9 78 88
            ba 4c c5 a3 6a 5e 9e 6e 7b e3 f2 02 41 0c 66 be
            ad fb ae a2 14 ce 92 f3 a2 34 8b b4 b2 b6 24 f2
            e5 d5 e0 c8 e5 62 6d 84 7b cb be bb 03 8b 7c 57
            ca f0 37 a9 90 af 8a ee 03 be 1d 28 9c d9 26 76
            a0 cd c4 9d 4e f0 ae 07 16 d5 be af 57 08 6a d0
            a0 42 42 42 1e f4 20 cc a5 78 82 95 26 38 8a 47
```

Figure 10.4: CA's Certificate; Details

Certificate Hierarchy

```
▷ Verisign Class 3 Public Primary CA
  ▷ www.verisign.com/CPS Incorp. by Ref. LIABILITY LTD.(c)97VeriSign
    ▷ online.wellsfargo.com
```

Certificate Fields

```
Verisign Class 3 Public Primary Certification Authority
  Certificate
    Version: Version 3
    Serial Number: 03:D7:98:CA:98:59:30:B1:B2:D3:BD:28:B8:E7:2B:8F
    Certificate Signature Algorithm: md5RSA
    Issuer: OU = www.verisign.com/CPS Incorp. ···
            OU = VeriSign International Server CA - Class 3
            OU = VeriSign, Inc.
            O = VeriSign Trust Network
            C = US
  Validity
    Not Before: Sunday, September 21, 2003 7:00:00 PM
    Not After: Wednesday, September 21, 2005 6:59:59 PM
  Subject: CN = online.wellsfargo.com
            OU = Terms of use at www.verisign.com.rpa (c)00
            OU = Class 1 Public Primary Certification Authority - G2
            OU = ISG
            O = Wells Fargo and Company
            L = San Francisco
            S = California
            C = US
    Subject Public Key Info: PKCS #1 RSA Encryption
    Subject's Public Key: 30 81 89 02 81 81 00 a9 ···
    Basic Constraints: Subject Type = End Entity,
                                     Path Length Constraint = None
    Subject's Key Usage: Digital Signature, Key Encipherment (AO)
    CRL Distribution Points: (1) CRL Distribution Point
                                 Distribution Point Name:
                                   Full Name:
                                     URL=http://crl.verisign.com/
                                         class3InternationalServer.crl
  Certificate Signature Algorithm: MD5 With RSA Encryption
  Certificate Signature Value: ······
```

Figure 10.5: A Client's Certificate

12, 13, 14, 15. Each pair of symbols is a byte, which is 8 bits. For example, *b6* represents *11, 6*, which is *10110110* in binary.

The last three bytes of the public key are `01 00 01`, which is $65537 = 2^{16} + 1$. This is a very common encryption exponent e for RSA, since raising something to this power by successive squaring (see Section 3.5) is fast. The preceding bytes `02 03` and the bytes `30 81 89 02 81 81 00` at the beginning of the key are control symbols. The remaining 128 bytes `aa d0 ba` $\cdots$ `6b e7 75` are the 1024-bit RSA modulus n.

7. *Signature:* The preceding information on the certificate is hashed using the hash algorithm specified – in this case, *SHA-1* – and then signed by raising to the CA's private RSA decryption exponent.

The certificate in Figure 10.5 has a few extra lines. One notable entry is under the heading *Certificate Hierarchy.* The certificate of the Wells Fargo has been signed by the Registration Authority (RA) on the preceding line. In turn, the RA's certificate has been signed by the root CA. Another entry worth noting is *CRL Distribution Points.* This is the Certificate Revocation List. It contains lists of certificates that have been revoked. There are two common methods of distributing the information from these lists to the users. Neither is perfect. One way is to send out announcements whenever a certificate is revoked. This has the disadvantage of sending a lot of irrelevant information to most users (most people don't need to know if the Point Barrow Sunbathing Club loses its certificate). The second method is to maintain a list (such as the one at the listed URL) that can be accessed whenever needed. The disadvantage here is the delay caused by checking each certificate. Also, such a web site could get overcrowded if many people try to access it at once. For example, if everyone tries to trade stocks during their lunch hour, and the computers check each certificate for revocation during each transaction, then a site could be overwhelmed.

When Alice (or, usually, her computer) wants to check the validity of the certificate in Figure 10.5, she sees from the Certificate Hierarchy that VeriSign's RA signed Wells Fargo's certificate and the RA's certificate was signed by the root CA. She verifies the signature on Wells Fargo's certificate by using the public key (that is, the RSA pair (n, e)) from the RA's certificate; namely, she raises the encrypted hash value to the eth power mod n. If this equals the hash of Wells Fargo's certificate, then she trusts Wells Fargo's certificate, as long as she trusts the RA's certificate. Similarly, she can check the RA's certificate using the public key on the root CA's certificate. Since she received the root CA's certificate from a reliable source (for example, it was packaged in her Internet browser, and the company doing this has a financial incentive to keep a good reputation), she trusts it. In this way, Alice has established the validity of Wells Fargo's certificate. Therefore, she can confidently do on-line transactions with Wells Fargo.

There are two levels of certificates. The **high assurance** certificates are issued by the CA under fairly strict controls. High assurance certificates are typically issued to commercial firms. The **low assurance** certificates are issued more freely and certify that the communications are from a particular source. Therefore, if Bob obtains such a certificate for his computer, the certificate verifies that it is Bob's computer, but does not tell whether it is Bob or Eve using the computer. The certificates on many personal computers contain the following line:

Subject: Verisign Class 1 CA Individual Subscriber - Persona Not Validated.

This indicates that the certificate is a low assurance certificate. It does not make any claim as to the identity of the user.

If your computer has Internet Explorer, click on *Tools*, then *Internet Options*, then *Content*. This will allow you to find the CA's whose certificates have been packaged with the browser. Usually, the validity of most of them has not been checked. But for the accepted ones, it is possible to look at the **Certification Path** that gives the path (often one step) from the user's computer's certificate back to the CA.

10.6 Pretty Good Privacy

Pretty Good privacy, more commonly known as *PGP*, was developed by Phil Zimmerman in the late 1980s and early 1990s. In contrast to X.509 certificates, PGP is a very decentralized system with no CA. Each user has a certificate, but the trust in this certificate is certified to various degrees by other users. This creates a **web of trust**.

For example, if Alice knows Bob and can verify directly that his certificate is valid, then she signs his certificate with her public key. Charles trusts Alice and has her public key, and therefore can check that Alice's signature on Bob's certificate is valid. Charles then trusts Bob's certificate. However, this does not mean that Charles trusts certificates that Bob signs – he trusts Bob's public key. Bob could be gullible and sign every certificate that he encounters. His signature would be valid, but that does not mean that the certificate is.

Each user, for example Alice, maintains a file with a **keyring**, containing the trust levels Alice has in various people's signatures. There are varying levels of trust that someone can assign: no information, no trust, partial trust, and complete trust. When a certificate's validity is being judged, the PGP program accepts certificates that are signed by someone Alice trusts, or a sufficient combination of partial trusts. Otherwise it alerts Alice and she needs to make a choice on whether to proceed.

The primary use of PGP is for authenticating and encrypting email. Suppose Alice receives an email asking for her bank account number so that Charles can transfer millions of dollars into her account. Alice wants to be sure that this email comes from Charles and not from Eve, who wants to use the account number to empty Alice's account. In the unlikely case that this email actually comes from her trusted friend Charles, Alice sends her account information, but she should encrypt it so that Eve cannot intercept it and empty Alice's account. Therefore, the first email needs authentication that proves that it comes from Charles, while the second needs encryption. There are also cases where both authentication and encryption are desirable. We'll show how PGP handles these situations.

To keep the discussion consistent, let's have Alice send a message to Bob in all cases.

Authentication:

1. Alice uses a hash function, usually *SHA-1*, and computes the hash of the message.

2. Alice signs the hash by raising it to her secret decryption exponent d mod n. The resulting hash code is put at the beginning of the message, which is sent to Bob.

3. Bob raises the hash code to Alice's public RSA exponent e. The result is compared to the hash of the rest of the message.

4. If the result agrees with the hash, and if Alice trusts Bob's public key, the message is accepted as coming from Bob.

This authentication is the RSA signature method from Section 9.1. Note the role that trust plays. If Alice does not trust Bob's public key as belonging to Bob, then she cannot be sure that the message did not come from Eve, with Eve's signature in place of Bob's.

Encryption:

1. Alice's computer generates a random number, usually 128 bits, to be used as the session key for a symmetric private key encryption algorithm such as *3DES*, *IDEA*, or *CAST-128* (these are block ciphers using 128-bit keys).

2. Alice uses the symmetric algorithm with this session key to encrypt her message.

3. Alice encrypts the session key using Bob's public key.

4. The encrypted key and the encrypted message are sent to Bob.

5. Bob uses his private RSA key to decrypt the session key. He then uses the session key to decrypt Alice's message.

The combination of a public key algorithm and a symmetric algorithm is used because encryption is generally faster with symmetric algorithms than with public key algorithms. Therefore, the public key algorithm RSA is used for the small encryption of the session key, and then the symmetric algorithm is used to encrypt the potentially much larger message.

Note that trust is not needed when only encryption is desired.

Authentication and Encryption:

1. Alice hashes her message and signs the hash to obtain the hash code, as in step (2) of the authentication procedure described above. This hash code is put at the beginning of the message.

2. Alice produces a random 128-bit session key and uses a symmetric algorithm with this session key to encrypt the hash code together with the message, as in the encryption procedure described above.

3. Alice uses Bob's public key to encrypt the session key.

4. The encrypted session key and the encryption of the hash code and message are sent to Bob.

5. Bob uses his private key to decrypt the session key.

6. Bob uses the session key to obtain the hash code and message.

7. Bob verifies the signature by using Alice's public key, as in the authentication procedure described above.

Of course, this procedure requires that Bob trusts Alice's public key certificate. Also, the reason the signature is done before the encryption is so that Bob can discard the session key after decrypting, and therefore store the plaintext message with its signature.

To set up a PGP certificate, Alice's computer uses random input obtained from keystrokes, timing, mouse movements, etc. to find primes p, q and then produce an RSA modulus $n = pq$ and encryption and decryption exponents e and d. The numbers n and e are then Alice's public key. Alice also chooses a secret passphrase. The secret key d is stored securely in her computer. When the computer needs to use her private key, the computer asks her for her passphrase to be sure that Alice is the correct person. This prevents Eve from using Alice's computer and pretending to be Alice.

The advantage of the passphrase is that Alice is not required to memorize or type in the decryption exponent d, which is probably more than one hundred digits long.

In the above, we have used RSA for signatures and for encryption of the session keys. Other possibilities are allowed. For example, Diffie-Hellman can be used to establish the session key, and DSA can be used to sign the message.

The software for PGP can be downloaded for free from many websites, including `http://www.mit.edu/network/pgp.html`. There is also a commercial version available through Network Associates.

10.7 SSL and TLS

If you have ever paid for anything over the Internet, your transactions were probably kept secret by SSL or its close relative TLS. Secure Sockets Layer (SSL) was developed by Netscape in order to perform http communications securely. The first version was released in 1994. Version 3 was released in 1995. Transport Layer Security (TLS) is a slight modification of SSL version 3 and was released by the Internet Engineering Task Force in 1999. These protocols are designed for commnications between computers with no previous knowledge of each other's capabilities.

In the following, we'll describe SSL version 3. TLS differs in a few minor details such as how the pseudo-random numbers are calculated.

Suppose Alice has bought something online from Gigafirm and wants to pay for her purchase. Alice's computer sends Gigafirm's computer a message containing the following:

1. The highest version of SSL that Alice's computer can support.

2. A random number consisting of a 4-byte timestamp and a 28-byte random number.

3. A Cipher Suite containing, in decreasing order of preference, the algorithms that Alice's computer wants to use for public key (for example, RSA, Diffie-Hellman, ...), block cipher encryption (3DES, DES, AES, ...), hashing (SHA-1, MD5, ...), and compression (PKZip, ...).

Gigafirm's computer responds with a random 32-byte number (chosen similarly) and its choices of which algorithms to use, for example, RSA, DES, SHA-1, PKZip.

Gigafirm's computer then sends its X.509 certificate (and the certificates in its certification chain). Gigafirm can ask for Alice's certificate, but this is rarely done for two reasons. First, it would impede the transaction, especially if Alice does not have a valid certificate. This would not help Gigafirm accomplish its goal of making sales. Secondly, Alice is going to send her credit card number later in the transaction, and this serves to verify that Alice (or the thief who picked her pocket) has Alice's card.

We'll assume from now on that RSA was chosen for the public key method. The protocol differs only slightly for other public key methods.

Alice now generates a 48-byte *pre-master secret*, encrypts it with Gigafirm's public key (from its certificate), and sends the result to Gigafirm, who decrypts it. Both Alice and Gigafirm now have the following secret random numbers:

1. The 32-byte random number r_A that Alice sent Gigafirm.
2. The 32-byte random number r_G that Gigafirm sent Alice.
3. The 48-byte pre-master secret s_{pm}.

Note that the two 32-byte numbers were not sent securely. The pre-master secret is secure, however.

Since they both have the same numbers, both Alice and Gigafirm can calculate the *master secret* as the concatenation of

$$\text{MD5}\big(s_{pm}||\text{SHA-1}(A||s_{pm}||r_A||r_G)\big)$$

$$\text{MD5}\big(s_{pm}||\text{SHA-1}(BB||s_{pm}||r_A||r_G)\big)$$

$$\text{MD5}\big(s_{pm}||\text{SHA-1}(CCC||s_{pm}||r_A||r_G)\big).$$

The A, BB, and CCC are strings added for padding. Note that timestamps are built into r_A and r_G. This prevents Eve form doing replay attacks, where she tries to use information intercepted from one session to perform similar transactions later.

Since MD5 produces a 128-bit (= 16-byte) output, the master secret has 48 bytes. The master secret is used to produce a *key block*, by the same process that the master secret was produced from the pre-master secret. Enough hashes are concatenated to produce a sufficiently long key block. The key block is then cut into six secret keys, three for communications from Alice to Gigafirm and three for communications from Gigafirm to Alice. For Alice to Gigafirm, one key serves as the secret key in the block cipher (3DES, AES, ...) chosen at the beginning of the communications. The second is a message authentication key. The third is the initial value for the CBC mode of the block cipher. The three other keys are for the corresponding purposes for Gigafirm to Alice.

Now Alice and Gigafirm are ready to communicate. When Alice sends a message to Gigafirm, she does the following:

1. Compresses the message using the agreed upon compression method.
2. Hashes the compressed message together with the message authentication key (the second key obtained from the key block). This yields the HMAC (= hashed message authentication code).
3. Uses the block cipher in CBC mode to encrypt the compressed message together with the HMAC, and sends the result to Gigafirm.

Gigafirm now does the following:

1. Uses the block cipher to decrypt the message received. Gigafirm now has the compressed message and the HMAC.
2. Uses the compressed message and the Alice-to-Gigafirm message authentication key to recompute the HMAC. If it agrees with the HMAC that was in the message, the message is authenticated.
3. Decompresses the compressed message to obtain Alice's message.

Communications from Gigafirm are encrypted and decrypted similarly, using the other three keys deduced from the key block. Therefore, Alice and Gigafirm can exchange information securely.

10.8 Secure Electronic Transaction

Every time someone places an order in an electronic transaction over the Internet, large quantities of information are transmitted. These data must be protected from unwanted eavesdroppers in order to ensure the customer's privacy and prevent credit fraud. Requirements for a good electronic commerce system include the following:

1. Authenticity: Participants in a transaction cannot be impersonated and signatures cannot be forged.

2. Integrity: Documents such as purchase orders and payment instructions cannot be altered.

3. Privacy: The details of a transaction should be kept as secure as possible.

4. Security: Sensitive account information such as credit card numbers must be protected.

All of these requirements should be satisfied, even over public communication channels such as the Internet.

In 1996, the credit card companies MasterCard and Visa called for the establishment of standards for electronic commerce. The result, whose development involved several companies, is called the SET, or Secure Electronic Transaction$^{\text{TM}}$ protocol. It starts with the existing credit card system and allows people to use it securely over open channels.

The SET protocol is fairly complex, involving, for example, the SSL protocol in order to certify that the cardholder and merchant are legitimate, and also specifying how payment requests are to be made. In the following we'll discuss one aspect of the whole protocol, namely the use of dual signatures.

There are several possible variations on the following. For example, in order to improve speed, a fast symmetric key system can be used in conjunction with the public key system. If there is a lot of information to be transmitted, a randomly chosen symmetric key plus the hash of the long message can be sent via the public key system, while the long message itself is sent via the faster symmetric system. However, we'll restrict our attention to the simplest case where only public key methods are used.

Suppose Alice wants to buy a book entitled *How to Use Other People's Credit Card Numbers to Defraud Banks*, which she has seen advertised on the Internet. For obvious reasons, she feels uneasy about sending the publisher her credit card information, and she certainly does not want the bank that issued her card to know what she is buying. A similar situation applies to many transactions. The bank does not need to know what the customer is ordering, and for security reasons the merchant should not know the card number. However, these two pieces of information need to be linked in some way. Otherwise the merchant could attach the payment information to another order. **Dual signatures** solve this problem.

The three participants in the following will be the Cardholder (namely, the purchaser), the Merchant, and the Bank (which authorizes the use of the credit card).

The Cardholder has two pieces of information:

- GSO = Goods and Services Order, which consists of the cardholder's and merchant's names, the quantities of each item ordered, the prices, etc.

- PI = Payment Instructions, including the merchant's name, the credit card number, the total price, etc.

The system uses a public hash function; let's call it H. Also, a public key cryptosystem such as RSA is used, and the Cardholder and the Bank have their own public and private keys. Let E_C, E_M, and E_B denote the (public) encryption functions for the Cardholder, the Merchant, and the Bank, and let D_C, D_M, and D_B be the (private) decryption functions.

The Cardholder performs the following procedures:

1. Calculates $GSOMD = H(E_M(GSO))$, which is the message digest, or hash, of an encryption of GSO.
2. Calculates $PIMD = H(E_B(PI))$, which is the message digest of an encryption of PI.
3. Concatenates $GSOMD$ and $PIMD$ to obtain $PIMD||GSOMD$, then computes the hash of the result to obtain the payment-order message digest $POMD = H(PIMD||GSOMD)$.
4. Signs $POMD$ by computing $DS = D_C(POMD)$. This is the Dual Signature.
5. Sends $E_M(GSO)$, DS, $PIMD$, and $E_B(PI)$ to the Merchant.

The Merchant then does the following:

1. Calculates $H(E_M(GSO))$ (which should equal $GSOMD$).
2. Calculates $H(PIMD||H(E_M(GSO)))$ and $E_C(DS)$. If they are equal, then the Merchant has verified the Cardholder's signature, and is therefore convinced that the order is from the Cardholder.
3. Computes $D_M(E_M(GSO))$ to obtain GSO.
4. Sends $GSOMD$, $E_B(PI)$, and DS to the Bank.

The Bank now performs the following:

1. Computes $H(E_B(PI))$ (which should equal $PIMD$).
2. Concatenates $H(E_B(PI))$ and $GSOMD$.
3. Computes $H\left(H(E_B(PI))||GSOMD\right)$ and $E_C(DS)$. If they are equal, the Bank has verified the Cardholder's signature.
4. Computes $D_B(E_B(PI))$, obtaining the payment instructions PI.
5. Returns an encrypted (with E_M) digitally signed authorization to the Merchant, guaranteeing payment.

The Merchant completes the procedure as follows:

1. Returns an encrypted (with E_C) digitally signed receipt to the Cardholder, indicating that the transaction has been completed.

The Merchant only sees the encrypted form $E_B(PI)$ of the payment instructions, and so does not see the credit card number. It would be infeasible for the Merchant or the Bank to modify any of the information regarding the order because the hash function is used to compute DS.

The Bank only sees the message digest of the Goods and Services Order, and so has no idea what is being ordered.

The requirements of integrity, privacy, and security are met by this procedure. In actual implementations, several more steps are required in order to protect authenticity. For example, it must be guaranteed that the public keys being used actually belong to the participants as claimed, not to impostors. Certificates from a trusted authority are used for this purpose.

10.9 Exercises

1. In a network of three users, A, B, and C, we would like to use the Blom scheme to establish session keys between pairs of users. Let $p = 31$ and let

$$r_A = 11 \quad r_B = 3 \quad r_C = 2.$$

Suppose Trent chooses the numbers

$$a = 8 \quad b = 3 \quad c = 1.$$

Calculate the session keys.

2. **(a)** Show that in the Blom scheme, $K_{AB} \equiv a + b(r_A + r_B) + cr_A r_B \pmod{p}$.
(b) Show that $K_{AB} = K_{BA}$.
(c) Another way to view the Blom scheme is by using a polynomial in two variables. Define the polynomial $f(x, y) = a + b(x + y) + cxy \pmod{p}$. Express the key K_{AB} in terms of f.

3. You (U) and I (I) are evil users on a network that uses the Blom scheme for key establishment with $k = 1$. We have decided to get together to figure out the other session keys on the network. In particular, suppose $p = 31$ and $r_U = 9, r_I = 2$. We have received $a_U = 18$, $b_U = 29$, $a_I = 24$, $b_I = 23$ from Trent, the trusted authority. Calculate a, b, and c.

4. Here is another version of the intruder-in-the-middle attack. It has the "advantage" that Eve does not have to intercept and retransmit all the messages between Bob and Alice. Suppose Eve discovers that $p = Mq + 1$, where q is prime and M is small. Eve intercepts α^x and α^y as before. She sends Bob $(\alpha^x)^q \pmod{p}$ and sends Alice $(\alpha^y)^q \pmod{p}$.

(a) Show that Alice and Bob each calculate the same key.
(b) Show that there are only M possible values for K, so Eve may find K by exhaustive search.

5. Bob, Ted, Carol, and Alice want to agree on a common key (cryptographic key, that is). They publicly choose a large prime p and a primitive root α. They privately choose random numbers b, t, c, a, respectively. Describe a protocol that allows them to compute $K \equiv \alpha^{btca} \pmod{p}$ securely (ignore intruder-in-the-middle attacks).

6. Suppose naive Nelson tries to implement an analog of the Diffie-Hellman key exchange as follows. Nelson wants to send the key K to Heidi. He chooses a one-time pad key K_N and XORs it with K. He sends $M_1 = K_N \oplus K$ to Heidi. She XORs what she receives with her one-time pad key K_H to get $M_2 = M_1 \oplus K_H$. Heidi sends M_2 to Nelson, who computes $M_3 = M_2 \oplus K_N$. Nelson sends M_3 to Heidi, who recovers K as $M_3 \oplus K_H$.
(a) Show that $K = M_3 \oplus K_H$.
(b) Suppose Eve intercepts M_1, M_2, M_3. How can she recover K?